The low temperature curvature is evident in much of the experimental data, but as has been stated previously, the θ/T_c ratios are considerably different from unity. Li has pointed out that if the susceptibility measurements are not taken at sufficiently high temperatures, an apparent θ/T_c will be obtained whose magnitude may be as high as 1.5–1.7. In this way, the data on some antiferromagnetic compounds, such as MnF_2 and FeF_2, might be fitted, but for most the θ/T_c values fall considerably outside the range allowed by Li's theory. Presumably a Bethe-Weiss treatment including both types of interactions and appropriate magnetic lattices should correct some of the deficiencies of both Li's theory and the present theory (at high temperatures) but the amount of calculation involved seems formidable.

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A Note on the Absorber Theory of Radiation

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The Wheeler-Feynman theory, starting with fields symmetric in time, obtained the Lorentz-Dirac equations of motion, which use retarded fields only, together with a condition on the total field of all particles in the universe. It is shown that if the field acting on a particle produced by all other particles is static but not zero, no motion of the system satisfying this condition exists. Some implications of this result for the physical interpretation of the Wheeler-Feynman theory are discussed.

RECENT attempts to obtain the force of radiative reaction on a moving charge in classical electrodynamics have proceeded from two different viewpoints. One is that of field theory, which considers the *total* field at all points in space to be the fundamental physical quantity. The other is that of action at a distance, which considers only the forces exerted on a charge by *other* charges to be physically meaningful.

The field theoretical point of view was first applied successfully to the problem of the motion of a point charge by Dirac,[1] who succeeded in obtaining the equations of motion first found by Lorentz on the basis of a model of an extended charge. These equations can be be written

$$m_a v_a'^{\mu} = e_a \sum_{k\neq a} {}_{\mathrm{ret}}F_{\nu}{}^{\mu(k)} v_a{}^{\nu} + \tfrac{2}{3} e_a (v_a''^{\mu} + v_a'^2 v_a{}^{\mu}), \qquad (1)$$

where ${}_{\mathrm{ret}}F^{(k)}$ is the retarded field of the kth particle evaluated at the position of the ath one, and where we have assumed that there are no fields present except those due to charges.

The first derivation of the Lorentz-Dirac equations on the basis of action at a distance is due to the absorber theory of radiation of Wheeler and Feynman.[2] This was achieved taking the forces on the charges as determined by half the sum of the retarded and the advanced field, while the previous field theoretical derivations all had been based on the use of retarded fields alone.

In a previous paper[3] it was pointed out that the need for the exclusive use of retarded fields for the explanation of the radiative reaction arose only in the field theoretical derivations for the one-particle problem, but that the considerations of Wheeler and Feynman on the total field due to all particles are applicable to field theory as well as to action at a distance. It was concluded that one can obtain the Lorentz-Dirac equations in both theories starting with fields symmetric in time, in spite of the fundamentally different underlying physical ideas.

It was noted, however, that there was not complete equivalence of the retarded and symmetric cases. The considerations of Wheeler and Feynman led to the symmetry condition

$$\sum_{\text{all } k} {}_{\mathrm{ret}}F^{(k)} = \sum_{\text{all } k} {}_{\mathrm{adv}}F^{(k)} \text{ everywhere.} \qquad (2)$$

Therefore the solutions of (1) are subject to this condition in the symmetric case both in field theory and in action at a distance, while there is no such restriction imposed upon them in the retarded case.

It appears that present mathematical methods are not powerful enough to enable us to discuss the effect of this restriction in the general case.[4] We shall only discuss the case that the effect on a single charge of all other charges in the universe is that of a static field, i.e., that

$$\sum_{k\neq a} {}_{\mathrm{ret}}F^{(k)} = \text{const} = \sum_{k\neq a} {}_{\mathrm{adv}}F^{(k)} \text{ everywhere.} \qquad (3)$$

From this and condition (2) we must have

$${}_{\mathrm{ret}}F^{(a)} = {}_{\mathrm{adv}}F^{(a)} \text{ everywhere.} \qquad (4)$$

[1] P. A. M. Dirac, Proc. Roy. Soc. (London) **A167**, 148 (1938).
[2] J. A. Wheeler and R. P. Feynman, Revs. Modern Phys. **17**, 157 (1945).
[3] P. Havas, Phys. Rev. **74**, 456 (1948).
[4] For a discussion of some of the difficulties of the two-body problem, see P. Havas, Acta Phys. Austriaca **3**, 342 (1949).

A necessary, but not sufficient, condition for the validity of (4) is that

$$ {}_{\rm ret}F^{(a)} - {}_{\rm adv}F^{(a)} = 0 \text{ on the world line of particle } a. \quad (5)$$

But according to Dirac[1] we have on the world line

$$\tfrac{1}{2}e_a({}_{\rm ret}F_\nu{}^{\mu(a)} - {}_{\rm adv}F_\nu{}^{\mu(a)})v_a{}^\nu = \tfrac{2}{3}e_a{}^2(v_a{}''^\mu + v_a{}'^2 v_a{}^\mu). \quad (6)$$

Therefore the condition (5) implies the vanishing of (6) or the vanishing of the radiation terms in (1).

Now the problem of determining the conditions under which Eq. (1) possesses solutions with vanishing radiation terms has already been solved by Schott.[5] He obtained the result that the only such solutions are motions in a plane; taking it as the x–y plane and having the particle at $t=0$ at $x=\frac{2}{3}ae_a{}^2/m_a$ and $y=0$, moving in the y direction with velocity $b(a^2+b^2)^{-\frac{1}{2}}$, such a motion will be produced by the combination of a uniform electric field E in the x direction of magnitude $\frac{3}{2}m_a{}^2(a^2+b^2)^{\frac{1}{2}}/(a^2e_a{}^3)$ and a uniform magnetic field H in the negative z direction of magnitude $\frac{3}{2}m_a{}^2b/(a^2e_a{}^3)$. The resultant motion is a hyperbola, with a constant y component of the velocity.

This result can be considerably simplified by changing to a frame of reference moving with a velocity $b(a^2+b^2)^{-\frac{1}{2}}$ in the y direction. In this frame the particle is moving in a straight line parallel to the electric field in the x direction; the magnetic field is zero. Therefore Schott's result can be expressed as follows: The only radiationless motions of a particle obeying the Lorentz-Dirac equations are those which in some frame of reference are parallel to a uniform electrostatic field causing the motion.

The motion of a charge in such a field has first been studied by Born,[6] who called it a "hyperbolic motion." It is given by

$$x = \pm(t^2 + \text{const})^{\frac{1}{2}}, \quad y = z = 0, \quad (7)$$

the upper and lower sign corresponding to the motion of a positive and a negative charge, respectively. This is a hyperbola with asymptotes $x=\pm t$ in the $x-t$ plane. In this plane the retarded field progresses parallel to these asymptotes in the direction of increasing t, and the advanced field similarly in the direction of decreasing t. Consequently for a positive charge the region to the left of the asymptote $x=t$ is not reached by the advanced field (similarly the region to the right of $x=-t$ for a negative charge), while the part of this region above the asymptotes is reached by the retarded field. Therefore the difference between the retarded and the advanced field does not vanish everywhere, i.e., Eq. (4) does not hold.

The only exception is provided by the case of zero field. In this case Schott's solution breaks down; however, this radiationless solutions is obviously given by

$$x = v_0 t, \quad y = z = 0 \quad (8)$$

if the reference frame is chosen appropriately. It can easily be shown that in this case Eq. (4) does hold.

Therefore the only motions of a particle obeying the Lorentz-Dirac equations in which the retarded field produced by the particle equals its advanced field everywhere are those with constant velocity.

Consequently, *if the field acting on a particle produced by all other particles is static but not zero, no motion of the system satisfying the symmetry condition (2) of the Wheeler-Feynman theory exists.* If we do not want to reject this theory, we have therefore to reject the original assumption of a static field; we have to assume that the field is only approximately static and therefore the retarded and advanced fields are not quite equal; their difference depends on the motion of all the particles in the universe.

This, however, amounts in effect to abandoning the application of the specific features of this theory to any practical problem, as we cannot hope ever to possess enough information about the initial conditions for such a problem even within classical theory. In practice we shall be forced to operate with the assumption of a knowledge of some average field acting on a particle. But then the symmetry condition (2) does not provide us with any information useful for the solution of our problem. If we accept the statistical interpretation suggested by Wheeler and Feynman we are in effect assuming that initially we possess knowledge of the retarded field only, but not of the advanced one.[7] Therefore, e.g., in the example treated above, we would mean by "static field" that the retarded field is static within the time and space interval of our experiment and nothing is known about the advanced one, and consequently we have to solve Eq. (1) without the restriction of the symmetry condition.

Therefore, at least for the one-particle problem, the absorber theory of radiation is not equivalent mathematically to a field theory using retarded fields only. An interpretation allowing operational equivalence appears to be possible, however.

[5] G. A. Schott, Phil. Mag. **45**, 769 (1923).
[6] M. Born, Ann. Physik **30**, 1 (1909).
[7] For a criticism of this interpretation see A. Landé, Phys. Rev. **80**, 283 (1950).