Cloud-Chamber Studies of the Production of Light "Positive Particles" near Negative Beta-Ray Emitters*

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A series of experiments was performed with a cloud chamber to investigate the nature of the particles producing the positive tracks near negative beta-ray emitters reported in various investigations. Special emphasis was laid on the questions of whether the tracks are (1) due to electrons reentering the source and (2) due to electrons emerging from the source multiply scattered to show an apparent positive curvature. While the second possibility can account for only a negligible fraction of the tracks, it is found that, depending upon the source arrangement and the manner of investigating cloud-chamber photographs, a considerable fraction of observed "positive" tracks may be due to electrons which reenter or appear to reenter the source. With stereoscopic analysis and an appropriate source arrangement, the fraction of spurious positive tracks may be reduced to a few percent. With these precautions we find for P^{32} a ratio of positive particles to electrons of 3×10^{-4}. This ratio is increased by covering the source with carbon or lead. It is shown that the estimate of two electron masses previously obtained for the mass of these particles by comparing their multiple scattering with that of electrons was not influenced by the difference in the momentum distributions of the two species of particles used in the determination.

I. INTRODUCTION

DURING the past twenty years, a number of investigators[1–14] have observed tracks in magnetic cloud chambers which they have attributed to positive particles produced in the vicinity of electron emitters which, on account of the requirements of conservation of energy, cannot alternatively decay by the emission of positrons. The positive particles in question have further been observed in much greater numbers[4,6,15,16] than would be expected theoretically[17] if they were positrons resulting from internal or external pair formation by the beta-particles or by the gamma-rays emitted by some of the sources. Furthermore, the apparent absence of annihilation radiation[18] makes it appear extremely unlikely that positrons are responsible for these tracks.

There has, however, been considerable doubt expressed that the tracks in question are due to positively charged particles at all. First, several investigators[19] have suggested that these tracks were due not to positive particles but rather to electrons traveling toward the source or to electron tracks having reversed curvatures as the result of multiple scattering. Second, the earlier attempts to detect positive particles produced by such emitters with magnetic spectrometers were generally unsuccessful.[8,20–24]

A number of recent experiments, however, have provided considerable evidence that such particles exist. Groetzinger and Kahn[25] for instance, hypothesized that the positive particles were not found with magnetic spectrometers because of the fact that the required path length in conventional instruments of this sort was too long (in view of the possibility that these particles may be unstable). Consequently, they constructed a (nonevacuated) spectrometer of reduced dimensions (4.5-cm path length) using a photographic

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plate as a detector and succeeded for the first time in finding the positive particles with a magnetic spectrometer. More recently, Yuasa[26] performed a similar experiment with a small evacuated spectrometer with a G-M counter as a detector and also obtained positive results. In addition, other measurements have made it possible to assign a definite mass to these particles. Smith and Groetzinger[12] studied the tracks of positive particles which had passed through a foil in the cloud chamber and deduced from the energy loss a mass between one-and-one-half and two electron masses. Groetzinger and Ribe[27] arrived at a value of about two electron masses by studying the multiple scattering of the particles in the cloud chamber according to the method developed by Groetzinger, Berger, and Ribe.[28] Champion and Ahmed[29] and Yuasa[26] have recently classified a large number of cloud-chamber tracks produced in the vicinity of negative β-emitters, and come to the conclusion that certain of these are fairly definitely associated with positive particles. Weinzierl[30] has reviewed the work on this subject and concluded that positive particles of this sort (distinct from positrons) probably exist.

The present paper is a report of some experiments we have performed regarding the occurrence of tracks which could be attributed to such particles in the magnetic cloud chamber. In particular, we were interested in eliminating the possibility that these tracks might have been produced by electrons which were either traveling toward the source or else were scattered in such a fashion as to achieve the same curvature as a positive particle. In the course of this work we also studied the effect of covering the source on the rate of production of the positive particles. Finally, we undertook to determine whether our estimate of the mass of

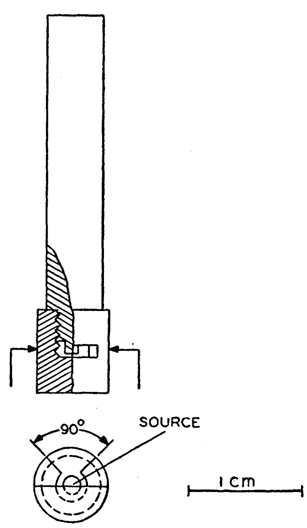

FIG. 1. Source arrangement No. 1.

[26] T. Yuasa, Compt. rend. **234**, 619 (1952).
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these particles, made on the basis of the determination of the multiple scattering,[27] might have been in error as the result of the difference in the momentum distribution of these particles and electrons used for the purpose of comparison.

II. CLOUD-CHAMBER EXPERIMENTS WITHOUT A COLLIMATING ARRANGEMENT

Our cloud chamber had a diameter of 24 cm and a height of 7.5 cm, of which about 3 cm was illuminated. Pictures were taken at a rate of one per minute showing a direct view inclined 3° to the axis of the chamber and a mirror view inclined 23° in the opposite direction. The magnetic field, which varied somewhat for different exposures, was produced by a set of Helmholtz coils and had a strength of from 325 to 355 gauss. The gas was a mixture of three parts helium and two parts argon at one atmosphere total pressure. The source was P^{32} (no nuclear gamma-radiation) obtained from the Isotope Branch of the Atomic Energy Commission as "separated radio isotope" in the chemical form $H_3P^*O_4$, which was evaporated in all our arrangements on an aluminum backing. In source arrangement No. 1, shown in Fig. 1, the radioactive phosphorus was deposited on the small cylindrical tip of the rod which was screwed into a cylindrical sleeve closed at the bottom. This served to suspend the source from the top glass plate at a height corresponding to the middle of the illuminated section and at a position 6 cm from the center of the chamber. The sleeve had a thickness of 1.9 mm and a horizontal slot 1.5 mm in height so that electrons emerging through the slot with a spread of approximately 120° in a horizontal plane, as well as those which had traversed the aluminum thickness of the sleeve in other directions, could be observed.

We will define here as a positive track, a track ending[31] in the source or the material covering the source or ending in the orifice of the collimating channel described in Sec. IV, which has a curvature in the magnetic field in the direction an (unscattered) positively charged particle moving away from the source would have. Such tracks were observed ending in the front (position of the slit) as well as the back of the source.

Only positive tracks of lengths in excess of 4 cm were considered. However, in the study of the effects of multiple scattering described in Sec. V, only tracks of length in excess of 7 cm were used. Similar restrictions concerning the length were imposed on electron tracks emerging from the source or its cover, whose number was determined in order to compare it with the number of positive tracks obtained under the same conditions. Furthermore, tracks with an ionization several times larger than the minimum ionization, which usually

[31] The meaning of the "end of a track" here and in the following does not imply any statement concerning the direction of motion of the particle producing the track. The method used to determine accurately the position of the end of a track is described below.

showed appreciable large angle single scattering, were eliminated from the considerations.

In order to classify a track as a "positive track" it is obviously essential to ascertain as accurately as possible that it really ends in the regions specified above. Tracks with a "positive curvature" satisfying the criteria of length and ionization density and ending in both views in the source we shall call candidates. In case it was doubtful whether a track lined up with the source in both views, it was not included among the candidates. This was the case when the end of a track near the source was covered by electron tracks or when the chamber was insensitive in the neighborhood of the source. The possible number of candidates lost by excluding these dubious cases amounted to about 50 percent. In the count of the electron tracks, on the other hand, no tracks were excluded due to the abovementioned circumstances. The total number of positive candidates ending in front and back of source arrangement No. 1 in 8800 pictures was 46. By counting the electron tracks emerging from the source in about five to ten out of every hundred pictures, a total number of electron tracks of 96,700 was derived. Since there is no doubt that the electrons emerge from the source, no further (stereoscopic) investigation of these tracks was made.

The criterion that a track ends in both views in the source (as is the case with the positive candidates) is, however, in the case of source arrangement No. 1 not yet sufficiently selective with respect to the height of the track near the aluminum sleeve of the source arrangement. Some of the tracks of particles traversing the chamber at random could satisfy this condition and still not line up vertically with the source. Therefore, all candidate tracks were investigated stereoscopically, using the camera with which the pictures were taken to project both views on a ground glass screen, which could be adjusted in order to bring the two images of the track near the source into coincidence. The position of the screen could be read from three attached scales. In order to determine the errors involved in this measurement of the track height, the tracks of 37 electrons emerging from the source were also examined stereoscopically. Figure 2 gives the results of this investigation, in which the measured height of the end of the electron tracks near the source with respect to the center of the slit in the source cover is indicated on the left side of the diagram. The spread in height is partly due to the extent in height of the radioactive P^{32}, partly due to the fact that the height is measured not at the position of the P^{32} but at the circumference of the aluminum sleeve surrounding it, and partly due to errors in the stereoscopic measuring. Figure 2 contains on the right side a similar plot for the 46 positive candidates. The peak, which is obtained near the center of the source has a shape similar to the electron peak and can only be explained as being due to emitted particles. Above the source the distribution is hardly

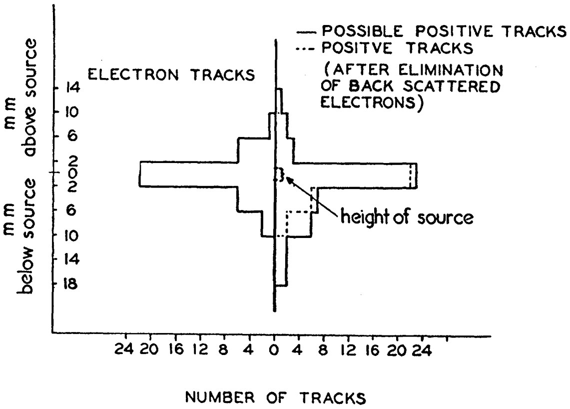

FIG. 2. Plot of the stereoscopically obtained height distribution of the electrons and positive particles obtained with source arrangement No. 1.

more spread out than that of the electrons. Below the source, however, the distribution extends beyond that of the electrons. This fact is so pronounced that it must be attributed to electrons re-entering the source rather than to a statistical fluctuation. The fact that back-scattered electrons are more numerous below the source than above it can be explained as follows: Positive candidates which are due to electrons scattered on the sidewalls of the chamber are eliminated by the fact that in most cases this process is directly observable. The spurious positive tracks are, therefore, more likely to be caused by electrons scattered on the top or bottom of the chamber which are outside the illuminated region and subtend furthermore a considerably larger solid angle with respect to the source. In our chamber the bottom consisted of velvet mounted on a thin (stationary) mesh above the piston, so that compared to those reflected by the glass top, particles scattered from the bottom are rather rare. The chance that an electron reflected at the top of the chamber has a track length exceeding four centimeters in the illuminated section will increase with the depth at which it hits or passes below the source. Of course, the number of backscattered electrons appearing in the distribution on the right side of Fig. 2 does not increase with the distance below the source due to the condition that the tracks have to end in the source in both views in order to be included among the candidates.

The dotted curve in Fig. 2 represents a distribution of the positive tracks corrected (somewhat arbitrarily) for the backscattered electrons, in such a way that it becomes similar to the distribution of the electrons emitted by the source. The stereoscopic investigation excludes in this way 11 candidates. It should be mentioned here that for all practical purposes elimination of tracks recorded as six millimeters above or below the center of the source leaves only about two reflected spurious positive particles out of a total of 33. The momentum spectrum of the positive particles shown in Fig. 3 is based on the cases contained in this interval.

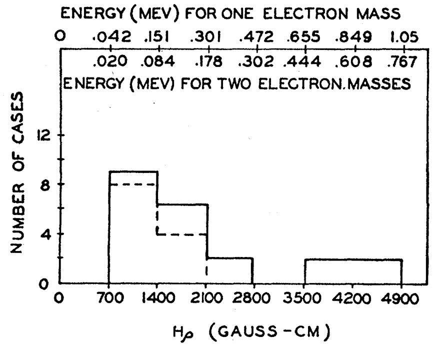

FIG. 3. Distribution of $H\rho$ for the positive tracks emerging from the uncovered portion (solid line) and from the covered portion (dashed line) of the P^{32} source in arrangement No. 1.

On the basis of the experiments described so far it can be seen that the error introduced by electrons re-entering the source arrangement No. 1 can be made approximately 6 percent. We should like to mention here that this arrangement is much more susceptible to the re-entering of electrons than the source arrangements No. 2 and No. 3 to be described below. According to the results described in Sec. V, it is extremely unlikely that the positive particles are due to electrons emerging from the source, deflected by multiple scattering in a direction corresponding to positively charged particles. We may talk, therefore, in the remainder of the paper, at least for the sake of expediency, about positive particles.

An investigation of the number of positive particles N^+ and the number of electrons N^- emerging from the slit in front (120° sector of the circumference) of the source arrangement and from the back (240° sector), which correspond to an uncovered source and one covered by 0.47 g/cm² of aluminum, shows a ratio of $N^+/N^-=2.9\times10^{-4}$ and 7.4×10^{-4}, respectively. It follows from a comparison of the two ratios that the aluminum cover increases the number of positive particles relative to the number of electrons. Since some of the positive particles and some of the electrons observed in front of the source might actually emerge from the aluminum sleeve rather than from the slit, it might well be that the ratio of the numbers of positive particles and electrons for a really bare source is even smaller.

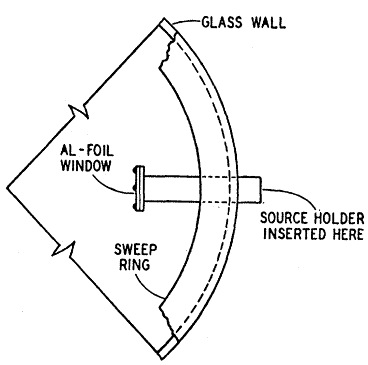

FIG. 4. Source arrangement No. 2.

III. CLOUD-CHAMBER EXPERIMENTS WITH THE SOURCE COVERED WITH CARBON OR LEAD

In order to study the influence of various materials covering the source on the number of the positive particles emerging from them, a different arrangement (source arrangement No. 2) was used. A brass tube of 1.3-cm outer diameter was extended along a radius 4.0 cm into the cloud chamber through a hole in its side wall. This tube was closed at the end inside the chamber by a 5-mil aluminum foil of 0.7-cm diameter. The source and the various covers were put into the tube through its outer (open) end in such a way that the source and the surface of the cover next to the source were in all cases in the same position. The source covers were constructed so that leakage of electrons around the cover was prevented. Pictures were taken without a cover (except for the window of thin aluminum foil) and with carbon and lead covers of various thicknesses. Arrangement No. 2 is shown in Fig. 4. The number of positive particles occurring with the various covers was again determined by the procedure outlined before. Table I contains the results of this investigation. The first two columns list the material covering the radioactive P^{32} and its thickness in g/cm²; the third and fourth, the number of the observed positive particles N^+ and electrons N^-; the fifth, the ratio N^- to the number of electrons N_0^- which would have been emitted into the same solid angle without the source cover; the sixth, the ratio N^+/N^-; and the seventh column, the ratio N^+/N_0^-, which represents a measure of the number of positive particles per disintegration. Owing to the fact that only one positive particle was obtained with arrangement No. 2 without a cover, the "bare source" results obtained with arrangement No. 1 are included in the table. As mentioned before, the number of positive particles per electron might be somewhat too high in this case, owing to the effects of the edges of the slit. Since with the large cover thicknesses fairly strong sources had to be used, it was not possible to take cloud-chamber pictures with these sources uncovered, which is necessary to obtain N_0^-. Therefore,

TABLE I. Influence of a source cover on the relative number of the positive particles, N^+; the number of electrons, N^-; and the number of electrons which would be emitted with the same solid angle with no cover, N_0^-.

Source cover						
Material	Thickness	N^+	N^-	N^-/N_0^- ×100	N^+/N^- ×10⁴	N^+/N_0^- ×10⁴
No cover		1	7300	100	1.2	1.2
No cover		22[a]	81,000	100	2.9	2.9
C	0.27 g/cm²	10	4400	4.8	23	1.1
C	0.46	8	9800	0.65	8.2	0.05
Pb	0.066	8	8300	36	9.6	3.4
Pb	0.34	5	12,000	0.42	4.3	0.02

[a] Obtained with arrangement No. 1.

in order to take pictures without a cover, the source was weakened by removing part of the deposit by an amount which was determined by the use of a G-M counter. In computing the various ratios the decay of the source was taken into account. The statistical accuracy of all the ratios depends almost entirely on the number N^+, since the numbers N^- and N_0^- are always based on the counting of a much higher number of tracks.

From these results we feel justified only in drawing the following qualitative conclusions: The effect of material covering the source is to increase the ratio of N^+/N^-, while N^+/N_0^-, after a possible initial increase, decreases. It follows from a consideration of the energy loss of the electrons that the positive tracks observed for the larger absorber thicknesses in the table cannot be due to electrons with energies in excess of 1 Mev.

IV. CLOUD-CHAMBER EXPERIMENTS WITH A COLLIMATING ARRANGEMENT

Figure 5 shows a part of the cloud chamber with a 3.3-cm long collimating tube which had a radius of curvature of 3 cm and thus discriminated to some extent against particles of either positive or negative charge traversing it, depending upon the direction of the magnetic field of approximately 350 gauss in the chamber and in the collimator. The source was placed on an aluminum backing at the end of the tube which was made of lead tubing of 0.15-cm wall thickness lined with 0.15 cm of aluminum so that a channel with a circular cross section of 0.4-cm diameter resulted. No window was used, so that the same mixture of helium and argon was present in the collimator as in the chamber. The discriminating power of the arrangement (which we shall call source arrangement No. 3) is rather poor. The distribution of the number of electrons as a function of $H\rho$ is shown in Fig. 6 for the case that the magnetic field supports their passing through the channel (curve A) and for the case that the field supports the passage of positively charged particles (curve B). These distributions are based on 62 and 63 particles, respectively. A count of several hundred tracks for each field direction shows that the number of electrons changes by a factor of 2.2 if the field is reversed. Curves A and B are drawn so as to take this factor into account. Figure 6 contains furthermore the P^{32} spectrum as obtained in a recent investigation by Agnew[32] with a beta-ray spectrometer (curve C). The spectrum is drawn to such a scale that the number of electrons per momentum interval at approximately 1100 gauss-cm coincides with that of the electrons emerging from the channel with the magnetic field supporting their passage, this momentum being the one the channel favors. It must be kept in mind that spectra obtained in a cloud chamber and in a spectrometer will differ slightly, owing to the effect of the backing of the source,

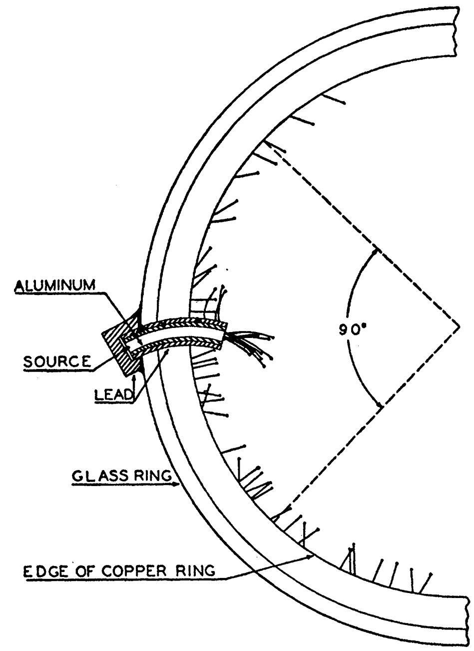

FIG. 5. Source arrangement No. 3 with the ends of tracks either due to positively charged particles emerging from or electrons entering into the orifice or the side wall of the collimator and the 3 cm high illuminated section of the side wall of the cloud chamber.

and that in the cloud chamber even without the collimator the obtained spectra will be somewhat influenced by such geometrical conditions as the required minimum track length, etc.

Figure 5 shows short portions of tracks at points where they end either in the orifice or in the wall of the collimator; these tracks—if we disregard the possibility of multiple scattering producing a curvature in a direction opposed to that which would occur under the influence of the magnetic field alone—being either due to positive particles emerging or electrons entering at these points. Similar tracks ending in the illuminated

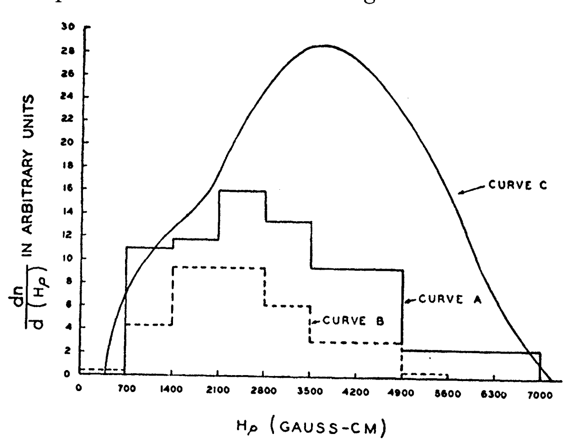

FIG. 6. Momentum distribution of the electrons emerging from source arrangement No. 3 with the magnetic field supporting (curve A) and opposing (curve B) the passage of negatively charged particles. Curve C represents the P^{32} electron spectrum.

[32] H. M. Agnew, Phys. Rev. 77, 655 (1950).

part of the sidewall of the chamber are only photographed to points 1.2 cm from the wall, the outer rim of the chamber being obscured by a brass ring on top. The figure contains the position and direction of these tracks where they become visible. None of all these tracks are due to electrons coming directly from the orifice of the channel without being reflected. Nine tracks end in the orifice, three in the outside wall of the collimator, and 26 in the illuminated part of the quarter of the sidewall next to the collimator. Six of the tracks ending in the orifice can be traced back to the source at the end of the channel while the other three, if they are due to positively charged particles, must be either scattered or be produced inside the collimator. The tracks of electrons emerging from the collimator, even in the case that the field has a direction which favors their passage, show a larger fraction of particles not traceable to the source and therefore scattered in the channel.

The area of the orifice is 0.12 cm^2 and the illuminated part of the quarter of the side wall next to the collimator is 48 cm^2 so that per unit area more than 100 times as many tracks of either electrons entering or positive particles emerging, end in the orifice than in the illuminated part of the glass wall next to the collimator. The possibility that an appreciable number of the tracks ending in the orifice are due to reflected electrons is therefore excluded; and, moreover, the direction of these tracks at the orifice is indicative of emerging positively charged particles.

Assuming that both objections against the cloud-chamber evidence of the existence of these positive particles are disproved,[33] one can obtain a ratio of the number of positive particles emerging from the collimator to the number of electrons emerging from it (with the field supporting the passage of positively charged particles). This ratio is 5×10^{-4}. By the use of the distribution functions (curves A and C) in Fig. 6 the ratio of the positive particles emerging from the collimator to the electrons of a spectral distribution as it occurs in the P^{32} spectrum emitted into the same solid angle as the positives can be estimated. This ratio is approximately 1×10^{-4}. In spite of the experimental uncertainties (e.g., the ratios are based on only nine positive particles), it is unlikely that the number of positive particles emerging from the channel can be accounted for by positrons due to the bremsstrahlung produced by the P^{32} electrons or due to pair production by the P^{32} electrons for which, experimentally, an upper limit of 1.3×10^{-5} per disintegration electron from P^{32} has been found.[8,23] The difference between the ratio determined here and that found with the bare source might be partly due to the fact that the additional length (of the channel) which the positives have to traverse, decreases their number. This would be consistent with the failure to detect these particles with beta-ray spectrometers, where they have to traverse longer distances.

In this connection it should be mentioned that out of the 34 positive tracks observed with source arrangement No. 1, three seemed definitely to end abruptly in the illuminated section of the cloud chamber. In none of these cases was a higher ionization density observed at the end of the track.

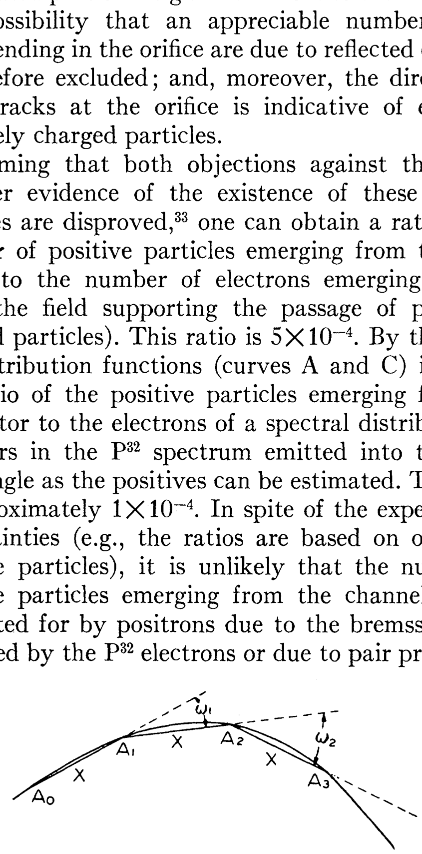

FIG. 7. Method of analysis of a cloud-chamber track.

[33] The objection against multiple scattering being responsible for the positive tracks is treated more fully in Sec. V.

V. ERRORS DUE TO MULTIPLE SCATTERING OF THE PARTICLES PRODUCING THE POSITIVE TRACKS

(a) General Considerations

We will be concerned with two sorts of errors introduced into the cloud-chamber study of the tracks attributed to positive particles as a result of multiple scattering. In the first place, we want to estimate the probability that these tracks are due to electrons which have been scattered in such a fashion as to have a curvature appropriate to positive particles. In the second place, we want to see whether the experimental scattering law and hence the mass found by Groetzinger and Ribe[27] for the particles associated with these tracks might be in error because of difficulties in determining the momenta of the particles arising from multiple scattering itself.

Our approach to this problem will be based on the analysis of multiple scattering in the magnetic cloud chamber carried out by Groetzinger, Berger, and Ribe[28] (in a publication referred to as GBR in the following). The tracks as projected onto a plane perpendicular to the magnetic field are divided into sections in the manner indicated in Fig. 7. A_0, A_1, $\cdots$, A_n, and A_{n+1} are dividing points separated by chords of an equal length x, here taken as one centimeter, which are very close to the length of a section of the track. The n angles between the $(n+1)$ successive chords are designated as ω_1, ω_2, $\cdots\omega_n$, while the deflections of the track between successive points are designated as ϕ_0, ϕ_1, $\cdots$, ϕ_n. It follows from the theory of multiple scattering that the angles ω_i and the angles ϕ_i obey a normal distribution about a magnetic mean deflection $\mu=x/\rho$ with a variance of ψ^2 and σ^2, respectively, where ρ is connected with the momentum p of the particle and the magnetic field H in the chamber by the relation

$$eH\rho=pc. \quad (1)$$

It can be shown that

$$\psi^2=(2/3)\sigma^2. \quad (2)$$

In case μ and ψ^2 are unknown, the best estimates for

TABLE II. Probabilities P and $\bar{P}$ (as defined in the text) which are a measure of the likelihood that positive tracks are due to electrons emerging from the source deflected in a positive sense by multiple scattering.

	Electrons					Positive track with $\bar{\omega}=10.4°$ (1967 gauss-cm)		Positive track with $\bar{\omega}=6.0°$ (4280 gauss-cm)	
k	E kev	μ_k	ψ_{exp} in degrees	ψ_{th}	α_k in %	P_{exp}	P_{th}	P_{exp}	P_{th}
					1.2[a]				
1	40	−29.8	18.0	21.8	2.3	3.3×10^{-7}	2.0×10^{-5}	6×10^{-6}	1.2×10^{-4}
2	50	−26.4	15.3	18.2	6.8	4.6×10^{-8}	3.1×10^{-6}	1.3×10^{-6}	3.5×10^{-5}
3	200	−12.5	5.2	6.19	65.0	$<10^{-23}$	1.1×10^{-11}	1.5×10^{-15}	8.0×10^{-11}
4	1000	− 4.32	2.68	...	24.7	$<10^{-23}$	...	3.6×10^{-18}	...
5	1730	− 2.81	...	...		...	...	...	...
						$\bar{P}_{exp}$	$\bar{P}_{th}$	$\bar{P}_{exp}$	$\bar{P}_{th}$
	40 to 1730				98.8	$<1.1\times10^{-9}$	$<6.7\times10^{-7}$	$<2.3\times10^{-7}$	$<5.1\times10^{-6}$

[a] Fraction of electrons with energies below 40 kev.

these quantities obtainable from the measurement of the ω_i's of a single track are, respectively,

$$\bar{\omega}=\frac{1}{n}\sum_{i=1}^{n}\omega_i \qquad (3a)$$

and

$$(\omega^2)_{Av}=[1/(n-1)]\sum_{i=1}^{n}(\omega_i-\bar{\omega})^2. \qquad (3b)$$

(b) Occurrence of "Positive Tracks" as the Result of Multiple Scattering of Electrons

The probability that a track with a mean deflection equal to or greater than a given value of $\bar{\omega}$ is due to a particle of a particular type which has a momentum corresponding to an average deflection μ can be estimated as follows. If tracks containing individual deflections through large angles are excluded, as was done here, then the distribution of the individual ω_i's for particles of a given type and momentum follows a Gaussian distribution with variance ψ^2. Consequently, the quantity $n\bar{\omega}$ for different tracks will have a normal distribution about the mean $n\mu$ with a variance $n\psi^2$. The quantity ψ^2 as a function of the momentum can be obtained from theoretical considerations or preferably from an experimental determination of the scattering of a large number of particles of the same kind with the conditions under which the track was obtained and using the same method of analysis. Thus, since μ, $\bar{\omega}$, and ψ^2 are known the probability in question can be calculated.

We are interested in determining the probability that the tracks attributed to positive particles actually be due to electrons. To carry out this computation for a given track, it is first necessary to determine the probability $P(\mu, \bar{\omega})$ that a track of an electron of a momentum corresponding to an average deflection μ have a deflection equal to or larger than $\bar{\omega}$, using as explained above either an experimental or theoretical value of ψ^2. In determining P, it must be remembered that $\bar{\omega}$ and μ have different signs for the "positive" tracks considered here. We make the convention that the $\bar{\omega}$'s are positive so that the μ's are negative. (This means that μ increases with increasing energy.) Then if the proportion of electrons emitted by P^{32} with momenta corresponding to values of μ between μ and $\mu+d\mu$ is $f(\mu)d\mu$, the total probability $\bar{P}(\bar{\omega})$ of the occurrence of such a track is

$$\bar{P}(\bar{\omega})=\int_{\mu_{min}}^{\mu_{max}} f(\mu)P(\mu, \bar{\omega})d\mu. \qquad (4)$$

μ_{max} is the value of μ associated with an electron of maximum energy (1700 kev for P^{32}), and μ_{min} is the value of μ associated with the least energy (here about 40 kev) which an electron can have without producing a track of such high ionization as to be excluded from consideration. In our calculations the integral in Eq. (4) was approximated by the sum

$$\bar{P}(\bar{\omega})=\sum_{k=1}^{m}\alpha_k P(\mu_k, \bar{\omega}), \qquad (5)$$

where α_k is the fraction of electrons with values of μ between μ_k and μ_{k+1}. The limits are chosen so that μ_1 and μ_{m+1} are equal to μ_{min} and μ_{max}, respectively. This way of choosing the intervals assures that the approximate value of $\bar{P}$ obtained is at least as high as the exact value since $P(\mu, \bar{\omega})$ decreases with increasing μ. The values μ we employed correspond to energies of 40, 50, 200, 1000, and 1700 kev.

The value of $\bar{P}$ was calculated for a pair of representative "positive" tracks each containing six sections ($n=5$). Table II gives the values of the quantities entering Eq. (5) for these tracks. The values of α_k shown are the approximate fractions of electrons in the beta-ray spectrum of P^{32} in energy intervals bounded by the energies $E=0$, 40, 50, 200, 1000 kev and the upper limit near 1700 kev, which were obtained from the investigation by Agnew[32] (curve C of Fig. 6). In order to calculate $P(\mu_k, \bar{\omega})$ it is necessary to know the root-mean-square angle of scattering ψ for a one-centimeter chord in the gas used. As remarked before, this quantity may be obtained either by a theoretical estimate or by a direct experimental determination of the scattering of electrons. The quantity ψ_{exp} given in Table II was deduced from an experimental scattering law obtained for electron tracks with six sections in the specified mixture of argon and helium by the method outlined in GBR. The value given for 1000 kev is a direct experimental result, while those for 40, 50, and 100 kev are difficult to determine directly (see GBR) and were, therefore, extrapolated from the value of ψ found for 400 kev by assuming an energy dependence for ψ^2 of the form:

$$\psi^2 \propto \frac{1}{p^2v^2}\log\frac{0.1}{(mc/p)(Z^{\frac{1}{3}}/181)}. \qquad (6)$$

Expression (6) is based on the theory of Williams[34] with limiting angles (contained in the logarithmic part) due to Bethe,[35] which showed, according to GBR, good agreement with the results of their experimental determination of the multiple scattering law for electrons of momenta between 2000 and 7000 gauss-cm in one atmosphere of argon. The quantity $Z^{\frac{1}{3}}$ occurring in expression (6) was obtained by averaging $Z^{\frac{1}{3}}$ for the proportions of helium and argon used in this investigation. The corresponding value of $P(\mu_k, \bar{\omega})$ for a given value of μ_k is given in Table II as P_{exp} and of $\bar{P}(\bar{\omega})$ as $\bar{P}_{\text{exp}}$. The theoretical values ψ_{th} of ψ given in Table II were obtained by using Eq. (6) with the factor of proportionality as given by Williams. The corresponding values of $P(\mu_k, \bar{\omega})$ and $\bar{P}(\bar{\omega})$ are given in the table as P_{th} and $\bar{P}_{\text{th}}$.

The values of $\bar{P}_{\text{th}}$ found in this fashion are much greater than those found for $\bar{P}_{\text{exp}}$, but each of these is much less than 10^{-4} which is roughly the actual proportion of positive tracks among those with a length in excess of 6 cm. Furthermore, basing one's arguments only on tracks of a larger number of sections, which leaves the ratio between the positive tracks and the electron tracks practically unchanged, reduces all the probabilities by a large factor.

The argument against the possibility that the positive tracks are due to electrons scattered in the "wrong" direction can be further strengthened by considering the distribution of the $\bar{\omega}$'s of the positive tracks observed in the course of an investigation. Whatever the energy distribution of the electrons assumed to be responsible for the positive tracks may be, the number of these tracks should be essentially a rapidly increasing function of the mean radius of curvature $\rho = x/\bar{\omega}$ or the apparent momentum. This is obviously not the case; the momentum spectra reported in various investigations contain only a very small fraction of particles of "positive" momenta exceeding 3800 gauss-cm and none in excess of 6400 gauss-cm.

[34] E. J. Williams, Proc. Roy. Soc. (London) **169**, 531 (1939); Phys. Rev. **58**, 292 (1940).
[35] H. A. Bethe, Phys. Rev. **70**, 821 (1946).

(c) Effect of the Difference in the Momentum Distribution of Electrons and the Positive Particles on the Determination of their Mass Ratio by Investigating the Multiple Scattering

In order to compare the mass of the positive particles with that of electrons, Groetzinger and Ribe[27] undertook to obtain ψ as a function of μ for both species of particles by measuring the tracks of such particles in a magnetic cloud chamber. Each pair of values (ψ, μ) representing a point of the curve was obtained by equating ψ to $[(\omega^2)_{\text{Av}}]^{\frac{1}{2}}$ and μ to $\bar{\omega}$ for the track in question. As remarked above, $\bar{\omega}$ is certainly the best estimate for μ for a given track in the case of a set of tracks all due to particles of the same momentum. This is also true in case the particles are distributed over a whole range of μ's provided that the number of particles per unit range of μ, $dN/d\mu$ is a constant independent of μ. Here the chance that the track be due to a particle with $\mu < \bar{\omega}$ is equal to the chance that it be due to a particle with $\mu > \bar{\omega}$ so that $\bar{\omega}$ remains the best estimate for μ. If, however, $dN/d\mu$ increases (decreases) with μ in the neighborhood of the obtained $\bar{\omega}$, it is more likely that $\bar{\omega}$ is due to a particle with a $\mu > \bar{\omega}(\mu < \bar{\omega})$. In the case that one is not interested in the best estimate of the μ of an individual particle but rather in establishing a scattering law for a group of particles, this fact is of no importance as long as the scattering does not depend strongly on μ or $H\rho$, as is the case for the higher momenta considered by us. For a comparison of the scattering of two groups of particles even at momenta where the scattering changes rapidly with μ, it is sufficient that the momentum spectra of the two groups are rather similar.

In order to make an estimate of this error in the determination of the momentum $H\rho$, we consider the case of a disintegration electron of P^{32} of an energy of 200 kev ($H\rho \sim 1650$ gauss-cm) in a magnetic field of 350 gauss in our mixture of helium and argon whose track has a length to give 8 individual deflections between chords of one centimeter length. (This is somewhat less than the average number of angles of the tracks whose scattering has been studied.) With the total mean deflection between the first and the last chord being 8μ and a variance of $8\psi^2$, the probable error r in μ is

$$r = 0.674\psi/(8)^{\frac{1}{2}}. \qquad (7)$$

Taking, for a conservative estimate of the error, for ψ the ψth for the energy of 200 kev from Table II which is somewhat too high, the limits of the momentum interval bounded by the probable error in μ are 1473 and 1880 gauss-cm. With a and b being the value of the distribution function $dN/d\mu$ at these points, a rough measure of the (systematic) error ϵ in the estimate of μ will be

$$\epsilon = r(b-a)/(a+b). \qquad (8)$$

Taking the values a and b from curve C in Fig. 6, considering that here the distribution $dN/d\mu$ is required rather than $dN/d(H\rho)$ which is shown in the figure, gives the result that in the case of P^{32} electrons of a length to give 8 angles between successive chords of one centimeter for the conditions prevailing in our chamber an $\bar{\omega}$ corresponding to an apparent momentum of 1587 gauss-cm will most likely be due to an electron of a μ corresponding to a momentum of 1650 gauss-cm. It follows from the curve in Fig. 1, reference 27, that this error of approximately 60 gauss-cm in $H\rho$ near the lower end of the curve will not essentially change the experimental scattering law for electrons above 1500 gauss-cm which is represented by this curve.

In the case of the positive particles, $dN/d\mu$ varies only slightly at momenta around 2000 gauss-cm so that the best estimate of μ is hardly affected by this error, while at higher momenta this error has little effect since the scattering is insensitive to a change in momentum.

In order to approach this source of error experimentally, 26 tracks of electrons of momenta between 1500 and 3700 gauss-cm emerging from the collimating arrangement shown in Fig. 5, with the field in a direction favoring particles of positive charge, have been analyzed As can be seen from a comparison of curves B and C. of Fig. 6, these particles have a different spectral distribution than the ones obtained without the collimator. Their distribution is, furthermore, more similar to that of the positives, since above an $H\rho=1600$ gauss-cm, the number of electrons per momentum interval is decreasing with increasing momentum while without the collimator this number is increasing up to momenta of approximately 3600 gauss-cm. The scattering of the electrons emerging from the channel showed very good agreement with that of the electrons obtained without the collimator, which proves again that the error introduced by the spectral distribution is not appreciable in our case.

VII. DISCUSSION AND SUMMARY OF THE RESULTS

The results described above indicate that positively charged particles originate from radioactive P^{32}, a substance which is energetically incapable of decaying by positron emission. Cloud-chamber evidence concerning these positive particles has in the past been criticized on the grounds that the tracks attributed to positively charged particles emerging from the source are in reality due to electrons entering the source, or due to electrons emerging from the source but with a positive curvature caused by strong multiple scattering. With special source arrangements and a stereoscopic method of analysis of the cloud-chamber tracks we have been able to reduce the number of spurious positive tracks due to electrons reentering the source to a few percent of the total observed positive tracks. The use of the collimator arrangement described in Sec. IV reduces this error even to approximately one percent. The fact that the reentering electrons can account for only a small fraction of the resulting observed tracks follows also from the amount of multiple scattering that these tracks show, which is different from that of electrons of the same momentum (mean curvature). Detailed analysis of multiple scattering showed that the appearance of scattering-produced positive curvatures has a negligible probability.

The ratio of positive tracks to electron tracks reported in this investigation is in general smaller than that reported in previous investigations for P^{32}. While we found this ratio for a bare source to be approximately 3×10^{-4}, Barlow and Rogers[14] report a value of 1.5 $\times10^{-3}$. Pi and Chao[13], on the other hand, find for a bare source a ratio which is of the order of magnitude of the experimental error caused by reflected electrons but with a cover of a light substance (aluminum or glass) of 0.1 g/cm^2 a ratio of 5×10^{-4} (after correcting for this error). In the investigation of Smith and Groetzinger[12] using a collimator which favored the appearance of positively charged particles in the chamber, a ratio of somewhat less than 1×10^{-2} was found which was, however, based on positive tracks with a length in excess of 4 cm and electron tracks in excess of 10 cm. The two older papers concerned with P^{32} sources[10,11] report ratios of the order of 1×10^{-2}, the authors being, however, at the time not aware of the error which could be introduced by reentering electrons.

By covering the P^{32} source with lead or carbon the ratio of the numbers of observed positive particles and electrons was increased. The number of positive particles, however, decreased with increasing absorber thickness (except for a possible increase for small absorber thicknesses). Although our experimental material concerning this point is rather limited, it suggests—together with the evidence concerning the ionization-loss, which was certainly not smaller than for electrons of comparable momentum—that a primary radiation with a range in matter greater than that of the positive particles may be responsible for their production. It also follows that P^{32} electrons with energies (at the point of production of the positive particles) in excess of one Mev are excluded as possible primaries.

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