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Analysis of the Nucleonic Component Based on Neutron Latitude Variations*

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A "specific yield" function $S_A(E, x)$ is defined which gives the production rate of disintegration neutrons at atmospheric depth x arising from a unit flux of vertically incident cosmic-ray primaries of atomic weight A and energy-per-nucleon E. The functions are obtained by comparing the latitude variations for the primary particle fluxes with the latitude variations for neutron production at various depths in the atmosphere. For atmospheric depths between at least 200 and 600 g/cm², it is found that the specific yields increase rapidly with energy up to about 4-Bev/nucleon and then become insensitive to energy up to at least 12.7-Bev/nucleon. It is also shown that this insensitivity probably extends up to considerably larger energies. The theoretical implications of these results are qualitatively discussed.

I. INTRODUCTION

It is known from cloud chamber and photographic emulsion experiments that in energetic nuclear disruptions particles are emitted which are capable of producing further nuclear interactions. This cascade-like development of nuclear processes has been discussed phenomenologically by various authors, in connection with studies of the cosmic radiation.[1–5]

The radiations which produce nuclear disruptions in the atmosphere consist mainly of nucleons. Although recent experiments indicate that the cross section for the interaction of fast pions with nuclei is geometrical,[6–8] the process of decay can be assumed to remove all but the most energetic pions before nuclear interactions can occur with appreciable probability. Very low energy nuclear disruptions may be produced to some extent by photons and muons, but the total contribution from this source has also been shown to be small, at least below the transition maximum in the atmosphere.[5] The disruption-producing nucleons in the atmosphere constitute the nucleonic component of the cosmic radiation. In discussing the series of correlated nuclear processes generated by a single primary cosmic-ray particle, we will use the term *nucleonic chain.*

The development of a nucleonic chain can be pictured in the following way. In its first interaction with an air nucleus, an energetic primary nucleon typically produces several classes of particles: low energy evaporation fragments and disintegration nucleons; particles which decay before they can produce nuclear disruptions (except at very large energies, the bulk of the pions fall into this class); and fast particles (mainly nucleons) which are capable of producing further nuclear disruptions. As the energy of the last-mentioned particles decreases, the probability for meson production in subsequent interactions also decreases. In every nuclear process, however, evaporation and slow recoil nucleons (which we class together as disintegration nucleons) are produced, and these can be used as indicators of the development of the nucleonic chain.[3] From an energy point of view these particles represent the end products of the chain, and a knowledge of how the production rates of disintegration nucleons depend on primary particle energy should prove useful in understanding the processes involved in the nucleonic component.

In particular, we will concern ourselves here with the disintegration neutrons, for which fairly extensive experimental data are available. We introduce a "specific yield" function $S_A(E, x)$, which gives the production rate per gram of air of disintegration neutrons at atmospheric depth x arising from a unit flux of vertically incident cosmic-ray primaries of atomic weight A and energy-per-nucleon E.

For a limited range of primary particle energies, the specific yield functions can be obtained by comparing the experimentally observed neutron latitude variations in the atmosphere with the latitude variations for the fluxes of primary particles.

II. THE GEOMAGNETIC ANALYSIS

The earth's magnetic field determines a minimum value of $N \equiv p/Z$ for arrival of a primary particle from a given direction at the top of the atmosphere, where p and Z are, respectively, the total momentum and charge of the particle. It is therefore convenient to express the specific yields as a function of N. Let $N_v(\lambda)$ be the minimum value of N for arrival from the vertical direction at geomagnetic latitude λ. (Because of the eccentricity of the earth's dipole field, N_v depends slightly on longitude; this dependence is not explicitly written, but it will be taken into account in the calculations.)

Let $-dF_T/dN$ be the total (differential) flux of primary particles per unit solid angle which have

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[2] M. Conversi, Phys. Rev. **79**, 749 (1950).
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[4] G. D. Rochester and W. G. V. Rosser, *Reports on Progress in Physics* (Physical Society, London, 1951), Vol. XIV, p. 227.
[5] J. A. Simpson, Phys. Rev. **83**, 1175 (1951).
[6] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950).
[7] A. J. Hartzler, Phys. Rev. **82**, 359 (1951).
[8] Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. **82**, 958 (1951).

TABLE I. Absorption mean free path for neutron production in the atmosphere at geomagnetic latitude λ, for atmospheric depths between 200 and 600 g/cm^2.

λ	L (g/cm^2)
0°	212±4
19	206±4
40	181±3
53	157±2
65	157±3

momentum-to-charge ratio N (where $F_T(N)$ is then the integral flux). Let $f_A(N)$ be the fraction of these which have atomic weight A, and let $S_A(N, x)$ be the specific yield, i.e., the neutron production rate at atmospheric depth x due to a unit vertical flux of primary particles of atomic weight A and momentum-to-charge ratio N. The total production rate at depth x and latitude λ, due to the primaries arriving within a unit solid angle from the vertical direction, is given by the expression

$$R_v(\lambda, x) = -\int_{N_v(\lambda)}^{\infty} \sum S_A(N, x) f_A(N)(dF_T/dN)dN. \quad (1)$$

We define an average specific yield function $S(N, x)$ by the equation[9]

$$S(N, x) \equiv \sum S_A(N, x) f_A(N); \quad (2)$$

and from Eq. (1) we obtain the result

$$S(N, x) = dR_v(\lambda, x)/dF_T(N), \quad (3)$$

where

$$N = N_v(\lambda). \quad (3')$$

R_v is treated here as a function of F_T, λ and x being parameters. It is understood that R_v and N_v are evaluated at the same longitude.

The quantity R_v is referred to as the "vertical" neutron production rate. Experimentally, of course, one normally measures the production rate $R(\lambda, x)$ due to primary particles which arrive from all directions at the top of the atmosphere; R_v and R are related approximately by the Gross transformation

$$2\pi R_v = R - xdR/dx. \quad (4)$$

III. DETERMINATION OF THE AVERAGE SPECIFIC YIELD

A. Results at 312 g/cm^2

In a recent paper Simpson[5] has presented curves showing the altitude and latitude variations of disintegration neutron intensity in the atmosphere. The neutrons detected are in the energy range ~2–30 Mev, and it has been shown that they are mainly produced in nuclear disruptions induced by higher energy nucleons. In the equilibrium region of the atmosphere, where these measurements were performed, the observed neutron intensities are proportional to the actual production rates in the vicinity of the detection apparatus. Throughout this paper, neutron production rates will be measured in counting rate units appropriate to the Simpson altitude and latitude curves. For a discussion of the absolute production rates, reference may be made to a number of recent papers.[5,10,11]

For atmospheric depths between 200 and 600 g/cm^2, where the measurements were performed, the production rates are observed to vary with depth in an approximately exponential manner. The absorption mean free path $L(\lambda)$, however, depends on latitude, as shown in Table I.[5] In discussing the altitude variations of various quantities, in what follows, it will always be understood that we restrict ourselves to the range of depths between 200 and 600 g/cm^2.

Our analysis will be based on the latitude curve for atmospheric depth 312 g/cm^2 and geographic longitude 80°W (see Fig. 4 in reference 5). The "vertical" production rate R_v is obtained by applying the Gross transformation of Eq. (4). Because of the exponential variation of $R(\lambda, x)$, this can be written

$$2\pi R_v(\lambda, x) = R(\lambda, x)[1 + x/L(\lambda)]. \quad (5)$$

The function $L(\lambda)$ is obtained by drawing a smoothed curve through the experimental points given in Table I.

It should be noted that if we assume $R_v(\lambda, x)$ to depend exponentially on depth, with absorption mean free path $L_1(\lambda)$, then Eq. (4) leads to the solution

$$R(x) = 2\pi R_v(0)[\exp(-x/L_1) + (x/L_1)E_i(-x/L_1)], \quad (6)$$

where we have omitted, for the moment, the explicit dependencies on λ. For $x \gtrsim L_1$, the function $R(x)$ of Eq. (6) does not differ appreciably from an exponential and, within the experimental uncertainties, can be adjusted to fit the observed neutron altitude curves by a suitable choice of $L_1(\lambda)$—at least for the limited range 200–600 g/cm^2. For example, at small latitudes (0°–19°) the choice

$$L_0 \equiv L_1(0°\text{–}19°) = 272 \text{ g/cm}^2 \quad (7)$$

leads to an approximately exponential function for R, where the apparent absorption mean free path L has the correct value 209 g/cm^2 (see Table I). We will later make use of this fact, although not in a critical way, by assuming that R_v is precisely exponential and that R is given by the corresponding (approximately exponential) function of Eq. (6).

It should also be noted here that the approximations involved in the derivation of the Gross transformation can be shown to lead only to negligible errors for the application considered in this paper.

For the primary cosmic-ray spectrum we adopt the curve given by Winckler *et al.*[12] (see Fig. 6 in reference

[9] The "yield" Y, introduced in reference 5, is related to the average specific yield S by the equation $2\pi Y = \int S dx$.

[10] L. Yuan, Phys. Rev. **81**, 175 (1951).
[11] S. Lattimore, Phil. Mag. **42**, 331 (1951).
[12] Winckler, Stix, Dwight, and Sabin, Phys. Rev. **79**, 656 (1950).

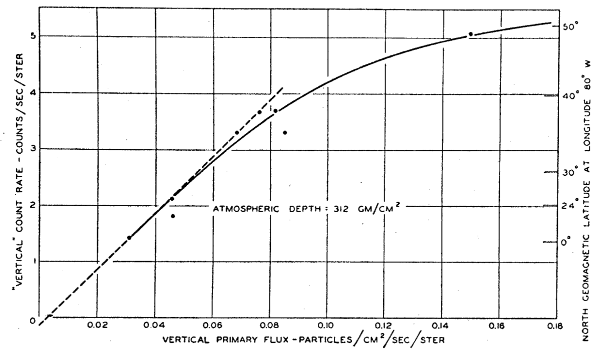

FIG. 1. "Vertical" neutron production rate at atmospheric depth 312 g/cm², plotted as a function of the total flux of primary particles which arrive from the vertical direction.

12). This curve, which includes the results of a number of investigators, gives the flux of particles with momentum-to-charge ratio greater than N, expressed as a function of the proton kinetic energy corresponding to N. The vertical cut-off value of momentum-to-charge ratio, $N_v(\lambda)$, has been given as a function of latitude for longitude 80°W by Neher.[13]

With latitude—or alternatively, the function $N_v(\lambda)$—serving as parameter, we can now obtain a plot of R_v (at 312 g/cm²) *vs* F_T. The result is shown in Fig. 1. The heavy points are the experimental flux values appearing on the Winckler curve. Two features of the $R_v - F_T$ curve should be noticed: (1) the curve approaches very closely a straight line for latitudes between 0° and about 24° (this corresponds to a range of N_v between 13.6 and 10 Bev/c); (2) the extrapolation of this straight line passes very close to the origin. Since the slope dR_v/dF_T gives the average specific yield, it appears that the latter is virtually independent of N in the interval 10–13.6 Bev/c. The significance of (2) will be discussed later. The slope dR_v/dF_T has been obtained graphically, and the result is shown in Fig. 2, where S is plotted as a function of N. The points shown on this curve represent the results of several determinations of the slope and give an indication of the purely graphical errors involved.

B. Discussion of Errors

The experimental errors in the curves of Figs. 1 and 2 are due chiefly to uncertainties in the primary spectrum. We are mainly interested in evaluating these errors for values of N in the range 10–13.6 Bev/c, where S appears to become independent of N. Between about 3 and 15 Bev/c the Winckler spectrum is well represented by the expression

$$F_T(N) = KN^{-1.0}, \tag{8}$$

where $K = 0.42$ within about 2 percent. (For smaller values of N the exponent decreases with decreasing N.) Vidale and Schein,[14] using a counter telescope, have recently determined the primary spectrum for values of N between 1.7 and 9.1 Bev/c. They find a power law with an exponent 1.0 ± 0.1 in excellent agreement with the Winckler exponent. Neher,[15] by the method of integrating ionization *vs* altitude curves, has obtained a spectrum which can be approximately represented by a power law for N between about 4 and 15 Bev/c; the exponent has the approximate value 1.1. This is considered to be in good agreement with the Winckler exponent, although the absolute flux values obtained by the ionization method are considerably smaller than the values obtained by direct measurements at the top of the atmosphere.

We are not concerned with the absolute flux of primary particles, which is subject to rather large experimental error, but rather with the shape of the primary spectrum curve. It can be shown that an error $\Delta\gamma$ in the exponent of Eq. (8) would lead to a difference ΔS between the values of S at the extremes of the interval 13.6–10 Bev/c, which is given by

$$\Delta S/S = 0.31\Delta\gamma.$$

Thus, even for an error as large as $\Delta\gamma = 0.2$, S would increase only by about 6 percent in going from 10 to 13.6 Bev/c.

It is probably correct, therefore, to conclude that the average specific yield function is at least very insensitive to N in the interval 10–13.6 Bev/c. For purposes of discussion we will refer to it as being constant. The choice of 10 Bev/c to mark the lower limit of the region of constant S is, of course, slightly arbitrary.

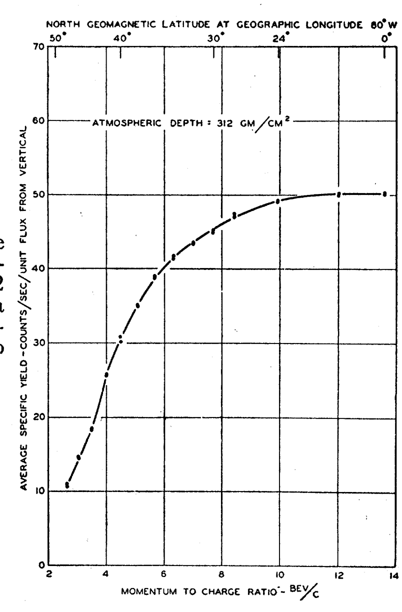

FIG. 2. Average specific yield at atmospheric depth 312 g/cm², plotted as a function of momentum-to-charge ratio N.

[13] H. V. Neher, Phys. Rev. **78**, 674 (1950).

[14] M. L. Vidale and M. Schein, Nuovo cimento **8**, 774 (1951).

[15] H. V. Neher, Phys. Rev. **83**, 649 (1951).

C. Results at Other Atmospheric Depths

The results thus far refer only to atmospheric depth 312 g/cm². For other depths in the range 200–600 g/cm², the shape of the average specific yield curve can be inferred from Table I. The absolute value of S for any value of N will, of course, decrease with increasing depth, and the relative shape of the curve will also, in general, change with depth. Thus, for high latitudes $L(\lambda)$ decreases rapidly with increasing λ; consequently, for small values of N the average specific yield will fall off the more rapidly with decreasing N, the greater is the depth.

However, for small latitudes $L(\lambda)$ is almost independent of λ; in fact, the values at 0° and 19° agree within the experimental errors. We will not be too greatly in error, therefore, in assuming that $L(\lambda)$ is independent of λ up to 24°. The constancy of S in the interval 10–13.6 Bev/c (which corresponds to the latitude interval 0°–24°) therefore holds for all depths, at least between 200 and 600 g/cm².

We denote by $S_0(x)$ the value of S in the interval 10–13.6 Bev/c. The altitude variation of $S_0(x)$ depends, through Eq. (3), on the altitude variation of R_v. The latter, as we have seen, can be taken to be an exponential function, where for small latitudes (0°–24°) the absorption mean free path has the value L_0 given by Eq. (7). The absorption mean free path for $S_0(x)$ is also, therefore, given by L_0.

TABLE II. Composition of primary radiation at small latitudes (0°–24°).

	Percentage of primary nuclei	Percentage of primary nucleons
Protons	87	58
Helium	12	32
C, N, O	0.64	6.0
$Z\geqslant 9$	0.19	3.6

IV. THE INDIVIDUAL SPECIFIC YIELDS

In a recent review article, Peters[16] has summarized the experimental information on the spectra of the heavy particles ($A>1$) in the primary cosmic radiation. When expressed as a function of N, the heavy particle spectra are found to have approximately the same form as the total primary spectrum given by Winckler, at least up to 15 Bev/c. (Beyond this value the Winckler spectrum is based on an uncertain extrapolation to the very high energy spectrum of Hilberry, so that no attempt is made here to compute the relative composition of the primary radiation above 15 Bev/c.) Between 3 and 15 Bev/c the fraction of heavy particles in the primary radiation changes only by about 12 percent, the fraction of heavy particles decreasing with increasing N. It is possible that this slow variation lies within the experimental errors. Between 10 and 13.6 Bev/c (0°–24°) the fraction of heavy particles appears to be almost completely independent of N. A breakdown of the composition of the primary radiation in this interval is given in Table II.

For the interval 10–13.6 Bev/c, therefore, we can take the fractions f_A to be independent of N. But for atmospheric depths between at least 200 and 600 g/cm², the average specific yield function has also been shown to be independent of N in this interval. If we assume that the individual specific yield functions S_A do not increase with increasing N for some species of primary particles and decrease for other species, then it follows from Eq. (2) that the specific yields are individually independent of N in the interval 10–13.6 Bev/c. The corresponding ranges of kinetic energy are 9.1–12.7 Bev for protons and 4.1–5.9 Bev/nucleon for heavy primaries ($A=2Z$).

We can see now the significance of the fact that the straight-line portion of the R_v-F_T curve extrapolates very close to the origin (Fig. 1). For the interval 10–13.6 Bev/c, Eq. (3) can be integrated to give

$$R_v(\lambda)=S_0F_T(N)_{N=N_v(\lambda)}+\text{constant}, \qquad (9)$$

where the constant of integration is nearly vanishing. If we assume that the fractional composition of the primary radiation remains independent of N for $N>13.6$ Bev/c, then it follows from Eqs. (1) and (9) that the specific yields also remain independent of N above 13.6 Bev/c. Actually, the composition of the primary radiation at large N is not known with any accuracy, and it is possible that the specific yields and the fractional fluxes f_A both vary with N in such a way as to accidentally produce the almost vanishing value observed for the constant of integration in Eq. (9). However, if we rule out a sudden large change in the composition of the primary radiation just above 13.6 Bev/c, we can probably correctly conclude that the specific yields remain fairly insensitive to N up to values considerably larger than 13.6 Bev/c. However, nothing can be said about the behavior of the specific yields at very large values of N, where the flux of primary particles is in any case small and therefore unimportant in contributing to neutron production in the atmosphere.

V. COMPARISON OF PROTON AND HEAVY PARTICLE SPECIFIC YIELDS

The results of the preceding discussion were obtained directly from the experimental data on neutron and primary particle latitude variations, without reference to any particular assumptions on the properties of nucleonic chains. It was shown that for atmospheric depths between at least 200 and 600 g/cm², the production rate of disintegration neutrons arising from a unit vertical flux of primary particles becomes insensitive

[16] B. Peters, *Progress in Cosmic Ray Physics* (North Holland Publishing Company, Amsterdam, 1952).

to primary energy in the ranges 9.1–12.7 Bev for protons and 4.1–5.9 Bev/nucleon for heavy primaries. It was also shown that this energy insensitivity probably extends up to considerably larger energies.

Because of their large collision mean free paths, the residue of heavy primary nuclei below 200 g/cm² is extremely small. The heavy primaries contribute to the neutron production at large depths through their break-up into fast nucleons in the upper parts of the atmosphere, these nucleons serving as the primary particles for the development of nucleonic chains. (Fragments of atomic weight greater than unity also result from the break-up, but these, in turn, yield smaller fragments and free nucleons; and after a very few collisions the break-up into nucleons is completed.) There must exist then a close relationship between the specific yield functions for heavy particles and for free nucleons. In the absence of a reliable theory of energetic nuclear interactions, however, it is not possible to deduce the exact nature of this relationship. Thus, we have seen that the specific yield functions do not depend appreciably on primary energy in the ranges 9.1–12.7 Bev for protons and 4.1–5.9 Bev/nucleon for heavy primaries, but the relative numerical values of the specific yields cannot be determined from the empirical analysis.

However, on the basis of fairly plausible assumptions on the properties of energetic nuclear interactions, we can establish the result that the free nucleon specific yield must become insensitive to energy in the same range where the heavy particle specific yields are energy-insensitive, i.e., that the proton specific yield is actually insensitive to energy down to 4.1 Bev.

Consider the interaction of an air nucleus with an incident primary particle of atomic weight A and energy-per-nucleon E. We fix our attention on a particular nucleon in the incident particle and view the interaction, for the moment, in the rest system of the incident particle. The following possibilities occur: (1) The nucleon may have its bond broken and emerge as a low energy evaporation or recoil particle. (2) It may emerge as part of a low energy fragment of atomic weight A' greater than unity. (3) It may suffer a direct nucleon-nucleon collision and receive a considerable amount of energy, in which case it is fairly certain to emerge as a free nucleon. As viewed in the laboratory system, the nucleon emerges in processes (1) and (2) with essentially the same velocity as that of the primary particle, i.e., with energy E. In the case of (3) the nucleon has appreciably smaller energy. Let p_{A1} and $p_{AA'}$ be the respective break-up probabilities, averaged over all values of the collision parameter, for processes (1) and (2); and let $p_A(E,E')dE'$ be the probability for process (3), where the nucleon emerges with energy E' in the interval dE'. If Λ_A is the collision mean free path for the heavy primary particle in air, then the specific yield at depth x is given by

$$S_A(E,x)=\Lambda_A^{-1}\int_0^x \exp(-x'/\Lambda_A)dx' \times\left[\sum_{A'} p_{AA'}S_{A'}(E,x-x')+p_{A1}S_p(E,x-x') +\int_0^E p_A(E,E')S_p(E',x-x')dE'\right], \quad (10)$$

where $S_p(E,x-x')$ is the specific yield at depth x due to a unit flux of nucleons of energy E produced at depth x'. We do not distinguish here between the fast neutrons and protons which emerge from the break-up of the heavy particle, although the latter, of course, immediately begin to lose energy by ionization. This may be justified by observing that for the large energies which we are considering (several Bev and greater), the ionization loss per collision mean free path is small compared to the proton kinetic energy. After several collisions, when the energy is sufficiently degraded, ionization loss becomes important; but because of the possibility of charge exchange, and because of the contribution to the nucleonic chain from both charged and uncharged recoils, the charge distribution among the particles of the chain should not depend strongly, after several collisions, on the state of charge of the primary nucleon.

As we have previously seen, the heavy particle specific yields become independent of energy above 4.1 Bev/nucleon (and below an upper limit which is probably much larger than 5.9 Bev/nucleon and which we take to be larger than 9.1 Bev/nucleon). If we assume that the collision mean free path Λ_A and the break-up probabilities $p_{AA'}$ and p_{A1} are independent of energy, then it follows that the sum of the last two terms on the right-hand side of Eq. (10) must become independent of energy in this interval. This result must hold for a considerable range of depths—at least between 200 and 600 g/cm². It seems reasonable to conclude, therefore, that the last two terms in Eq. (10) are individually independent of energy above 4.1 Bev, unless we suppose that there occurs an accidental cancellation of energy and spatial dependencies. It appears then that the proton specific yield S_p, which we have previously shown to be energy-insensitive above 9.1 Bev, actually remains insensitive to energy down to 4.1 Bev.

It is apparent from Fig. 2 that for energies much below 4.1 Bev/nucleon, the proton and heavy particle specific yields both fall off rapidly with decreasing energy.

The details of the collision process are compressed into the break-up probabilities $p_{AA'}$ and p_{A1} and the distribution function $p_A(E,E')$. (For complete generality, the latter should include the fast recoil nucleons coming from the target nucleus.) An upper limit for the heavy particle specific yields, relative to the proton specific yield, can be obtained by neglecting the last term in Eq. (10), i.e., by assuming that the collision with an air nucleus merely serves to break up the incident heavy particle into free nucleons and fragments having the same energy per nucleon as the incident particle. We have seen that in the energy-insensitive region the specific yields vary with depth in an approximately exponential manner, with absorption mean free path L_0. In this range of energies, therefore, an upper limit on S_A is obtained by setting these exponential functions into Eq. (10) and observing that for large depths ($x \gtrsim 200$ g/cm^2), $\exp(-x/L_0) \gg \exp(-x/\Lambda_A)$. We obtain the result

$$S_A < (1-\Lambda_A/L_0)^{-1}[\sum p_{AA'}S_{A'} + p_{A1}S_p]. \quad (11)$$

VI. DISCUSSION

Simpson[17] has shown that disintegration neutrons in the atmosphere are produced mainly in small stars (nuclear bursts) and that the production rates of neutrons and small stars are substantially in equilibrium, at least below 200 g/cm^2; i.e., within the experimental errors both show the same latitude and altitude variations. We can therefore redefine our specific yield functions to refer to the production of small stars without changing any of our results.

Our conclusions on the energy-insensitivity of the specific yield functions above 4.1 Bev/nucleon followed essentially from the fact that the neutron and small star latitude variations are proportional to the latitude variation of primary particle intensities in the interval 0°–24°. It would obviously be of interest to test whether or not this is also true for the production of large stars.

A question of considerable importance in the study of the nucleonic component is that of the role played by mesons. In the interaction of an energetic primary nucleon with a nucleon in an air nucleus, a pair of recoils and possibly one (or more) pions are produced, where we refer to both the target and incident nucleons as recoils after they emerge from the collision. The recoils can presumably suffer further collisions within the same nucleus, and in each elementary act, pion production presumably occurs with a probability which depends on the energies involved. If we restrict ourselves to primary energies below $\sim$12.7 Bev, then because of the large probability for decay, the pions which are produced will not contribute appreciably to the further development of the nucleonic chain once they leave the air nucleus. On the other hand, a pion produced in an elementary act might be expected to interact within the same nucleus, producing fast recoil nucleons which can contribute to the chain.

Camerini *et al.*[18] have shown that pions of energy less than 1 Bev which are incident on the nuclei of photographic emulsions emerge with greatly reduced energy in about 50 percent of the cases. (In the remaining cases it is not possible to determine whether complete absorption or charge exchange has taken place.) This result, taken together with the evidence previously cited for a large interaction cross section between pions and nuclei, indicates a strong interaction of pions with nuclear matter. It has also been shown,[18] however, that the average number of "black" and "grey" prongs emerging from nucleon-produced stars increases only very slowly with increasing number of shower particles. If the shower particles, which consist mainly of pions, were strongly interacting within the nucleus in which they are created, one would expect to find a strong dependence of "black" and "grey" prongs on the number of shower particles. From this evidence, therefore, it appears that pions do not contribute appreciably to the production of recoil nucleons in the parent nucleus.

Our specific yield results lead to a similar conclusion. For primary proton energies between 4.1 and at least 12.7 Bev, the production rate of small stars at large atmospheric depths becomes insensitive to primary energy. On the other hand, the production of pions in the atmosphere is known to increase rapidly with increasing primary energy.[18,19] If the pions contributed appreciably to the nucleonic chains, the specific yield would also be expected to increase appreciably with primary energy. Therefore, despite the evidence of a strong interaction with nuclear matter for single, fast pions incident on nuclei, it seems to be necessary to conclude that pions do not contribute appreciably to the development of nucleonic chains, at least for primary energies below 12.7 Bev.

Theoretical treatments of the nucleonic component, in which the development of nucleonic chains is traced down to small energies, have been given by several authors.[20–22] For large energies (greater than several Bev) the calculations are based on the Heitler-Janossy theory,[23] in which it is assumed that (1) the contribution of mesons to the nucleonic component can be neglected; (2) the interaction of a nucleon with a nucleus can be decomposed into a series of independent nucleon-nucleon collisions which develop within the nucleus in a cascade-like series of events; (3) the total nucleon-nucleon collision cross section is independent of energy; (4) the distribution function for the recoil nucleon energies is a homogeneous function of the primary energy; (5) the angular deviation of the recoils with respect to the direction of the primary nucleon can be neglected.

In detailed calculations based on the above model, Messel[22] has obtained fairly good agreement between

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[19] K. Sitte, Phys. Rev. **81**, 484 (1951).
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[22] H. Messel, Phys. Rev. **83**, 21 (1951); **83**, 26 (1951).
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theory and experiment for the latitude variations of small stars. This must be considered to be spurious, however, since the primary proton energy spectrum adopted by Messel (integral power law exponent = 1.7) is in serious disagreement with the spectrum deduced from the data of Winckler and Peters (exponent ≈ 1.0); and furthermore, the presence of heavy particles in the primary radiation has been neglected. Actually, the results obtained by Messel predict a specific yield function for small star production which depends strongly on primary proton energy, even for energies above 4.1 Bev.

Instead, we have found that the proton specific yield at large atmospheric depths increases rapidly with energy up to about 4 Bev but then becomes relatively insensitive to energy up to at least 12.7 Bev. If one retains all of the assumption of the Heitler-Janossy theory, with the exception of (4), then in order to explain our results it is necessary to suppose that the energy distribution of the recoil nucleons becomes insensitive to primary energy in the range from 4.1 to at least 12.7 Bev. This would imply that nucleon-nucleon collisions are on the average fairly elastic at low energies but that the elasticity begins to fall off rapidly with increasing energy in the vicinity of 4 Bev, owing, presumably, to a strong increase in meson production. The fraction of cases in which a proton-nucleus interaction leads to meson production is indeed known to increase rapidly with proton energy up to about 4 Bev, beyond which energy nearly every interaction results in meson production.[18] However, the above description of nucleon-nucleon interactions requires further that the energy carried away by the mesons increase very rapidly with primary energy above 4.1 Bev (in order that the energy remaining to the recoil nucleons be an insensitive function of primary energy). A test of this possibility must await further experiments.

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Phenomenological Relationships between Photomeson Production and Meson-Nucleon Scattering*

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Photomeson production is studied on the basis of the hypothesis of charge independence combined with the suggestion from meson theory that the meson-nucleon state of angular momentum and isotopic angular momentum equal to $\frac{3}{2}$ is one of strong, resonant interaction. Reasonable agreement with experiment is obtained, both as to the energy dependence of the cross sections and as to their ratios. Similar considerations lead to a possible explanation for the angular dependence of π^+ mesons produced in p–p collisions.

I. INTRODUCTION

Our present theoretical understanding of meson-nucleon interactions is based upon general considerations of parity and angular momentum and upon the more detailed and less reliable models of field theory. The latter have been, perhaps, surprisingly successful in many cases in predicting orders of magnitude and qualitative features of the interactions of mesons and nucleons. One of the most interesting predictions of meson theories is that meson-nucleon interactions involve relatively few states of angular momentum. This is consistent with present experimental evidence for meson-nucleon processes. However, the lack of quantitative agreement with experiment has led one to the conclusion that (among other difficulties) neither the weak nor the strong coupling limit for meson theories is correct. In view of these shortcomings of explicit models, it seems desirable to fall back on more general phenomenological analyses of the experimental data with the hope of establishing contact with meson theory where possible.

An analysis of meson nucleon scattering along these lines was made by one of us.[1] To the usual requirements of conservation of angular momentum and parity was added that of conservation of "isotopic angular momentum."[2] The suggestion of strong P-state interactions between mesons and nucleons was borrowed from meson theory and from strong-coupling theory the notion of nucleon isobars, as well as their classification in terms of spin and isotopic spin. The resulting analysis

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