

Molecular Field Treatment of Ferromagnetism and Antiferromagnetism*

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A modified Weiss treatment of magnetism in crystals which considers both first and second nearest neighbor interactions with all four combinations of signs has been carried out. The usual results for ferromagnetism are obtained for both interactions positive. For the other cases, various types of magnetic ordering below the Curie temperature are predicted, the exact kind depending on the signs and magnitudes of γ_1 and γ_2 , the Weiss field coefficients for first and second nearest neighbor interactions, respectively. Explicit results are given for the kind of ordering as functions of θ/T_c for bcc and fcc lattices. Expressions are derived for the susceptibility of an antiferromagnet above and below the Curie temperature, the latter calculation being given for only a single axis of spontaneous magnetization only. The theory gives a good qualitative description of the behavior of an antiferromagnet for all temperatures, although some of the detailed results are in disagreement with experiment.

THE usual Weiss molecular field treatment of ferromagnetism and antiferromagnetism¹⁻³ considers only interactions between nearest neighbors. Also, for the antiferromagnetic case, it is usually assumed that the lattice of magnetic ions may be divided into two equivalent sublattices as in the simple cubic and body-centered cubic structures. The high temperature molar susceptibility calculated in this way is given by

$$\chi = C_M / (T - \theta), \quad (1)$$

where C_M is the molar Curie constant and $\theta = \pm T_c$, depending on whether the interaction is ferromagnetic or antiferromagnetic. The experimental data on most antiferromagnetic compounds can be fitted reasonably well to an equation of this form. However, the value of θ/T_c , instead of being -1 as predicted, varies between rather wide limits, the observed values ranging all the way from $+0.7$ for MnAs to -5.0 for MnO.

Recent work by Néel⁴ and Anderson⁵ has shown that theoretical values of θ/T_c more in line with the experimental results may be obtained by including second nearest neighbor interactions and by considering magnetic lattices appropriate to actual antiferromagnetic compounds. The calculations of Anderson were carried out only for the case of both first and second nearest neighbor interactions antiferromagnetic while Néel's results are in a form not easily applied to antiferromagnetism. It seems worthwhile to generalize the method of Anderson to include all four possible combinations of sign.

THE HIGH TEMPERATURE SUSCEPTIBILITY AND THE MAGNETIC ORDERING

The starting point for calculating the high temperature susceptibility is the high temperature approximation of Langevin's equation of state for a paramagnetic

material.

$$M = CH/T, \quad (2a)$$

where M is the magnetization in the direction of the applied field H . We assume the material contains N magnetic atoms, each with a magnetic moment $\mu = g\beta S_0$, where S_0 is the spin quantum number, g the gyromagnetic ratio, and β the Bohr magneton. Then (2a) can be written in a form more convenient for our purposes:

$$S = C'H/T, \quad (2b)$$

where $g\beta S$ is the component of μ in the direction of H and C' is the "spin Curie constant," $g\beta S_0(S_0+1)/3k$.

The Weiss procedure is to redefine H to include the effects of internal fields due to neighboring atoms. We shall denote the Weiss field coefficients for first and second nearest neighbor interactions by γ_1 and γ_2 respectively. It is convenient to define the γ_i as positive quantities and to write in the signs of the interactions explicitly. In terms of the Heisenberg theory, the γ_i are given by

$$\gamma_i = 2z_i |J_i| / g\beta, \quad (3)$$

where z_i is the number of i th nearest neighbors and J_i is the exchange interaction between electrons on i th neighbors. However, for the purpose of this paper, the γ_i may be considered merely as phenomenological constants with the J 's being a measure of the difference in interaction energy of parallel and antiparallel spins.

In substituting into Eq. (2), γ_i will be multiplied by $\epsilon_i = \pm 1$, which specifies whether the i th neighbor interaction is ferromagnetic or antiferromagnetic. For convenience in discussion, the various combinations of signs of the ϵ_i are classified as indicated below.

Case	I	II	III	IV
ϵ_1	+1	-1	-1	+1
ϵ_2	+1	-1	+1	-1

Case I should of course lead to the usual results for ferromagnetism in the Weiss theory, except that the Curie temperature is defined in terms of two Weiss coefficients instead of one. Anderson has worked out Case II for a face-centered cubic lattice, and analogous

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¹ L. Néel, *Ann. Phys.* **18**, 64 (1932); **5**, 256 (1936).

² F. Bitter, *Phys. Rev.* **54**, 79 (1938).

³ J. H. Van Vleck, *J. Chem. Phys.* **9**, 85 (1941).

⁴ L. Néel, *Ann. Phys.* **3**, 137 (1948).

⁵ P. W. Anderson, *Phys. Rev.* **79**, 705 (1950).

results for a body-centered cubic lattice are given by Van Vleck.⁶ Their results will be rederived here along with new ones for Cases III and IV.

In problems involving antiferromagnetic interactions, it is necessary to divide the lattice of magnetic atoms into sublattices to take account of various possibilities of magnetic ordering. The method of subdivision depends on the symmetry of the specific lattice, but there should be at least division into enough sublattices so that a given atom has neither first nor second neighbors on its own sublattice. Also, we consider only cases where the sublattices have the same number of atoms. In general, for both first and second nearest neighbor interactions, there will be at least two kinds of magnetic ordering, one favorable to each type of interaction. The possibility of intermediate kinds of magnetic ordering is certainly not excluded.

We first give the calculations for a body-centered cubic lattice. In accordance with the discussion above, it is subdivided into four fcc lattices. We denote the four lattices by the subscripts 1, 2, 3, 4 at the risk of causing confusion with the subscripts 1 and 2 for the γ 's. The division is made so that sublattices 1 and 2 contain the center atoms and sublattices 3 and 4 the corner atoms. Then an atom on sublattice 1 has all its second nearest neighbors each on sublattice 2 and half of its first nearest neighbors each on sublattices 3 and 4. The equations corresponding to (2b) are

$$S_k = (C'/T)[H_0 + \sum_{l \neq k} \epsilon_{kl} \gamma_{kl} S_l], \quad (4a, b, c, d)$$

$k, l = 1, 2, 3, 4.$

where H_0 is the applied magnetic field and $\gamma_{kl} S_l$ is the internal field acting on an atom on the k th sublattice due to its neighbors on the l th sublattice. The γ_{kl} are simply related to γ_1 and γ_2 , the Weiss field coefficients for first and second nearest neighbor interactions. For example, $\gamma_{12} = \gamma_2$; $\gamma_{13} = \gamma_{14} = \frac{1}{2}\gamma_1$. The ϵ_{kl} are ± 1 , depending on whether the given interaction is ferromagnetic or antiferromagnetic.

From Eqs. (4) we can easily calculate the molar susceptibility

$$\chi = (Ng\beta/4H_0) \sum_k S_k = C_M/(T - \theta), \quad (5)$$

where N is Avogadro's number, $C_M = Ng\beta C'$, and

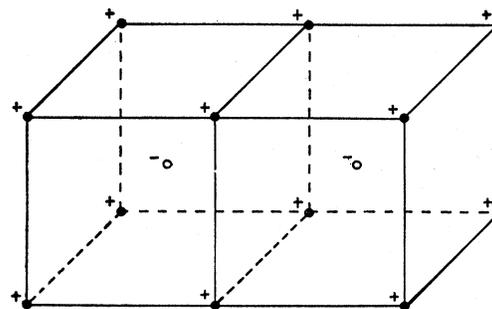
$$\theta = C' \sum_{l \neq k} \epsilon_{kl} \gamma_{kl} = C' \sum_i \epsilon_i \gamma_i. \quad (6)$$

The factor $\frac{1}{4}$ appears in Eq. (5) because the sum is over four atoms, one on each sublattice. The second summation in Eq. (6) is over the different shells of neighbors ($i = 1, 2$, in this case).

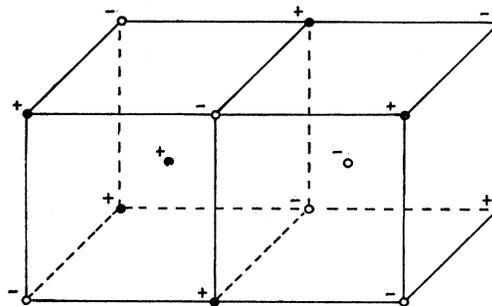
The possible Curie temperatures can be found by setting $H_0 = 0$ in Eqs. (4) and finding what temperature allows nonzero solutions for the S_k . This procedure leads to a determinantal equation of the form

$$\begin{vmatrix} 1 & a_2 & \frac{1}{2}a_1 & \frac{1}{2}a_1 \\ a_2 & 1 & \frac{1}{2}a_1 & \frac{1}{2}a_1 \\ \frac{1}{2}a_1 & \frac{1}{2}a_1 & 1 & a_2 \\ \frac{1}{2}a_1 & \frac{1}{2}a_1 & a_2 & 1 \end{vmatrix} = 0 \quad (7)$$

⁶ J. H. Van Vleck, *J. phys. et radium* **12**, 262 (1951).



ORDERING OF THE FIRST KIND



ORDERING OF THE SECOND KIND

FIG. 1. Possible kinds of magnetic ordering for a body-centered cubic lattice.

where $a_i = -C' \epsilon_i \gamma_i / T$. By the usual methods, this determinant may be factored into

$$[(1 + a_2)^2 - a_1^2][(1 - a_2)^2] = 0$$

giving

$$T_{c1} = -C'(\epsilon_1 \gamma_1 - \epsilon_2 \gamma_2), \quad T_{c2} = -C' \epsilon_2 \gamma_2, \quad (8a, b, c)$$

$$T_{cF} = C'(\epsilon_1 \gamma_1 + \epsilon_2 \gamma_2).$$

Resubstituting the values of T_c , we find that T_{c1} corresponds to $S_1 = S_2 = -S_3 = -S_4$, which is just the condition for ordering with respect to nearest neighbors, or ordering of the first kind, as Van Vleck has designated it. T_{c2} corresponds to $S_1 + S_2 = 0$, $S_3 + S_4 = 0$, which is the second nearest neighbor ordering, or ordering of the second kind. Figure 1 shows the two kinds of magnetic ordering for a bcc lattice. T_{cF} corresponds to $S_1 = S_2 = S_3 = S_4$ or the ferromagnetic case. For each combination of signs of ϵ_1 and ϵ_2 , one or more of the Curie temperatures may be negative, indicating that for this case, the particular kind of ordering has a higher energy than the random state.

The kind of magnetic ordering which actually exists below the Curie temperature will be that which corresponds to the highest value of T_c . For example, in Case II with $\epsilon_1 = \epsilon_2 = -1$, we have $T_{c1} = C'(\gamma_1 - \gamma_2)$ and $T_{c2} = C' \gamma_2$. Then the ordering will be of the first or

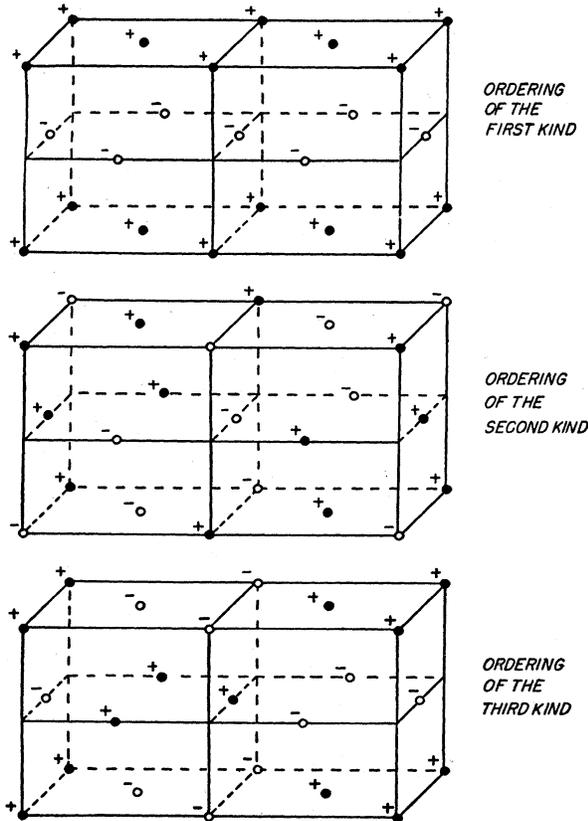


FIG. 2. Possible kinds of magnetic ordering for a face-centered cubic lattice.

second kind depending on whether γ_2 is less than or greater than $\frac{1}{2}\gamma_1$.

We should be able to obtain the Curie temperature for a specified type of magnetic ordering without solving

TABLE I. Values of θ and T_c for a body-centered cubic lattice.

Case	θ/C'	T_c/C'			Magnetic structure
		Ferro	1st	2nd	
I	$\gamma_1 + \gamma_2$	$\gamma_1 + \gamma_2$			ferro
II	$-\gamma_1 - \gamma_2$		$\gamma_1 - \gamma_2$	γ_2	{1st for $\gamma_2 < \frac{1}{2}\gamma_1$ 2nd for $\gamma_2 > \frac{1}{2}\gamma_1$ }
III	$-\gamma_1 + \gamma_2$		$\gamma_1 + \gamma_2$		1st
IV	$\gamma_1 - \gamma_2$	$\gamma_1 - \gamma_2$		γ_2	{1st for $\gamma_2 < \frac{1}{2}\gamma_1$ 2nd for $\gamma_2 > \frac{1}{2}\gamma_1$ }

TABLE II. Values of θ and T_c for a face-centered cubic lattice.

Case	θ/C'	T_c/C'			Magnetic structure
		Ferro	1st	2nd	
I	$3\gamma_1 + \gamma_2$	$3\gamma_1 + \gamma_2$			ferro
II	$-3\gamma_1 - \gamma_2$		γ_2	$\gamma_1 - \frac{1}{3}\gamma_2$	{3rd for $\gamma_2 < \frac{3}{2}\gamma_1$ 2nd for $\gamma_2 > \frac{3}{2}\gamma_1$ }
III	$-3\gamma_1 + \gamma_2$		$\gamma_1 + \gamma_2$		1st
IV	$3\gamma_1 - \gamma_2$	$3\gamma_1 - \gamma_2$		γ_2	{ferro for $\gamma_2 < \frac{3}{2}\gamma_1$ 2nd for $\gamma_2 > \frac{3}{2}\gamma_1$ }

Eq. (7). Suppose the ordering is specified by $S_k = \eta_{kl} S_l$ where $\eta_{kl} = \pm 1$. Then we note that T_c is given by

$$T_c = C' \sum_{l \neq k} \eta_{kl} \epsilon_{kl} \gamma_{kl}, \quad (9)$$

an expression equivalent to Eqs. (8).

One significant result of the calculations with different signs of first and second nearest neighbor interactions is that they help clarify the difference in the physical interpretations of the quantities θ and T_c . The quantity θ appearing in the susceptibility formula indicates the effect of the internal field on the susceptibility. If the internal field favors ferromagnetism, then it aids the applied field in aligning the moments. For this case, $\theta > 0$, and for a given temperature (above T_c) χ is greater than its "pure paramagnetic" value C/T . On the other hand, if the internal field favors antiferromagnetism, then it hinders the applied field by tending to keep the total moment zero. In this case $\theta < 0$, and for a given temperature χ is less than C/T . θ for a particular material is obtained by adding algebraically the contributions from first and second nearest neighbor interactions (and others, if more are considered) as indicated by Eq. (6).

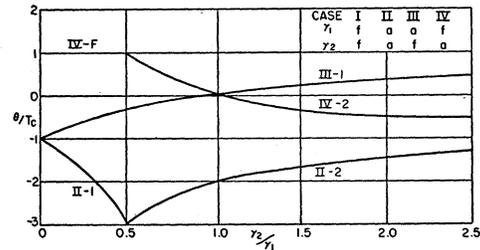


FIG. 3. θ/T_c values for a body-centered cubic lattice. The Roman numerals on each curve refer to the signs of the interactions; the Arabic numerals refer to the kind of magnetic ordering.

The quantity T_c depends on the actual magnetic configuration existing below the Curie temperature— θ , on the other hand, is independent of what happens below the Curie temperature. Essentially, kT_c is a measure of the energy which has to be supplied to bring the material from a state of perfect order to one where the long-range order is destroyed. To find T_c , we combine the same terms as were used in finding θ but with plus or minus signs according to whether or not the magnetic structure is favorable or unfavorable to a given interaction, as indicated by Eq. (9).

The previous considerations enable us to write down θ and T_c for any magnetic lattice by inspection, once the molecular field coefficients and the magnetic structure are assigned. Thus, in considering the fcc lattice, we find that it is convenient to divide it into eight fcc sublattices. Then Eqs. (6) and (9) with appropriate generalizations to the eight sublattice case may be used to obtain θ and T_c directly. Figure 2 shows three possible kinds of antiferromagnetic ordering for the fcc lattice, the third kind being a compromise version

suggested by Anderson⁵ which under certain conditions has a lower energy than either of the other two.

Tables I and II give values of θ and T_c for the bcc and fcc lattices, respectively. Only values of T_c which are largest for some range of γ_2/γ_1 and thus may correspond to actual structures, are listed.

θ and T_c are easily determined from susceptibility measurements, so that γ_1 and γ_2 can be calculated from the relations in Tables I and II, except for Case I where θ and T_c are identical. However, a convenient way of demonstrating the relation between the experimental results and the Weiss field coefficients is to plot θ/T_c as a function of γ_2/γ_1 . These curves are shown in Figs. 3 and 4. The Roman numerals labeling the curves refer to the type of interactions and the Arabic numerals to the kind of ordering. Thus, for Case III, the ordering is of the first kind for all values of γ_2/γ_1 . For Case IV, the material is ferromagnetic for low values of γ_2/γ_1 and antiferromagnetic with ordering of the second kind for high values of γ_2/γ_1 . These two cases extend the theoretical upper limit for θ/T_c for an antiferromagnet from -1 to $+1$, and cover some experimental results which were not allowed by Case II alone. In general, there are two possible values of γ_2/γ_1 for a given θ/T_c ratio; thus there is an ambiguity in the theory. In order to determine the correct value of γ_2/γ_1 the magnetic structure must be established by neutron diffraction.

Table III gives experimental values of θ and T_c for some antiferromagnetic compounds. The columns designated "Low" and "High" give the two values of γ_2/γ_1 , corresponding to the observed θ/T_c ratio and list the curves of Figs. 3 and 4 from which they were taken. In some cases, the kind of ordering has been determined

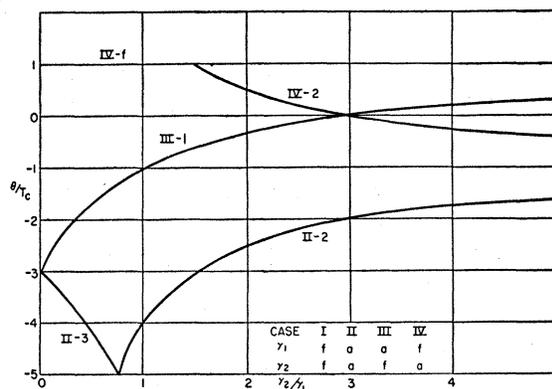


Fig. 4. θ/T_c values for a face-centered cubic lattice. The Roman numerals on each curve refer to the signs of the interactions; the Arabic numerals refer to the kind of magnetic ordering.

by neutron diffraction; this information is recorded in the last column.

No antiferromagnetic compounds are known in which the magnetic lattice is body-centered cubic. However, in the rutile structure the magnetic lattice is body-centered tetragonal. In this case, an atom has two neighbors along the c axis at distance c , eight neighbors on the corners of the rectangular parallelepiped at distance $(a^2+c^2)^{1/2}/2$, and four neighbors along the a axes at distance a . We may divide this structure so that the central atom is on the A sublattice, the corner atoms on the B sublattice, and the other two sets of neighbors on the A sublattice. For convenience, we will assign the coefficient γ_1 to the corner atoms and γ_2 to the combined effect of the other two sets. Actually, the atoms along the c axis are nearer the central atom than

TABLE III. Experimental data on antiferromagnetic compounds.

Compound	Crystal structure	Magnetic lattice structure	$-\theta$	T_c	$-\theta/T_c$	γ_1/γ_2		Ordering from neutron diffraction
						Low	High	
MnF ₂	Rutile	bc tetragonal	113	72	1.57	II-1 0.22	II-2 1.75	1st ^a
FeF ₂	Rutile	bc tetragonal	117	79	1.48	II-1 0.19	II-2 2.10	1st ^a
MnO ₂	Rutile	bc tetragonal		92				2nd ^b
VO ₂	Rutile	bc tetragonal		338				
MnO	NaCl	fcc	610	122	5.0	II-3 0.75	II-2 0.75	2nd ^c
FeO	NaCl	fcc	570	198	2.9	III-1 0.02	II-2 1.57	2nd ^c
CoO	NaCl	fcc	280 ^d	292 ^d	0.96	III-1 1.05	IV-2 75	2nd ^c
NiO	NaCl	fcc	2470 ^e	647 ^{d,f}	3.8	II-3 0.35	II-2 1.07	2nd ^c
MnS	NaCl	fcc	435	140	3.1	II-3 0.05	II-2 1.43	
FeCO ₃	Calcite		14	57	0.25			

^a R. A. Erickson and C. G. Shull, Bull. Am. Phys. Soc. 26, No. 3, 17 (1951).

^b R. A. Erickson, Phys. Rev. 85, 745 (1952).

^c Shull, Strauser, and Wollan, Phys. Rev. 83, 333 (1951).

^d C. Henry La Blanchetais, J. phys. et radium 12, 765 (1951).

^e T. R. McGuire, unpublished work.

^f Approximate temperature of the susceptibility maximum. A crystal structure transition occurs about 520°K. The remainder of the data in this table were taken from the review article of H. Bizette, Ann. phys. 1, 306 (1946).

are the corner atoms for the usual c/a ratios in the rutile structure.

Data are available on two fluorides with this structure, MnF_2 and FeF_2 . As indicated in Table III, both have θ/T_c values of about -1.5 , corresponding to $\gamma_2/\gamma_1 \sim 0.2$ with the first kind of ordering or $\gamma_2/\gamma_1 \sim 2.0$ with the second kind of ordering. However, neutron diffraction experiments have shown that the ordering is of the first kind; this determines the ratio unambiguously, as far as this simple theory goes, and the correct value is given in italics in Table III. MnO_2 and VO_2 are interesting because the magnetic ordering of MnO_2 has been shown to be of the second kind.

Experimental data are available on five antiferromagnetic compounds with the NaCl structure. For four of these, the magnetic structure has been determined to be of the second kind. For MnO , FeO , and NiO the second kind of ordering is allowed for the observed θ/T_c values with $\gamma_2/\gamma_1 = 0.75, 1.57,$ and 1.07 , respectively. In the case of CoO , however, where the ordering is also of the second kind, the value of γ_2/γ_1 is about 75, which seems unreasonably large. It is not clear whether the theory fails here or whether some special feature of the Co^{++} ion such as the relatively large spin-orbit coupling acts to give a very large effective γ_2/γ_1 ratio. The magnetic structure of MnS has not been determined but from the data it appears to be of the second kind with $\gamma_2/\gamma_1 = 1.43$.

FeCO_3 is an interesting case because it has a value of θ/T_c of -0.25 . Such values are not allowed by the previous calculations of Néel and Anderson which consider only Case II. FeCO_3 has the calcite structure, a distortion of NaCl which suggests the subdivision into eight sublattices. If the results for the fcc lattice are used as a guide, the θ/T_c value could be explained either by Case IV with ordering of the second kind or Case III with ordering of the first kind. In view of the lack of experimental data on materials with the calcite structure, it does not seem worthwhile to make special calculations for this case.

THE SUSCEPTIBILITY BELOW THE CURIE TEMPERATURE

In an antiferromagnetic material below the Curie temperature, each of the sublattices acquires a spontaneous magnetization and the internal field becomes very large. This means we can no longer use the approximation $\mu H/kT \ll 1$ which leads to Eq. (2). Also the susceptibility depends upon the direction of the applied magnetic field with respect to the antiferromagnetic axis. Van Vleck³ has calculated the susceptibility of a simple AB type antiferromagnet considering only first nearest neighbor interactions. For this case χ_{\perp} is constant for $T < T_c$ and χ_{\parallel} is zero at absolute zero, rising to χ_{\perp} at $T = T_c$ where χ_{\parallel} and χ_{\perp} refer to the susceptibility with the magnetic field applied respectively parallel and perpendicular to the antiferromagnetic axis. To obtain the powder susceptibility, χ_{\parallel} and

χ_{\perp} are weighted in the ratio 1:2 giving the results that $\chi(0) = \frac{2}{3}\chi(T_c)$ and that the susceptibility should have a cusp at the Curie temperature. In this way, the qualitative features of the $\chi-T$ curve for an antiferromagnet are explained although the experimental value of $\chi(0)/\chi(T_c)$ is not always close to $\frac{2}{3}$. Anderson⁵ has pointed out that if there is more than one antiferromagnetic axis within a single domain, then the $\chi(0)/\chi(T_c)$ ratio should be expected to be different from $\frac{2}{3}$. This seems to be actually the case in MnO_2 where both the susceptibility data⁷ and the neutron diffraction data referred to in Table III suggest that the antiferromagnetic axis for sublattices 1 and 2 is perpendicular to that for sublattices 3 and 4. However, for most antiferromagnetic materials, the ratio is reasonably close to $\frac{2}{3}$, suggesting that there is only one antiferromagnetic axis; also, most of the observed crystal structure changes⁸ at the Curie temperature support this idea. In the calculations to be given here, we consider only the case of a single antiferromagnetic axis.

As in the preceding section, the calculations are given explicitly for the bcc lattice. Let \mathbf{S}_k be the average spin per atom on the k th sublattice with an external magnetic field applied and \mathbf{S}_k^0 be the spontaneous value in zero applied field. Then

$$\begin{aligned} \mathbf{H}_k &= \mathbf{H}_0 + \sum_{l \neq k} \epsilon_{kl} \gamma_{kl} \mathbf{S}_l \\ &= \mathbf{H}_0 + \sum_{l \neq k} \epsilon_{kl} \gamma_{kl} [\mathbf{S}_l^0 + \delta \mathbf{S}_l], \end{aligned} \quad (10)$$

where $\delta \mathbf{S}_l$ is the change produced in \mathbf{S}_l by application of the external magnetic field. (In the previous section, the quantities \mathbf{S}_k were written as scalars because it was implicitly assumed they were either parallel or antiparallel to \mathbf{H}_0 . For $T < T_c$ it is necessary to take into account differences in direction between \mathbf{H}_0 and \mathbf{S}_k^0 .) Let the type of magnetic ordering existing be given by $\mathbf{S}_k^0 = \eta_{kl} \mathbf{S}_l^0$ where $\eta_{kl} = \pm 1$. Also, we assume that all of the $\delta \mathbf{S}_l$ are equal. Then Eq. (10) may be rewritten

$$\begin{aligned} \mathbf{H}_k &= \mathbf{H}_0 + \mathbf{S}_k^0 \sum_{l \neq k} \eta_{kl} \epsilon_{kl} \gamma_{kl} + \delta \mathbf{S} \sum_{l \neq k} \epsilon_{kl} \gamma_{kl} \\ &= \mathbf{H}_0 + (T_c/C') \mathbf{S}_k^0 + (\theta/C') \delta \mathbf{S}. \end{aligned} \quad (11)$$

Then straightforward application of the Langevin-Weiss theory gives

$$S = |\sum_k \mathbf{S}_k| = 4\delta S = S_0 [\sum_k \cos(\mathbf{H}_k, \mathbf{H}_0) B(y_k)], \quad (12)$$

where B is the Brillouin function for spin S_0 and

$$y_k = g\beta S_0 |\mathbf{H}_k| / kT. \quad (13)$$

When $\mathbf{H}_0 = 0$, $\delta \mathbf{S} = 0$ and

$$y_k = y^0 = 3S_0(S_0 + 1)^{-1} T_c T^{-1} |\mathbf{S}_k^0 / S_0|. \quad (14)$$

Now consider separately the cases where \mathbf{H}_0 is parallel and perpendicular to \mathbf{S}_k^0 .

$$T < T_c, \mathbf{H}_0 \parallel \mathbf{S}_k^0$$

⁷ H. Bizette and B. Tsai, *Colloque sur la polarisation de la matiere* (Paris, 1949).

⁸ J. S. Smart and S. Greenwald, *Phys. Rev.* **82**, 113 (1951).

The applied field is assumed to be small in comparison with the internal field so that the Brillouin function may be expanded about $H_0=0$, giving

$$B(y_i) = B(y^0) + \eta_{ki} g \beta S_0 [H_0 + (\theta/C') \delta S] B'(y^0) / kT. \quad (15)$$

Also $\cos(\mathbf{H}_i, \mathbf{H}_0) = \eta_{ki}$. Substituting into Eq. (12) we find

$$S = \frac{4g\beta S_0^2}{kT} H_0 B'(y^0) \left[1 - \frac{g\beta S_0^2 \theta}{kT C'} B'(y^0) \right]^{-1}. \quad (16)$$

Expressing the result in terms of the molar susceptibility,

$$\chi_{11} = C_M \frac{3S_0(S_0+1)^{-1} B'(y^0)}{T - 3S_0(S_0+1)^{-1} B'(y^0) \theta}. \quad (17)$$

Equation (17) reduces to Van Vleck's result³ for his approximations ($S_0 = \frac{1}{2}$, $\gamma_2 = 0$).

$$T < T_c, \mathbf{H}_0 \perp \mathbf{S}_k^0$$

Let the z axis be the axis of spontaneous antiferromagnetism and the x axis be the direction of the applied magnetic field. Then since $|\mathbf{H}_0| \ll |\mathbf{H}_k|$, we may write

$$S = 4\delta S = 4|\mathbf{S}_k^0| \frac{|\mathbf{H}_x|}{|\mathbf{H}_z|} = 4|\mathbf{S}_k^0| \frac{|\mathbf{H}_0 + (\theta/C') \delta \mathbf{S}|}{|(T_c/C') \mathbf{S}_k^0|}, \quad (18)$$

and

$$S = 4C'H_0 / (T_c - \theta). \quad (19)$$

The molar susceptibility is then

$$\chi_{11} = C_M / (T_c - \theta) = \chi_{11}(T_c) = \chi(T_c). \quad (20)$$

T_c is determined experimentally as the point at which the maximum in the $\chi-T$ curve occurs, while θ may be found by fitting the high temperature ($T > T_c$) susceptibility data to Eq. (5). If the theory is consistent, then we should be able to use the values of θ and T_c determined in this way to calculate the susceptibility for $T < T_c$ from Eqs. (17) and (20). Figure 5 shows the results of such a calculation for MnO which has $\theta/T_c = -5$. The agreement between the theoretical and experimental curves is not very good, their shapes being considerably different. Actually, the curve for $\theta = -T_c$ has more nearly the shape of the experimental curve. However, Fig. 5 is plotted on a reduced scale; on an absolute scale the curve for $\theta = -T_c$ gives a susceptibility which is much too large.

One reason for the discrepancy between theory and experiment may be the neglect of anisotropy effects. Recently, Nagamiya⁹ has calculated the susceptibility for a simple AB type antiferromagnet, including an anisotropy field. If this type of calculation were extended to second nearest neighbors and to more complicated lattices, the agreement might be improved. It is also possible that the reason for the discrepancy lies in

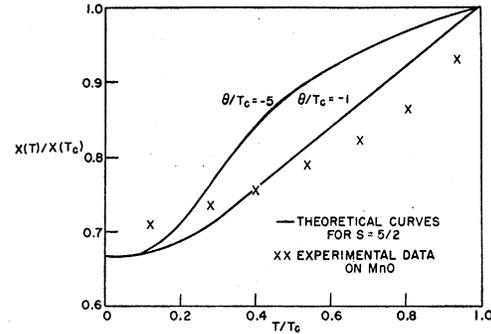


FIG. 5. $\chi(T)/\chi(T_c)$ vs T/T_c for a material with a single antiferromagnetic axis. The experimental data are taken from Bizette, Squire, and Tsai, *Compt. rend.* **207**, 449 (1938).

a basic defect of the Weiss theory itself. Holstein and Primakoff¹⁰ have shown that the Weiss theory and the spin-wave theory give quite different results for the intrinsic susceptibility of a ferromagnet below its Curie temperature.

GENERAL DISCUSSION

This simple extension of the Weiss theory given above is a good qualitative description of the behavior of an antiferromagnet for all temperatures although some of the detailed results are in disagreement with the experiments. In this respect, it is quite similar to the usual Weiss theory of ferromagnetism. In particular the theory gives an adequate account of the θ/T_c values although it is uncertain as to just how much significance can be attached to the corresponding γ_2/γ_1 . Since these quantities cannot be computed theoretically at the present time, we are forced to consider them merely as adjustable constants. The theory also gives explicit conditions for the different kinds of magnetic ordering, although here it is necessary to have the magnetic lattice subdivided properly.

It is an interesting question as to whether a simple Weiss theory which takes account of both first and second nearest neighbor interactions or a more sophisticated theory which considers only nearest neighbor interactions gives the better results in antiferromagnetism. Certainly the fact that both types of ordering have been observed in compounds with the same magnetic lattice (rutile) points up the importance of considering both interactions. It is interesting to compare the results of the Weiss field treatment to those of Li,¹¹ who applied the more elegant Bethe-P. R. Weiss method to the problem of a simple AB type antiferromagnet with first neighbor interactions only. His results indicate that there should be some curvature in the $1/\chi-T$ curve just above the Curie temperature (due to short range ordering) but that at high temperatures, the susceptibility should be given by Eq. (1) with $\theta \cong -T_c$.

¹⁰ T. Holstein and H. Primakoff, *Phys. Rev.* **58**, 1098 (1940).

¹¹ Y.-Y. Li, *Phys. Rev.* **84**, 721 (1951).

⁹ T. Nagamiya, *Prog. Theoret. Phys.* **6**, 342 (1951).

The low temperature curvature is evident in much of the experimental data, but as has been stated previously, the θ/T_c ratios are considerably different from unity. Li has pointed out that if the susceptibility measurements are not taken at sufficiently high temperatures, an apparent θ/T_c will be obtained whose magnitude may be as high as 1.5–1.7. In this way, the data on some antiferromagnetic compounds, such as

MnF_2 and FeF_2 , might be fitted, but for most the θ/T_c values fall considerably outside the range allowed by Li's theory. Presumably a Bethe-Weiss treatment including both types of interactions and appropriate magnetic lattices should correct some of the deficiencies of both Li's theory and the present theory (at high temperatures) but the amount of calculation involved seems formidable.

A Note on the Absorber Theory of Radiation

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The Wheeler-Feynman theory, starting with fields symmetric in time, obtained the Lorentz-Dirac equations of motion, which use retarded fields only, together with a condition on the total field of all particles in the universe. It is shown that if the field acting on a particle produced by all other particles is static but not zero, no motion of the system satisfying this condition exists. Some implications of this result for the physical interpretation of the Wheeler-Feynman theory are discussed.

RECENT attempts to obtain the force of radiative reaction on a moving charge in classical electrodynamics have proceeded from two different viewpoints. One is that of field theory, which considers the *total* field at all points in space to be the fundamental physical quantity. The other is that of action at a distance, which considers only the forces exerted on a charge by *other* charges to be physically meaningful.

The field theoretical point of view was first applied successfully to the problem of the motion of a point charge by Dirac,¹ who succeeded in obtaining the equations of motion first found by Lorentz on the basis of a model of an extended charge. These equations can be written

$$m_a v_a'^{\mu} = e_a \sum_{k \neq a} \text{ret} F_{\nu}^{\mu(k)} v_a^{\nu} + \frac{2}{3} e_a (v_a'^{\mu} + v_a'^2 v_a^{\mu}), \quad (1)$$

where $\text{ret} F^{(k)}$ is the retarded field of the k th particle evaluated at the position of the a th one, and where we have assumed that there are no fields present except those due to charges.

The first derivation of the Lorentz-Dirac equations on the basis of action at a distance is due to the absorber theory of radiation of Wheeler and Feynman.² This was achieved taking the forces on the charges as determined by half the sum of the retarded and the advanced field, while the previous field theoretical derivations all had been based on the use of retarded fields alone.

In a previous paper³ it was pointed out that the need for the exclusive use of retarded fields for the explana-

tion of the radiative reaction arose only in the field theoretical derivations for the one-particle problem, but that the considerations of Wheeler and Feynman on the total field due to all particles are applicable to field theory as well as to action at a distance. It was concluded that one can obtain the Lorentz-Dirac equations in both theories starting with fields symmetric in time, in spite of the fundamentally different underlying physical ideas.

It was noted, however, that there was not complete equivalence of the retarded and symmetric cases. The considerations of Wheeler and Feynman led to the symmetry condition

$$\sum_{\text{all } k} \text{ret} F^{(k)} = \sum_{\text{all } k} \text{adv} F^{(k)} \text{ everywhere.} \quad (2)$$

Therefore the solutions of (1) are subject to this condition in the symmetric case both in field theory and in action at a distance, while there is no such restriction imposed upon them in the retarded case.

It appears that present mathematical methods are not powerful enough to enable us to discuss the effect of this restriction in the general case.⁴ We shall only discuss the case that the effect on a single charge of all other charges in the universe is that of a static field, i.e., that

$$\sum_{k \neq a} \text{ret} F^{(k)} = \text{const} = \sum_{k \neq a} \text{adv} F^{(k)} \text{ everywhere.} \quad (3)$$

From this and condition (2) we must have

$$\text{ret} F^{(a)} = \text{adv} F^{(a)} \text{ everywhere.} \quad (4)$$

¹ P. A. M. Dirac, Proc. Roy. Soc. (London) **A167**, 148 (1938).

² J. A. Wheeler and R. P. Feynman, Revs. Modern Phys. **17**, 157 (1945).

³ P. Havas, Phys. Rev. **74**, 456 (1948).

⁴ For a discussion of some of the difficulties of the two-body problem, see P. Havas, Acta Phys. Austriaca **3**, 342 (1949).