TABLE V. Location of showers within the chamber.

Plate No.	Material	Thickness g/cm²	No. showers
1 (top)	C	3.67	132
2	C	3.47	113
3	Pb	15.5	204
4	C	3.29	78
5	C	3.40	64
6	C	3.46	55
7	Pb	19.0	191

a mean free path for production of 76 ± 8 g/cm². This value is probably slightly too high because showers in lead have more secondaries and are possibly recorded with greater efficiency.

DISCUSSION

From the comparison of showers starting in carbon, iron, and lead it seems apparent that in the more energetic showers secondary multiplication occurs inside of the struck nucleus. Not many of the details of the multiplication process can be derived from these experiments. The number of particles formed in the initial encounter of the primary with the nucleus is not known. However, the fact that a large fraction of the showers starting in hydrogen were missed would indicate that the average number of fast particles produced in a single nucleon-nucleon encounter must be smaller than the number observed in the average shower from carbon in this experiment (about 5). The

ratio of the normalized rates for large showers of eight or more penetrating particles is 4:1 for lead as compared to carbon and is 2:1 for iron as compared with carbon. This indicates that in a large fraction of the showers in iron the cascade process has not gone to completion. This fact seems to indicate that the secondary particles which produce the later multiplication have a long mean free path in nuclear matter for doing so.

Practically no elastic collisions occurred in carbon in which a small amount of energy was transferred. This would seem to indicate that whenever a primary particle encounters a nucleus a shower is produced with high probability and there is only very small probability of a collision with small energy transfer.

ACKNOWLEDGMENTS

The Liquid-Solid Transformation in Helium from 1.6° to 4°K

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The transformation from liquid to solid helium behaves quite differently above and below the intersection of the λ-point-pressure curve with the melting pressure curve. The experimental data given here extend previous work from below 1.8° to about 4°, and show the effect of the λ-transformation on the latent heat of melting, the change in molar volume on melting, and the melting pressure curve. When these data are combined with a new determination of the λ-point-pressure curve, the indication seems to be that the λ-transformation is not truly of second order; that is, that it does not obey the Ehrenfest equations.

I. INTRODUCTION

THE phase diagram for the liquid and solid phases of helium, shown in Fig. 1, is quite unique in its general characteristics. Helium normally remains a liquid to absolute zero, with the vapor and solid phases never coexisting. However, upon the application of moderate pressures to the liquid helium at these very low temperatures, solid helium is formed, and the liquid-solid equilibrium curve is shown in Fig. 1. At higher temperatures, a "triple point" is found between solid helium, liquid helium I, and liquid helium II. It is not a triple point in the usual sense of the word, since

the helium I–helium II transformation (called the λ-transformation) is believed to be of the second order, and, as a consequence, the slope of the melting pressure curve should be continuous, with a discontinuous second derivative.

Since the properties of solid helium are quite regular, the changes in the properties of liquid helium at the λ-point should be reflected quantitatively in the experimental values of the molar change in volume on melting $(\Delta V = V_L - V_S)$, the heat of melting $[\rho = T(S_L - S_S)]$, and the melting pressure curve. A detailed study of these quantities in the region of the "triple point"

should give a bit more insight as to the nature of the λ-transformation in liquid helium, and also should furnish additional opportunity to check the applicability of the Ehrenfest equations, which, presumably, are obeyed in this case.

The liquid-solid transformation in helium has been investigated in some detail below 1.8°,[1,2] with the emphasis placed on the region near 1°K. The thermodynamics of the "triple point" were also discussed qualitatively in this work, although the only available melting data for the region above 1.8° were doubtful, since they had to be extrapolated from 2.2°, and showed deviations from the Clausius-Clapeyron equation above 3°. Thus, it seemed worth while to extend the work from below 1.8° to higher temperatures, both to investigate the effect of the λ-transformation on the melting phenomenon, and also to investigate the inconsistencies in the data above 3°.

The techniques used were similar to those described previously and will be outlined in the section on experimental details. In the past, melting pressure curves at low temperatures have been determined almost invariably through the use of the blocked capillary technique.[3] This technique has been open to criticism, mostly owing to the fact that pressure gradients exist across the sample and could give melting pressures that are too low.[4] In addition to these objections, preliminary experiments showed that in this work, at any rate, supercooling effects became very troublesome, so that it was hard to reproduce points accurately. A version of the piston displacement method used by Bridgman and others was found to be more satisfactory and was adopted for these experiments.[5]

In essence, this method consisted of determining the temperature at which the liquid-solid transformation took place under constant pressure in an isolated cryostat. The helium in the cryostat was connected directly to a pressure balance (dead-weight piston gauge) by means of an oil transmitter,[6] and the temperature of the cryostat was slowly decreased. At the melting point the increase in the density of the helium on solidification caused the pressure balance piston to drop slowly, while the temperature of the cryostat remained constant. Thus, supercooling effects were avoided, and by cooling very slowly, readings could be taken in a leisurely manner. The response of the temperature of the isolated cryostat to changes in pressure was very rapid.

The ΔV and ρ measurements were performed in exactly the same manner, except that they involved warming up a vessel full of solid helium, rather than cooling liquid helium. One run could in principle give
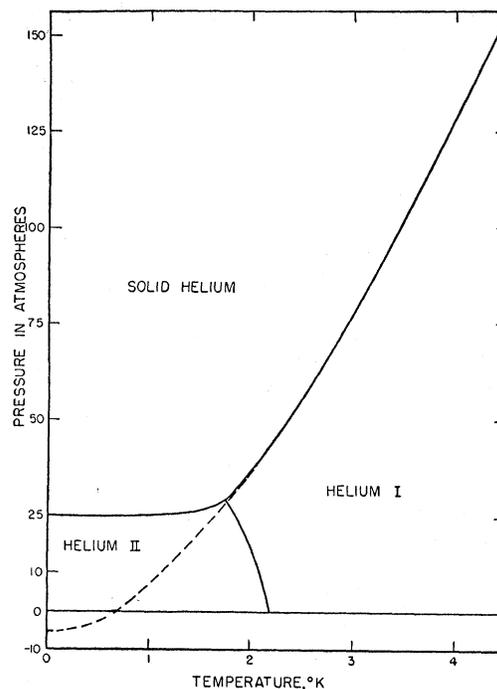

FIG. 1. The phase diagram of helium below 4°K.

all three pieces of data, but, in fact, the experimental difficulties increased markedly from P vs T to ΔV vs T to ρ vs T, and the number of points taken decreased in this order.

The above measurements cast some doubt as to the coordinates of the "triple point" as given by earlier workers,[7] so the λ-point-pressure curve was redeter-
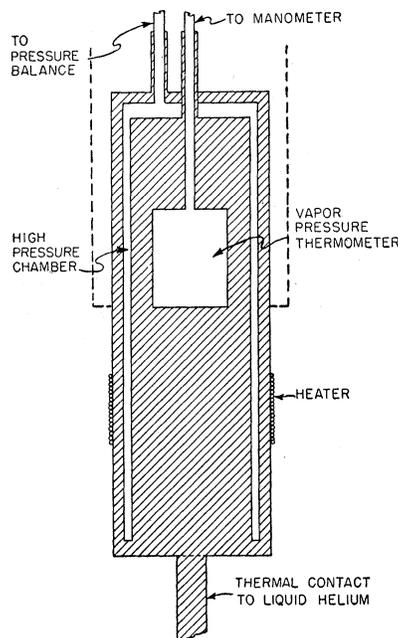

FIG. 2. A schematic diagram of the cryostat used.

[1] F. E. Simon and C. A. Swenson, Nature 165, 829 (1950).
[2] C. A. Swenson, Phys. Rev. 79, 626 (1950).
[3] W. H. Keesom, Helium (Elsevier, Amsterdam, 1942), p. 180.
[4] P. W. Bridgman, Revs. Modern Phys. 18, 28 (1946).
[5] P. W. Bridgman, The Physics of High Pressure (G. Bell, London, England, 1949), p. 190.
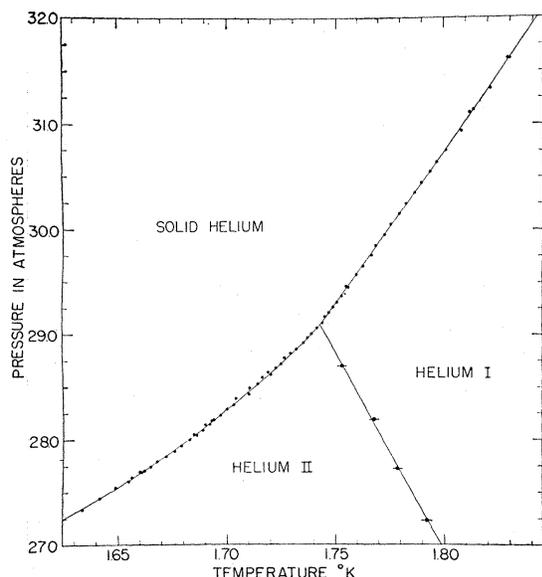[6] C. A. Swenson, Rev. Sci. Instr. 21, 22 (1950).
[7] Reference 3, p. 225.

FIG. 3. The melting pressure and λ-point-pressure data in the neighborhood of the "triple point."

mined to complete the phase diagram. The method used was essentially the determination of the temperature at which the thermal conductivity of liquid helium under constant pressure increased suddenly as the temperature was decreased. This information was necessary to determine the slope of the λ-line at the upper "triple point," and to check the other data for their consistency.

II. EXPERIMENTAL DETAILS

The cryostat used for these measurements was unusual enough to be described in some detail, whereas the remainder of the apparatus was more or less standard. The main features of the cryostat, which was mounted in a vacuum jacket, are shown in Fig. 2. It consisted of two separate sections: a high pressure chamber and a vapor pressure thermometer bulb. A vapor pressure thermometer could be used directly for this work, since all measurements were made at a constant equilibrium temperature. Vapor pressures were measured directly using either a mercury manometer or an Apiezon oil manometer, and temperatures were calculated using the 1949 Cambridge scale.

The top of the vacuum jacket was maintained at a temperature slightly greater than 4°K, so that a temperature gradient always existed down the thin stainless steel high pressure (0.3-mm i.d.) and vapor pressure tubes (2-mm i.d.). The bottom half of the vacuum jacket was immersed in a bath of liquid helium, the temperature of which could be varied from 4° to about 1.4°K. The cryostat itself, which was constructed entirely from copper to minimize temperature gradients, was connected to the bottom of the vacuum jacket by a thin copper rod, and its rate of cooling could be controlled by varying the temperature of the helium bath.

The annular high pressure space was 1 mm wide and

15 cm long, with a total volume of about 6 cc. The object of this type of construction was twofold: first, to have as large a ratio of surface area to volume as possible, since solid helium is a relatively poor heat conductor; and secondly, to surround the vapor pressure thermometer completely with a relatively constant temperature. The actual success of this design is shown in Fig. 3, where points taken at about 30-atmos pressure are shown, reproducible (but not accurate) to 0.01 atmos and 0.001°K. These points represent several runs made with different diameter high pressure capillaries, two different size copper connections to the helium bath, and various rates of cooling.

The relative values of melting pressures are accurate to within about 0.01 atmos, since the pressure balance was able to reproduce pressures this well from run to run. Two different pistons were used: one of 0.9828 sq cm area for pressures up to 45 atmos, and the other, 0.3296 sq cm area, for pressures up to 145 atmos. The pressures were calculated from the weights used and the areas of the pistons, and the calibrations were checked against the vapor pressure of carbon dioxide at 0°C.[8] The agreement was closer than one tenth of one percent.

The heater on the outer surface of the cryostat was used to ascertain that a reversible freezing process was being observed. If the temperature of the cryostat remained constant with the pressure balance piston dropping slowly, then, presumably, solidification was taking place. To verify this, the addition of heat to the helium should cause only a reversal of the piston motion and have no effect on the temperature. This was checked in each case.

For the ΔV measurements, the cryostat was first filled with solid helium, after which heat was applied and the piston kept at constant height by bleeding gas through a needle valve at room temperature into a calibrated volume. The two main sources of error here were incomplete filling of the cryostat, which could be checked as above, and the natural leak rate of the oil past the pressure balance piston. The annular shape of the high pressure space and the fact that all the cooling was from the bottom helped to reduce the filling factor error. The second error depended directly on the time needed for melting, and, fortunately, ΔV depends so slightly on pressure that rather rapid rates of warming could be used. These two sources of error are perhaps equal in magnitude, and set the limit of accuracy.

The heat-of-melting measurements involved a modification of the cryostat shown in Fig. 2. The copper connection to the helium bath was removed, and a second vacuum jacket was added inside the original one (see the dotted lines in Fig. 2). The heat of solidification was removed from the cryostat by a few mm pressure of helium gas in the outer vacuum jacket, while a good vacuum in the inner one kept the high pressure tube from being blocked. This exchange gas

[8] O. C. Bridgeman, J. Am. Chem. Soc. 49, 1174 (1927).

TABLE I. The smoothed thermodynamic functions for the liquid-solid transformation in helium between 1.6° and 4.0°K. For data below 1.6°, see reference 2, and for data above 4.0°, see reference 13.

T (°K)	P(atmos)	dP/dT (atmos/°K)	ΔV (cc/mol)	ρ(calc, cal/mol)
4.0	128.6	56.0	1.03	5.56
3.5	102.0	51.0	1.10	4.75
3.0	78.1	46.5	1.19	4.02
2.5	56.3	41.6	1.29	3.26
2.0	37.10	34.15	1.41	2.35
1.9	33.90	...	...	...
1.8	30.76	30.0	1.45	1.80
1.75	29.31	29.0	1.48	1.82
1.72^5	28.75	18.7	1.52^5	1.19
1.70	28.30	16.6	1.60	1.09
1.6	27.01	10.2	1.89	0.75

was then pumped out, and, with the outer bath at roughly the melting temperature, the amount of heat needed to melt the solid at constant pressure was measured. A ΔV measurement, as outlined above, was made simultaneously. The oil leak past the piston does not affect the accuracy of the heat of melting measurement, so the main error here is in the filling factor. It cannot be eliminated completely.

III. RESULTS

The melting pressure data fall roughly into two groups, above and below 2.1°K, and the agreement with earlier and less precise work is, in general, satisfactory.[2] The 22 experimental points in the high temperature group could be represented within 0.1 percent by the equation,

$$p + 5.6 = 13.458T^{1.66074} \text{ atmos.} \tag{1}$$

This curve is shown in Fig. 1, together with its extrapolation to absolute zero.

Figure 1 shows that below 2.1° the melting pressures begin to fall off less rapidly, and the data did not fit an empirical curve of the above type. These data, representing about 65 separate points, are tabulated in column 2 of Table I as a smooth curve. The reproducibility of these measurements is shown in Fig. 3, where the vicinity of the upper "triple point" is shown in some detail. Whereas a gradual change in curvature begins at about 2.1°, an abrupt change takes place at 1.743°, and this temperature corresponds almost exactly with the termination of the λ-point-pressure curve on the melting pressure curve.

The complete λ-point-pressure curve, as redetermined in these experiments, is given in Fig. 4. The agreement with earlier measurements is quite good, except for the point of intersection, which was, according to Professor Clusius, obtained by extrapolation from the other points. These data are summarized in the smoothed curve of Table II, and the coordinates of the upper triple point can be given as

$$P_\lambda = 29.12(\pm 0.05) \text{ atmos}, \quad T_\lambda = 1.743(\pm 0.003)°K.$$

The less accurate values given in reference 2 were based

almost entirely on the ΔV data, and were weighted by the data given by Keesom.

The results of the ΔV measurements are shown in Fig. 5, together with the previously determined curve for temperatures below 1.75°. The agreement above 2.2° with previous work by Keesom[9] (not shown) is quite good, although fundamentally different methods were used.[10] Since the experimental error involved in these measurements is about three percent, it is impossible to make any decision as to whether or not a discontinuity exists in the slope of the ΔV curve at the λ-point. Possibly the easiest check of this point would be to redetermine the density or thermal expansion of liquid helium in the vicinity of the λ-point, since the rapid increase of ΔV is due to the change in the thermal expansion coefficient of liquid helium from positive to negative as its temperature is decreased through the λ-point.[11]

The slope of the melting pressure curve (dP/dT) was then combined with the ΔV data using the Clausius-Clapeyron equation to obtain the heat of melting, ρ. These calculations are shown in the solid curve of Fig. 6, and they are tabulated in column 5 of Table I. Previous experimental determinations of the heat of melting are also shown in Fig. 6, and the agreement between 3° and 4° is very poor.[12,13]

Using the technique outlined previously, ρ was remeasured, and the data are indicated in Fig. 6 also.
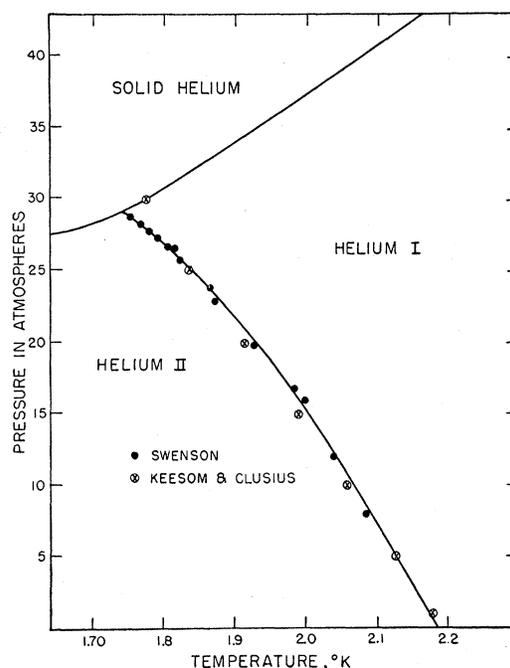

FIG. 4. The λ-point-pressure curve.

[9] Reference 3, p. 210.
[10] See Table I, column 4 of reference 2.
[11] Reference 3, pages 206 and 245.
[12] W. H. Keesom and A. P. Keesom, Leiden comm. 240b (1936).
[13] R. Kaischew and F. E. Simon, Nature 113, 460 (1934).

TABLE II. Smoothed values for the coordinates of the λ-point-pressure curve.

T_λ (°K)	P_λ (atmos)
1.743	29.12
1.750	28.78
1.85	24.4
1.95	18.8
2.05	11.3
2.15	3.0

The agreement is well within experimental error. No effort was made to obtain extreme accuracy, and the possible error of these measurements is of the order of five percent. The calculated values for ρ are probably the more reliable.

The reason for these discrepancies with earlier work is not obvious. The results quoted by Keesom rely on two values for the absolute entropy of solid helium, and one of these might have been in error due to an incorrect extrapolation. The data due to Kaischew and Simon were essentially preliminary in nature, although the deviation at 4° seems high. It is unlikely that these deviations have any significance, since the melting pressure measurements have been so easily reproducible, with no points significantly off the curve. These data, both ΔV and ρ, agree well with recent work at higher temperatures and pressures by Dugdale and Simon at Oxford.[14]

IV. DISCUSSION

Qualitatively, these results confirm the second-order nature of the λ-transformation. The first derivatives of both ΔV and ρ seem to be, if not discontinuous, changing very rapidly. Quantitative verifications are much more satisfactory, however, and by using Ehrenfest's relations[15] and certain other thermodynamic formulas, the consistency of the results can be checked in the vicinity of the upper "triple point." Actually, as Fig. 6 shows, the heats of melting are in good agreement with the ΔV and melting pressure data above this point.

Using the second Ehrenfest equation, and the fact that the thermodynamic properties of solid helium are well-behaved, the change in the slope of the ΔV vs T curve at 1.74° can be written as

$$\Delta_{I,II}\left(\frac{d(\Delta V)_M}{dT}\right)_M$$

$$=\Delta_{I,II}\left(\frac{\partial V}{\partial T}\right)_P+\Delta_{I,II}\left(\frac{\partial V}{\partial P}\right)_T\left(\frac{dP}{dT}\right)_M$$

$$=\Delta_{I,II}\left(\frac{\partial V}{\partial T}\right)_P\left[\frac{(dP/dT)_\lambda-(dP/dT)_M}{(dP/dT)_\lambda}\right]. \quad (2)$$

[14] J. S. Dugdale and F. E. Simon, Proceedings of the International Conference on Low Temperature Physics, Oxford, 1951, p. 25.

[15] Reference 3, p. 255.

Here, $\Delta_{I,II}(\partial V/\partial T)_P$ is the difference in the thermal expansions of helium I and helium II at the upper triple point, and $(dP/dT)_\lambda$ and $(dP/dT)_M$ are the slopes of the λ-point-pressure and melting pressure curves, respectively, at this point.

Using $(\partial V_I/\partial T)_P \doteq 0$, $(\partial V_{II}/\partial T)_P = -2.5$ cc/mol-deg, $(dP/dT)_\lambda = -37.5$ atmos/deg, $(dP/dT)_M = 29$ atmos/deg, calculation gives $\Delta_{I,II}(d\Delta V_M/dT)_M = -4.5$ cc/mol-deg. This agrees well with an experimental value of $-4(\pm 1)$ cc/mol-deg, obtained from Fig. 5.

A second relationship can be derived in much the same way for the difference in the slope of the dP/dT curve on the two sides of the "triple point," $\Delta_{I,II}(d^2P/dT^2)_M$. This derivation involves using the Clausius-Clapeyron equation, the first Ehrenfest equation, the above relationship, and

$$c_{PL}-c_{PS}=\frac{dp}{dT}\frac{\rho}{T}+\frac{\rho}{\pi V}\left[\left(\frac{\partial V_L}{\partial T}\right)_P-\left(\frac{\partial V_S}{\partial T}\right)_P\right].^{16}$$

The result is

$$\Delta_{I,II}\left(\frac{d^2P}{dT^2}\right)_M$$

$$=\frac{1}{(\Delta V)_M}\Delta_{I,II}\left(\frac{d(\Delta V)_M}{dT}\right)_M\left[\left(\frac{dP}{dT}\right)_\lambda-\left(\frac{dP}{dT}\right)_M\right]. \quad (3)$$

The data from the previous paragraph, and $\Delta V = 1.48$ cc/mol from this equation, gives $\Delta_{I,II}(d^2P/dT^2)_M = -180$ atmos/deg².

The experimental dP/dT curve, obtained by a graphical differentiation of Fig. 3 and confirmed by differencing the data and obtaining average slopes by least square straight lines, is shown in Fig. 7. The gist of the evidence is that dP/dT at 1.74° is about 20 atmos/deg and that it rises to about 29 atmos/deg at 1.75°. The shape of the curve is doubtful in this interval and is indicated by a dotted curve, The
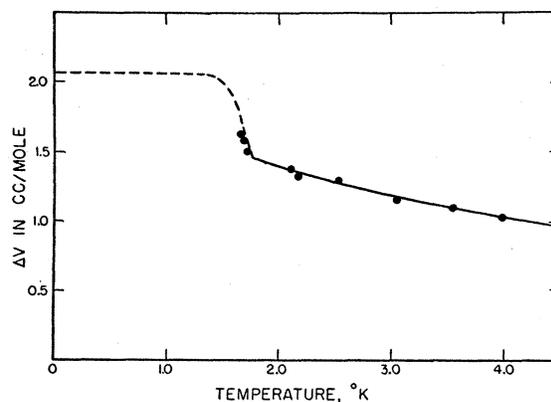

FIG. 5. The change in molar volume of helium on melting $(\Delta V = V_L - V_S)$.

[16] See, for instance, E. A. Guggenheim, Thermodynamics (Interscience Publishers, Inc., New York, 1949), p. 126.

average value of $\Delta_{I,II}(d^2P/dT^2)_M$ over this interval is about -900 atmos/deg^2, and the actual value just below $1.743°$ is most likely two or three times this figure. The predicted slope is also shown in Fig. 7 for comparison.

The discrepancy between the calculated and experimental values is very large and cannot be explained readily by experimental error in any of the quantities involved, especially in view of the good agreement with the ΔV curve. When these melting pressure data are compared with earlier data, the deviations are always in such a direction so as to decrease the slope of the dP/dT curve below the λ-point, although the discrepancies in melting pressures are of the order a few tenths of a percent. Further experiments are planned, using the blocked capillary technique, to check the accuracy of these melting pressure data by an independent method.

An alternative suggestion is that there is something incorrect in the application of second-order phase transition theory to liquid helium, although Eq. (2) seems to hold quite well. The usual "proof" of the second-order nature of the λ-transformation uses the first Ehrenfest equation,[15]

$$\left(\frac{dP}{dT}\right)_\lambda = \frac{\Delta C_P}{T\Delta(\partial V/\partial T)_P},\qquad(4)$$

and the experimental values of $(dP/dT)_\lambda$ and ΔC_P to calculate $\Delta(\partial V/\partial T)_P$. The agreement with experiment, usually quoted as "good," is of the order of fifty percent or less, if the original calculation is checked. Since it is doubtful that the accuracy of the melting pressure measurements can be pushed any farther, it would seem that a new and much more accurate determination
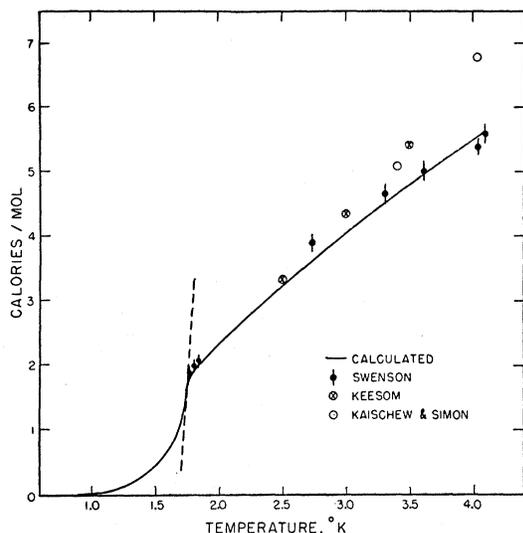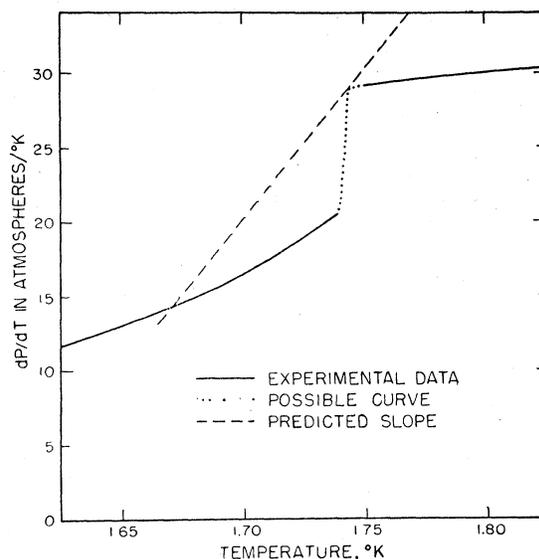
FIG. 7. Experimental values for the slope of the melting pressure curve in the vicinity of the "triple point."

of $\Delta(\partial V/\partial T)_P$ would be of value to help decide whether or not the Ehrenfest relations apply to liquid helium. It is obviously quite important that the Ehrenfest equations should be checked closely in the one case in which they are believed to apply accurately.

The greater part of this work was made possible by a grant from the Harvard Foundation for Advanced Study and Research. I wish to express my appreciation to Mr. J. A. Stewart for his help in taking the data and to Mr. Gösta Bjorklund for his technical assistance.

Note added in proof:—Recent experiments at this laboratory cast doubt on the λ-point-pressure curve of Fig. 4, and it was redetermined using a more accurate technique. These measurements show that below $1.85°$ the λ-temperatures are appreciably higher than given above, and the coordinates of the upper "triple point" on the helium phase diagram should be given as

$$P_\lambda = 29.64(\pm.03) \text{ atmos}, \quad T_\lambda = 1.764(\pm.003) \text{ °K}.$$

The discrepancy is most likely due to the rather insensitive method used previously, which, it would seem, gave a point of high thermal conductivity, rather than the onset of superfluidity.

There is no evidence from the data of Fig. 3 that $(dP/dT)_M$ begins to decrease until below $1.75°$, although one would expect the change to occur at the "triple point." New melting pressure data, which were obtained by using the blocked capillary technique, confirm this curve to within $\pm.001°$ below $1.75°$ but indicate that the melting temperatures of Fig. 3 are low (by $0.001°$ to $0.002°$) above this point. Thus, it is suggested that this obvious break in the curve is due to the onset of the high thermal conductivity of helium II, the maximum value of which occurs a few hundredths of a degree below the λ-point.

FIG. 6. The latent heat of fusion of helium, as calculated from the Clausius-Clapeyron equation and as determined experimentally.

The most important effect of these new data is on the discussion in Sec. IV. The "possible" curve in Fig. 7 can now be constructed by extrapolating the $(dP/dT)_M$ curve in helium II from $1.74°$ to intersect the $(dP/dT)_M$ curve in helium I at $1.764°$, so that a better agreement with the "predicted" slope is obtained (using a new $(dP/dT)_\lambda = 54.5$ atmos/deg). A difference of a factor of three or more still exists in the slopes, however, and this seems to be outside experimental error.

Details of the capillary blocking and λ-point-pressure experiments will be published elsewhere.

Second-Order Radiative Corrections to Hyperfine Structure*

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In this paper the effect of second-order radiative displacements of the energy levels of a bound electron on hyperfine structure separation is examined, and a correction to the Fermi formula for s levels is obtained. The treatment is based on an approximate evaluation of a finite, completely renormalized, and exact expression for the second order energy. The correction, which is restricted to atoms of small αZ, is found to be $[\alpha/2\pi - \alpha^2 Z(5/2 - \ln 2)]E_H$, where E_H is the Fermi energy. The effect of this and other corrections to the Fermi formula on the presently accepted value of the fine structure constant is discussed.

INTRODUCTION

THE interaction of the quantized electromagnetic field with the electron field modifies the properties of the electron and thus causes displacement of the energy levels of electrons bound in atomic or prescribed external fields. The existence of electromagnetic displacements was first recognized through observation of energy level separations, and it is through precise measurement of these separations that the various predictions of quantum electrodynamics are accessible to detailed experimental investigation. Previous theoretical treatments of this problem have been limited by the fact that weak field approximations are made in the process of recognizing and removing charge and mass renormalizations. In the case of the hyperfine structure, the radiative corrections have never been calculated directly, previously obtained results having been inferred from the anomalous magnetic moment of the electron.[1] It has long been realized, however, that a direct evaluation of the second order radiative displacement might yield, in addition to the effect of the second order moment, corrections to the hyperfine structure frequency of order $\alpha^2 Z$. As such corrections arise from the high momentum parts of the external field and electron wave function, their evaluation and comparison with experiment may serve to extend the domain in which the theory has been investigated. Furthermore, in view of the important role played by the hyperfine structure formula in recent determinations of the fine structure constant, it is clear that such corrections may significantly affect the numerical value which one obtains.[2-4]

THE SELF-ENERGY FORMULA

In order to deal systematically with the energy level displacement problem it is desirable to begin with an exact expression for the second order correction. Such expressions in noncovariant form were in fact the starting point of the earliest calculations of the level shift[5,6] and have been given in covariant form by Feynman[7] and Schwinger.[8] We shall briefly derive these here following the methods of Dyson.[9]

As a starting point we consider our theory to be cast in a modified interaction representation in which the effects of the external potential appear in the equations of motion for the field variables, while that of the interaction between the electron and photon fields appears in the interaction Hamiltonian.[10] That is, the development of the wave functional in time is described by

$$\frac{\delta \Psi[\sigma]}{\delta \sigma(x)} = -\frac{i}{\hbar c} H_i(x)\Psi[\sigma], \qquad (1)$$

where

$$H_i(x) = -j_\mu(x)A_\mu(x)/c, \qquad (2)$$

and

$$j_\mu(x) = \tfrac{1}{2}iec[\bar{\psi}(x)\gamma_\mu\psi(x) - \psi(x)\gamma_\mu{}^T\bar{\psi}(x)]. \qquad (3)$$

The operators $\psi(x)$, $\bar{\psi}(x)$, and $A_\mu(x)$, which describe the electron-positron and electromagnetic fields, respectively, satisfy the equations of motion

$$\gamma_\mu\frac{\partial\psi}{\partial x_\mu} + \left[-\frac{ie}{\hbar c}\gamma_\mu A_\mu{}^e(x) + \kappa_0\right]\psi(x) = 0, \qquad (4)$$

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