theory and experiment for the latitude variations of small stars. This must be considered to be spurious, however, since the primary proton energy spectrum adopted by Messel (integral power law exponent = 1.7) is in serious disagreement with the spectrum deduced from the data of Winckler and Peters (exponent $\approx 1.0$); and furthermore, the presence of heavy particles in the primary radiation has been neglected. Actually, the results obtained by Messel predict a specific yield function for small star production which depends strongly on primary proton energy, even for energies above 4.1 Bev.

Instead, we have found that the proton specific yield at large atmospheric depths increases rapidly with energy up to about 4 Bev but then becomes relatively insensitive to energy up to at least 12.7 Bev. If one retains all of the assumption of the Heitler-Janossy theory, with the exception of (4), then in order to explain our results it is necessary to suppose that the energy distribution of the recoil nucleons becomes insensitive to primary energy in the range from 4.1 to at least 12.7 Bev. This would imply that nucleon-nucleon collisions are on the average fairly elastic at low energies but that the elasticity begins to fall off rapidly with increasing energy in the vicinity of 4 Bev, owing, presumably, to a strong increase in meson production. The fraction of cases in which a proton-nucleus interaction leads to meson production is indeed known to increase rapidly with proton energy up to about 4 Bev, beyond which energy nearly every interaction results in meson production.[18] However, the above description of nucleon-nucleon interactions requires further that the energy carried away by the mesons increase very rapidly with primary energy above 4.1 Bev (in order that the energy remaining to the recoil nucleons be an insensitive function of primary energy). A test of this possibility must await further experiments.

The writer wishes to thank Professor J. A. Simpson for his valuable guidance and for many illuminating discussions. Thanks are also due to Professor M. Schein for several helpful discussions.

PHYSICAL REVIEW VOLUME 86, NUMBER 6 JUNE 15, 1952

# Phenomenological Relationships between Photomeson Production and Meson-Nucleon Scattering*

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(Received February 25, 1952)

Photomeson production is studied on the basis of the hypothesis of charge independence combined with the suggestion from meson theory that the meson-nucleon state of angular momentum and isotopic angular momentum equal to $\frac{3}{2}$ is one of strong, resonant interaction. Reasonable agreement with experiment is obtained, both as to the energy dependence of the cross sections and as to their ratios. Similar considerations lead to a possible explanation for the angular dependence of $\pi^+$ mesons produced in $p$–$p$ collisions.

## I. INTRODUCTION

Our present theoretical understanding of meson-nucleon interactions is based upon general considerations of parity and angular momentum and upon the more detailed and less reliable models of field theory. The latter have been, perhaps, surprisingly successful in many cases in predicting orders of magnitude and qualitative features of the interactions of mesons and nucleons. One of the most interesting predictions of meson theories is that meson-nucleon interactions involve relatively few states of angular momentum. This is consistent with present experimental evidence for meson-nucleon processes. However, the lack of quantitative agreement with experiment has led one to the conclusion that (among other difficulties) neither the weak nor the strong coupling limit for meson theories is correct. In view of these shortcomings of explicit models, it seems desirable to fall back on more general phenomenological analyses of the experimental data with the hope of establishing contact with meson theory where possible.

An analysis of meson nucleon scattering along these lines was made by one of us.[1] To the usual requirements of conservation of angular momentum and parity was added that of conservation of "isotopic angular momentum."[2] The suggestion of strong $P$-state interactions between mesons and nucleons was borrowed from meson theory and from strong-coupling theory the notion of nucleon isobars, as well as their classification in terms of spin and isotopic spin. The resulting analysis

* Supported in part by the joint program of the ONR and AEC.

[1] K. A. Brueckner, Phys. Rev. **86**, 106 (1952).
[2] K. M. Watson and K. A. Brueckner, Phys. Rev. **83**, 1 (1951); K. M. Watson, Phys. Rev. **86**, 852 (1952), discussion of the applications of the concept of "isotopic angular momentum" conservation for meson-nucleon phenomena.

led to a simple interpretation of presently known experimental cross sections.[3]

This treatment[1] began by resolving the scattering into substates of angular momentum and isotopic angular momentum. For $P$-state interaction (orbital angular momentum of unity), the angular momentum of the meson-nucleon system can be resolved into states of $J=\frac{1}{2}$ and $J=\frac{3}{2}$. Similarly, the meson carries one unit of isotopic angular momentum and the nucleon one-half unit, so the total isotopic angular momentum, $I$, can be resolved into the states $I=\frac{1}{2}$ and $I=\frac{3}{2}$. From the isobaric nucleon levels in meson theory, one would be led to expect that the state $I=\frac{3}{2}$, $J=\frac{3}{2}$ is responsible for most of the scattering, except at low energies.

This suggestion seems to be borne out by the experimental results. However, the agreement depended upon there being strong scattering in the $I=\frac{3}{2}$ state rather than in the $J=\frac{3}{2}$ state, so it cannot be taken as direct evidence for stronger scattering in the $J=\frac{3}{2}$ state than in the $J=\frac{1}{2}$ state. Measurements of the angular distribution of the scattering should help to answer this question.

Similar considerations should apply to meson production by $\gamma$-rays. An analysis by Brueckner and Case[4] of the photoproduction of mesons did indeed lend support to the idea that nucleon isobars may play an important role in photomeson production, but their analysis was made on the basis of classical meson theory and involved an average over unquantized spins and isotopic spins. In the present paper an analysis of the photomesons cross sections will be made along the lines of the previous analysis[1] of meson-nucleon scattering. One might expect the same substates of spin and isotopic spin to be of importance in both processes.[5]

A formal analysis[6] of the role played by the isotopic spin in photomeson production suggests that such a study may be promising. The admixture of the $I=\frac{3}{2}$ state is twice as great in the neutral photomeson cross section as in the charged-meson cross section. This can be expected to help increase the $\pi^0$ cross sections at the expense of the $\pi^{\pm}$ cross sections. Again, the angular distribution (in particular of the $\pi^0$'s) can be expected to give a means of determining the angular momentum states of the meson-nucleon system which are of importance in contributing to the cross sections.

These considerations can also be applied to meson production in nucleon-nucleon collisions. Here, because of the greater complexity, the model is less specific.

[3] Lundby, Fermi, Anderson, Nagle, and Yodh, Bull. Am. Phys. Soc. **27**, No. 1, 28 (1952); and Nagle, Anderson, Fermi, Long, and Martin, Bull. Am. Phys. Soc. **27**, No. 1, 28 (1952).

[4] K. A. Brueckner and K. M. Case, Phys. Rev. **83**, 1141 (1951).

[5] H. Miyazawa, private communication, has studied the photomeson production according to charge-symmetric pseudoscalar meson theory in the limit of almost strong coupling. His results, when interpreted on the basis of the present analysis, show that there is indeed strong interaction in the $J=\frac{3}{2}$, $I=\frac{3}{2}$ state according to meson theory. His ratios of the photo cross sections represent a special case of the general isotopic spin restrictions, which are imposed in the present paper.

[6] K. M. Watson, reference 2.

## II. THE ANALYSIS OF THE PHOTOMESON CROSS SECTIONS

### A. Formal Considerations

It will be simplest to calculate first the radiative absorption of mesons by nucleons and then to obtain the photoproduction cross sections by detailed balancing.

We suppose the meson absorption to take place from $S$- and $P$-states of orbital angular momentum. For charged mesons the absorption is expected to take place primarily from $S$-states (this is the prediction of both weak- and strong-coupling meson theory and is not in disagreement with the experimental results). For neutral mesons the strong energy dependence of the cross sections suggests that $P$-waves rather than $S$-waves are important (this also is expected from meson theory).

We shall take then the mechanism of $S$-wave absorption as predicted by either weak- or strong-coupling meson theory. For the $P$-wave absorption, the angular momentum of the system can be either $J=\frac{1}{2}$ or $\frac{3}{2}$ and the emission of a $\gamma$-ray may go by either magnetic dipole or electric quadrupole interaction (in the latter case there is only the $J=\frac{3}{2}$ state). From meson theory, we would expect magnetic dipole radiation. We shall suppose this to be the case for the present, including the other possibility later.

We suppose the meson momentum to be $\mathbf{q}$ and that this is directed along the $z$-axis (i.e., $\mathbf{q}=q\mathbf{k}$, where $\mathbf{k}$ is the unit vector along the $z$-axis). $\mathbf{K}$ is the photon momentum vector and $\hat{e}$ is its polarization vector. The $z$-component of the angular momentum of the system is $J_z=M$, and this is just the $z$-component of the nucleon spin initially since the meson is moving along the $z$-axis.

The outgoing photon wave corresponding to an initial total isotopic spin $I$ for the meson and nucleon and a $z$-component of isotopic spin $m$ will have the form

$$\psi_0(I,m)=-\frac{\hbar\pi^{\frac{1}{2}}}{q}\sum_{\nu}\frac{e^{i\mathbf{K}r}}{r}\xi^{\nu}\{u_0{}^{\frac{1}{2}Im}(\nu|x_0{}^{\frac{1}{2}}|M)-3^{\frac{1}{2}}u_1{}^{\frac{1}{2}Im}(\nu|x_1{}^{\frac{1}{2}}|M)-3^{\frac{1}{2}}u_1{}^{\frac{3}{2}Im}(\nu|x_1{}^{\frac{3}{2}}|M)\}, \quad (1)$$

where the typical term $u_l{}^{JIm}(\nu|x_l{}^J|M)$ is equivalent to an element of the general $U$ matrix given by Wigner and Eisenbud.[7] Here the $x_l{}^J$ involve spherical harmonics of the outgoing photon angular variables and polarization, $\nu$ is the $z$-component of the nuclear spin in the final state, $\xi^{\nu}$ is the final nucleon spin function. The $u_l{}^{JIm}$ are independent of angle but depend on energy. The superscripts $J$ and $I$ refer to the total angular momentum and total isotopic spin of the initial wave; the subscript $l$ refers to the orbital angular momentum of the meson.

According to the previously mentioned analysis[6] of the role played by the isotopic spin in photomeson production, there will be one $\psi_0$ for the $I=\frac{3}{2}$ state, say $\psi_0(\frac{3}{2})$, and two sets of $\psi_0$'s for the $I=\frac{1}{2}$ state which

[7] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).

depend not only on the total isotopic spin but also on the total charge (related to the $z$-component of the isotopic spin). These we denote by $\psi_0(\frac{1}{2}+)$ and $\psi_0(\frac{1}{2}-)$, the first state corresponding to a total charge of zero and the second to a total charge of one. Thus for each angular momentum state there are three $\psi_0$'s, corresponding to the three isotopic states, there being 9 of these quantities. From the general theory[8] we can obtain the form of the outgoing waves for initial charge eigenstates of the meson and nucleon.

$$\begin{aligned}\psi_0(\pi^0+n\rightarrow n+\gamma)&=[\sqrt{2}\psi_0(\tfrac{3}{2})-\psi_0(\tfrac{1}{2}+)]/\sqrt{3},\\ \psi_0(\pi^0+p\rightarrow p+\gamma)&=[\sqrt{2}\psi_0(\tfrac{3}{2})-\psi_0(\tfrac{1}{2}-)]/\sqrt{3},\\ \psi_0(\pi^-+p\rightarrow n+\gamma)&=[\psi_0(\tfrac{3}{2})+\sqrt{2}\psi_0(\tfrac{1}{2}+)]/\sqrt{3},\\ \psi_0(\pi^++n\rightarrow p+\gamma)&=[\psi_0(\tfrac{3}{2})+\sqrt{2}\psi_0(\tfrac{1}{2}-)]/\sqrt{3}.\end{aligned} \tag{2}$$

We note that the $u$'s in Eq. (1) are the only quantities depending upon the isotopic spin, so the transformation (2) is really a transformation of the $u$'s alone.

We observe two interesting qualitative features of Eq. (2).[9] First, if the absorption from $P$-states is predominantly due to the $I=\frac{3}{2}$ state, it will lead to an absorption rate for neutral mesons which is twice as great as that for charged mesons (from $P$-states only, of course). This can be expected to help increase the $\pi^0$ photoproduction cross section at the expense of that for the $\pi^+$ mesons, except near threshold, which is in agreement with the measured cross section.[10] Second, the $I=\frac{3}{2}$ state enters with equal weight into the cross sections for photoproduction of mesons on both neutrons and protons, so we would expect approximately equal cross sections in both cases for those energies at which the $I=\frac{3}{2}$ state gives the major contribution. This too seems to be in agreement with recent experiments.[11]

We turn next to the analysis of the angular momentum states occurring in Eq. (1). Since $\psi_0$ must remain invariant if the entire scattering system is rotated in space, we know that the $x$'s must be pseudoscalar combinations (i.e., supposing the meson to be pseudoscalar) of the vectors $\mathbf{k}=\mathbf{q}/q$, $\hat{e}$, $\mathbf{K}$, and $\boldsymbol{\sigma}$, the nucleon spin. For capture of the meson from an $S$-state we have the unique form

$$(\nu|x_0^{\frac{1}{2}}|M)=(8\pi)^{-\frac{1}{2}}(\xi^\nu, \boldsymbol{\sigma}\cdot\hat{e}\xi^M), \tag{3}$$

which is also that given by meson theory. The $\xi$'s are spin wave functions, and the factor $(8\pi)^{-\frac{1}{2}}$ is chosen to agree with the Wigner-Eisenbud[7] normalization.

[8] The correspondence between the $\psi_0$'s of Eq. (2) and the matrix elements $T_1$, $T_2$, and $S_0$ of reference 6 is

$$\psi_0(\tfrac{3}{2})\rightarrow\sqrt{2}T_2,\quad \psi_0(\tfrac{1}{2}+)\rightarrow\tfrac{1}{2}T_1+S_0,\quad \psi_0(\tfrac{1}{2}-)\rightarrow\tfrac{1}{2}T_1-S_0.$$

[9] Heckrotte, Henrich, and Lepore [Phys. Rev. **86**, 490 (1952)] have remarked that there should be constructive interference of the meson wave amplitudes for neutral photomeson production from deuterium if one assumes the couplings of symmetric-meson theory and the results of a perturbation calculation. Reference to Eq. (2) shows that this would be expected quite generally if production into the $I=\frac{3}{2}$ state is predominant.

[10] Panofsky, Steinberger, and Stellar, Phys. Rev. **86**, 180 (1952).

[11] A. Silverman and G. Cocconi, Bull. Am. Phys. Soc. **27**, No. 1, 27 (1952).

For the capture of a meson from an orbital $P$-state, the incoming wave function has the form

$$\varphi=\mathbf{k}\cdot\mathbf{r}\xi^M \text{ times a function of } |\mathbf{r}|. \tag{4}$$

To resolve this into eigenstates of $J$, we note that $\xi^M$ corresponds to $J=\frac{1}{2}$ and so does

$$\boldsymbol{\sigma}\cdot\mathbf{r}\xi^M,$$

since the latter form transforms under rotations like $\xi^M$. Then

$$\varphi_{\frac{3}{2}}{}^M=\mathbf{k}\cdot(\mathbf{r}-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{r}\boldsymbol{\sigma})\xi^M \tag{5}$$

is a state of $J=\frac{3}{2}$ since it is orthogonal to $\boldsymbol{\sigma}\cdot\mathbf{r}\xi^M$. The appropriate $J=\frac{1}{2}$ state is

$$\varphi_{\frac{1}{2}}{}^M=\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{r}\boldsymbol{\sigma}\cdot\mathbf{k}\xi^M \tag{6}$$

(being just $\pm\frac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{r}\xi^M$), since the sum of Eqs. (5) and (6) gives us a wave function of the type (4).

The outgoing photon wave function is also to be resolved into eigenstates of $J$. For magnetic dipole radiation it is simpler to use the pseudovector $\mathbf{K}\times\hat{e}$ rather than the spherical harmonics. From Eqs. (5) and (6), we see that

$$\begin{aligned}K(\nu|x_1^{\frac{3}{2}}|M)&=(3/8\pi)^{\frac{1}{2}}(\xi^\nu, [\mathbf{K}\times\hat{e}\cdot\mathbf{k}-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{K}\times\hat{e}\boldsymbol{\sigma}\cdot\mathbf{k}]\xi^M),\\ K(\nu|x_1^{\frac{1}{2}}|M)&=(3/8\pi)^{\frac{1}{2}}(\xi^\nu, [\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{K}\times\hat{e}\boldsymbol{\sigma}\cdot\mathbf{k}]\xi^M).\end{aligned} \tag{7}$$

That these are the correct expressions can be seen if one substitutes them into Eq. (1). The resulting expression for the outgoing photon wave contains terms of the form of (5) and (6), in which $\mathbf{K}\times\hat{e}$ replaces $\mathbf{r}$.

For electric quadrupole radiation the symmetric tensor $\frac{1}{2}(K_ie_j+e_iK_j)$ enters into the $x$'s, and we have only $J=\frac{3}{2}$. Thus

$$K(\nu|x,^{\frac{3}{2}}(\text{e.q.})|M)=(8\pi)^{-\frac{1}{2}}(\xi^\nu, \tfrac{1}{2}[\boldsymbol{\sigma}\cdot\mathbf{K}\hat{e}\cdot\mathbf{k}+\boldsymbol{\sigma}\cdot\hat{e}\mathbf{K}\cdot\mathbf{k}]\xi^M). \tag{8}$$

The cross section for absorption (neglecting electric quadrupole radiation) is obtained from Eq. (1),

$$\frac{d\sigma(\text{abs})}{d\Omega}=\frac{\pi\hbar^2}{2q^2}\sum_{\nu,M,\hat{e}}|A(\nu|x_0^{\frac{1}{2}}|M)-3^{\frac{1}{2}}B(\nu|x_1^{\frac{1}{2}}|M) -3^{\frac{1}{2}}C(\nu|x_1^{\frac{3}{2}}|M)|^2, \tag{9}$$

where $A$, $B$, and $C$ are those linear combinations of the $u$'s in Eq. (1) which are given by the appropriate expression (2). By detailed balancing we obtain from Eq. (9) the photoproduction cross sections:

$$\begin{aligned}d\sigma/d\Omega=(\hbar^2/8K^2)\{&|A|^2+|B|^2+\tfrac{1}{2}|C|^2(5-3\cos^2\theta)\\ &-2\,\mathrm{Im}[A^*(B-C)]\cos\theta\\ &-\tfrac{1}{2}\,\mathrm{Re}(B^*C)(3\cos^2\theta-1)\},\end{aligned} \tag{10}$$

where $\theta$ is the angle between $\mathbf{K}$ and $\hat{e}$. For the total cross section, we have

$$\sigma=(\pi\hbar^2/4K^2)\{2|A|^2+2|B|^2+4|C|^2\}. \tag{11}$$

For pure electric quadrupole radiation [Eq. (8)], the angular distribution would be of the form

$$1+\cos^2\theta. \tag{12}$$

### B. Detailed Development of the Model

We can now introduce considerable simplification into the general expression for the cross section of Eq. (10) if we make use of the results of the analysis of the scattering of mesons by nucleons. There it was shown that when the meson was in a $P$-state, the assumption of strong interaction in the $I=\frac{3}{2}$ state gave a good description of the ratios observed for the various processes of meson scattering. The assumption was also made that the interaction was strong in the $J=\frac{3}{2}$ state, since then the strength of the interaction could be associated with the excitation of a nucleon isobar with $I=\frac{3}{2}$, $J=\frac{3}{2}$ which is in accord with the predictions of strong coupling meson theory. We shall assume that this is also so for the photoproduction. The resulting angular dependence of the cross section can then of course be subjected to experimental verification. We shall also draw on one of the most striking features of the neutral photomeson cross section: that it increases rapidly with energy from very small values at low energies. This is due to the near absence of a contribution from meson $S$-states, presumably, and to the strong energy dependence to be expected from states of higher angular momentum. This feature is also given by meson theory where the large $S$-state contribution is present only for charged mesons. From Eqs. (1) and (2), the $S$-state term for neutral mesons is proportional to

$$-[u_0^{\frac{1}{2}\frac{1}{2}m}+\sqrt{2}u_0^{\frac{1}{2}\frac{3}{2}}]/\sqrt{3},$$

which is to vanish, fixing the relation

$$u_0^{\frac{1}{2}\frac{3}{2}}=u_0^{\frac{1}{2}\frac{1}{2}m}/\sqrt{2}, \tag{13}$$

which also shows that $u_0^{\frac{1}{2}\frac{1}{2}m}$ is independent of $m$ because of the independence of $m$ in the $I=\frac{3}{2}$ states.

Collecting these results, we find for the differential cross sections

$$\begin{aligned}\frac{d\sigma(\text{charged})}{d\Omega}&=\frac{\hbar^2}{24K^2}\{(9/2)|u_0^{\frac{1}{2}\frac{1}{2}}|^2+\tfrac{1}{2}|u_1^{\frac{3}{2}\frac{3}{2}}|^2(5-3\cos^2\theta)\\&\qquad+3\sqrt{2}\,\mathrm{Im}[(u_0^{\frac{1}{2}\frac{1}{2}})^*u_1^{\frac{3}{2}\frac{3}{2}}]\cos\theta\},\\\frac{d\sigma(\text{neutral})}{d\Omega}&=\frac{\hbar^2}{24K^2}|u_1^{\frac{3}{2}\frac{3}{2}}|^2(5-3\cos^2\theta).\end{aligned} \tag{14}$$

The elements of the $u$ matrix which enter into these expressions can now be expressed in terms of the generalized one-level formulas of Wigner and Eisenbud. The result is

$$u_l^{JI}=\frac{[\Gamma_l^{JI}(\pi)]^{\frac{1}{2}}[\Gamma_l^{JI}(\gamma)]^{\frac{1}{2}}}{E_l^{JI}-E-i\frac{1}{2}\Gamma_l^{JI}}, \tag{15}$$

where $\Gamma_l^{JI}(\pi)$ is the meson width, $\Gamma_l^{JI}(\gamma)$ is the gamma-width, $E_l^{JI}$ is the resonance energy, and $\Gamma_l^{JI}$ is the total width. The meson width is much greater than the gamma-width, corresponding to the much larger probability of scattering than of radiation, so the total width can be taken equal to the meson width. This equation, which is of the form given by Feshbach, Peaslee, and Weisskopf,[12] is valid only near resonance; the more general form of Wigner and Eisenbud contains explicitly a level shift but can be approximated by a formula of this type.

The resonance energy $E_1^{\frac{3}{2}\frac{3}{2}}$ and level width $\Gamma_1^{\frac{3}{2}\frac{3}{2}}$ have been given previously[1] in connection with the analysis of meson scattering. The assignment made there, which gave a good description of the scattering, was that

$$\begin{gathered}E_1^{\frac{3}{2}\frac{3}{2}}\equiv E_R=137\text{ Mev},\\\frac{\Gamma_1^{\frac{3}{2}\frac{3}{2}}}{2E_1^{\frac{3}{2}\frac{3}{2}}}\equiv\rho=\rho_0\frac{q^3/\mu^3c^3}{1+(q^2a^2/\hbar^2)},\quad\rho_0=0.23,\end{gathered} \tag{16}$$

where the parameter $\rho_0$ is independent of energy and is related to the reduced width of Wigner and Eisenbud; $\mu$ is the mesonic rest mass; $a$ is the radius of the internal region of strong interaction. For the meson emitted into an $S$-state, both weak and strong coupling theory give an assignment to the energy $E_0^{\frac{1}{2}I}$ which is

$$|E_0^{\frac{1}{2}I}|\gg\mu c^2,$$

so that the kinetic energy $E$ can be neglected relative to $E_0^{\frac{1}{2}I}$. For this state we shall in addition neglect the level width relative to $E_0^{\frac{1}{2}I}$. The energy variation of the $u_l^{IJ}$ also depends on the level widths; the widths depend on $K^{2l+1}$, where $K$ is the momentum. Accordingly, we can write

$$\begin{gathered}u_0^{\frac{1}{2}\frac{1}{2}}\simeq\alpha q^{\frac{1}{2}}K^{\frac{1}{2}}/\mu c,\\u_1^{\frac{3}{2}\frac{3}{2}}=\beta\frac{q^{\frac{3}{2}}K^{\frac{3}{2}}}{\mu^3c^3}\frac{1}{1-E/E_R-i\rho},\end{gathered} \tag{17}$$

where the new dimensionless parameters $\alpha$ and $\beta$ defined by these equations are independent of the energy.

The differential cross sections of Eq. (14) can now be expressed entirely in terms of these two constants (in general complex), with the energy and angular dependence completely specified. The constants $\alpha$ and $\beta$ can be determined from experiment. It is interesting, however, to point out that they are given by meson theory and that their ratio is independent of the meson coupling constant. The weak coupling perturbation treatment, including the effects of the nucleon magnetic moments in nonrelativistic approximation gives

$$\alpha=-\frac{2}{\sqrt{3}\pi}ge,\quad\beta=+\frac{\sqrt{2}}{6}\frac{\mu}{M}(|\gamma_N|+|\gamma_P|)\alpha. \tag{18}$$

Here $M$ is the nucleonic rest mass. $\gamma_N$ and $\gamma_P$ are the respective values of the neutron and proton magnetic moments in units of $e\hbar/2Mc$, where $e$ is the charge on

[12] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).

a proton. The total cross sections, with these values of $\alpha$ and $\beta$, are

$$\sigma(\text{charged})=\frac{g^2e^2}{\pi}\frac{q}{K}\frac{\hbar^2}{\mu^2c^2}+\tfrac{1}{2}\sigma(\text{neutral}),$$

$$\sigma(\text{neutral})=\frac{g^2e^2}{\pi}\frac{q}{K}\frac{\hbar^2}{\mu^2c^2}\left\{\frac{4}{81}\frac{q^2K^2}{M^2\mu^2c^4}(|\gamma_N|+|\gamma_P|)^2\times\frac{1}{(1-E/E_R)^2+\rho^2}\right\}. \quad (19)$$

In Fig. 1 are plotted the cross sections (19), with $g$ chosen to fit the magnitude of the $\pi^+$ cross section to the observed value.[13] The contribution from the remaining spin and isotopic spin states was estimated by the method used[1] in connection with meson-nucleon scattering (i.e., by fitting at low energies the formal expression (1) to that obtained from perturbation theory). These further terms gave only a very small contribution to the cross sections.

We note that if we increase the nucleon magnetic moments by a factor of about 1.5 (corresponding to an energy dependence of the moments) we can make the neutral cross section larger than the charged near the resonance energy, in contrast to the predictions from classical meson theory.[4]

In Fig. (2) are plotted the predicted differential cross sections at 90° in the laboratory system. It is to be noted that the only arbitrary parameter used was $g$ [Eq. (19)], which was chosen to fit the magnitude of the total charged meson cross section at 255 Mev. The ratio of the two cross sections is given by the theory. The fit to the experimental data of Steinberger and Bishop[13] and of Silverman and Stearns[14] seems reasonable, considering that the static values of the nucleon magnetic moments were used.

The angular distribution of the $\pi^0$ mesons from the $J=\frac{3}{2}$ state is $5-3\cos^2\theta$ in the center-of-mass system. This term is also present for the charged mesons, but is less important because of the large $S$-state contribution. The interference of these two terms is such as to shift the peak at 90° to somewhat larger angles. This is in qualitative agreement with the angular distribution of Steinberger and Bishop,[13] but the resulting angular asymmetry seems to be somewhat less than they find. Use of the complete meson current interaction as given by meson theory would probably help in obtaining more precise agreement.[15] Because of the inherent crudity of the present model we have not considered such refinements of the theory (the effects of which are expected from meson theory to be small).

[13] J. Steinberger and A. S. Bishop, Phys. Rev. **86**, 171 (1952).
[14] A. Silverman and M. Stearns, Phys. Rev. **83**, 206 (1951).
[15] K. A. Brueckner, Phys. Rev. **79**, 641 (1950).

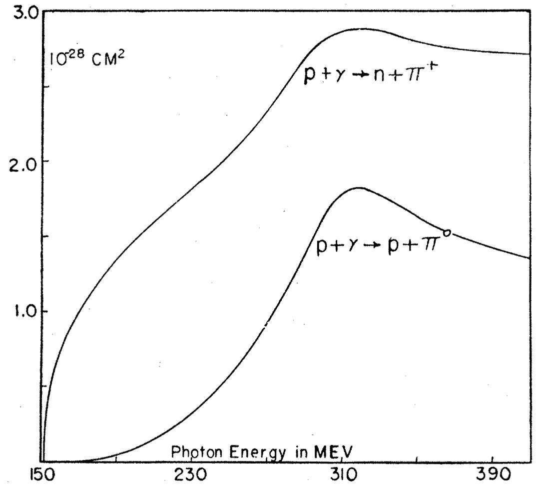


FIG. 1. The total cross sections as a function of photon energy in the laboratory system for the production of mesons from protons as predicted by the model. The only arbitrary parameter was a normalization factor used to fit the $\pi^+$ cross section at 255 Mev to that measured by Steinberger and Bishop.

## III. MESON PRODUCTION IN NUCLEON-NUCLEON COLLISIONS

We shall be particularly concerned with the reaction[16] $p+p\rightarrow\pi^++d$ and its inverse.[17] An analysis[18] of the experimental results indicated that the meson is emitted predominantly into $P$-states. From this it was concluded that the $^1S$ and $^1D$ states of the initial $p$–$p$ system contribute to the cross section. The angular distribution of the mesons (which looks approximately like $\cos^2\theta$) is then determined by the admixture of $^1S(J=0)$ and $^1D(J=2)$ states. The outgoing wave is then of the form[19]

$$(e^{iqr}/r)[Q_0+\eta Q_2{}^0]\sum\nolimits_0 \omega(1^+), \quad (20)$$

where $Q_0$ and $Q_2{}^0$ are those linear combinations of the spherical harmonics of order one and the deuteron wave function which correspond to states of angular momentum with $J=0$ and $J=2$, respectively. $\Sigma_0$ is the isotopic spin (singlet) part of the wave function of the nucleons and $\omega(1^+)$ is a symbolic factor in the wave function which implies that the meson is positive. $\eta$ is a complex parameter.

The formal requirements of charge independence for the processes of meson production in nucleon-nucleon collisions have been given previously.[18] These are expected to constitute an exceedingly severe test of the hypothesis of charge independence when sufficient experimental data are available. Aside from this, however, if the suggestion introduced[1] in connection with

[16] Cartwright, Richman, Whitehead, and Wilcox, Phys. Rev. **78**, 823 (1950); C. Richman and M. H. Whitehead, Phys. Rev. **83**, 855 (1951); Peterson, Iloff, and Sherman, Phys. Rev. **84**, 372 (1951).
[17] Durbin, Loar, and Steinberger, Phys. Rev. **84**, 581 (1951); Clark, Roberts, and Wilson, Phys. Rev. **83**, 649 (1951).
[18] K. Watson and K. Brueckner, Phys. Rev. **83**, 1 (1951).
[19] K. Watson and C. Richman, Phys. Rev. **83**, 1256 (1951).

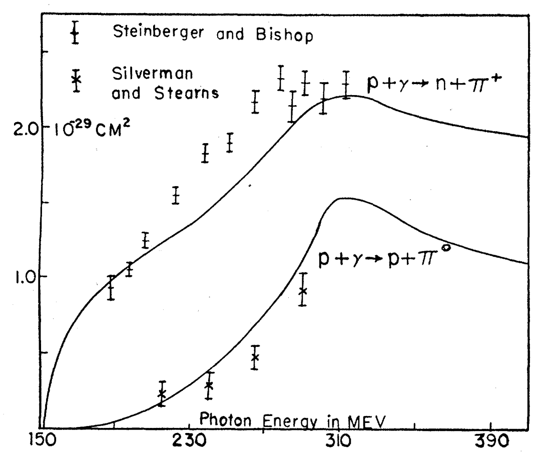


FIG. 2. The differential cross sections for the production of $\pi^+$ and $\pi^0$ mesons at 90° in the laboratory system. The only arbitrary parameter used was that of Fig. 1, which normalized the total cross section at 255 Mev to the observed value. The ratio of the two cross sections is given by the theory. The experimental data are from Steinberger and Bishop (reference 13) and Silverman and Stearns (reference 14).

meson scattering that the state ($J=\frac{3}{2}$, $I=\frac{3}{2}$) of a meson and single nucleon is a state of strong interaction, one would expect to find it also playing an important role here.

The situation is complicated by the presence of the second nucleon and consequently our model will not be so specific as before. We can expect, in general, nonlinear effects in the meson field which might tend to obscure the details of the individual meson-nucleon interaction. On the other hand, predominantly strong interactions, say in the $I=\frac{3}{2}$, $J=\frac{3}{2}$ state of the meson and either nucleon, would be expected to lead to anomalously large production cross sections from this state. To investigate such an effect in a qualitative manner, we can resolve Eq. (20) into eigenstates of angular momentum and isotopic angular momentum of the meson with respect to one of the nucleons. We use the symbols $j_1$, $j_2$ and $i_1$, $i_2$ to denote these respective quantities with respect to the nucleons "1" and "2" in the deuteron.

In considering only the reaction $p+p\rightarrow\pi^++d$ the isotopic spin properties will give us no information. However, the angular momentum considerations will give conditions on the angular distribution of the mesons.

We introduce the projection operators

$$E_{\frac{1}{2}}^{(i)}=-\tfrac{2}{3}[\mathbf{l}\cdot\mathbf{s}_i-\tfrac{1}{2}],\quad E_{\frac{3}{2}}^{(i)}=\tfrac{2}{3}[\mathbf{l}\cdot\mathbf{s}_i+1],\qquad(21)$$

where the index $i$ refers to nucleon "1" or nucleon "2". $\mathbf{l}$ is the orbital angular momentum of the meson with respect to either nucleon (assumed to a sufficient approximation to be the same as that with respect to the deuteron). $\mathbf{s}_i$ is the spin operator of the nucleon "$i$". $E_{\frac{1}{2}}^{(i)}$ is zero or unity, depending upon whether $j_i=\frac{3}{2}$ or $\frac{1}{2}$, respectively. $E_{\frac{3}{2}}^{(i)}$ is the corresponding projection operator for the $j_i=\frac{3}{2}$ state.

Returning to Eq. (20), we observe that

$$\begin{aligned}E_{\frac{1}{2}}^{(i)}Q_0&=Q_0,\quad E_{\frac{3}{2}}^{(i)}Q_0=0,\\ E_{\frac{1}{2}}^{(i)}Q_2^0&=0,\quad E_{\frac{3}{2}}^{(i)}Q_2^0=Q_2^0.\end{aligned}\qquad(22)$$

Thus if the $j=\frac{3}{2}$ state is one of strong interaction, one might expect the term $Q_2^0$ to be predominant in Eq. (20)—i.e., $\eta$ large. For strong interaction in the $j=\frac{1}{2}$ state only, one would expect $\eta$ to be small and the $Q_0$ term to contribute. The angular distribution from either of these terms alone is

$$\begin{aligned}Q_0&:\ \text{spherically symmetric},\\ Q_2^0&:\ 3\cos^2\theta+1.\end{aligned}\qquad(23)$$

The first possibility is in definite disagreement with experiment, so we conclude that the $j=\frac{1}{2}$ interaction is not predominant in this case. The second possibility is perhaps a little more spherically symmetric than is indicated by present experiments, but is roughly correct. We further note that $|\eta|$ can be taken of the order of 5 to 10 (with the right phase) to give quite satisfactory agreement with experiment. This then is consistent with the previous conclusion[1] of a strong interaction in the $j=\frac{3}{2}$ state—and may indeed be a first step in the explanation of the observed angular distribution.

## IV. CONCLUSIONS

A general analysis of the photomeson cross sections has been made on the assumption that only meson $S$- and $P$-waves are of importance. These have been given a simple form on basis of the joint hypotheses of charge independence and resonant interaction of the meson and nucleon in the $J=\frac{3}{2}$, $I=\frac{3}{2}$ state. Specific predictions are then obtained as to the energy and angular dependence of the cross sections. The agreement with present experimental measurements of these cross sections is reasonable (see Fig. 2).

These same considerations can be applied in a somewhat more qualitative manner to the production of mesons in nucleon-nucleon collisions. Here we obtain what may be a partial explanation of the angular distribution of $\pi^+$ mesons produced in $p$-$p$ collisions.