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Shape of Cosmic-Ray Star-Size Distributions in Nuclear Emulsions*

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The occurrence of a break in integral star-size distributions at a prong number of approximately 7, previously observed with nuclear emulsions in the lower atmosphere, has also been found at several stratosphere altitudes under a variety of conditions. This shows that the effect is due mainly to the composition of the nuclear emulsion. Although the shape of the distribution curves is thus strongly influenced by the properties of the detector, the altitude dependence of the nucleonic energy spectrum is clearly reflected in the progressive change in the slopes of these curves.

INTRODUCTION

In reporting on measurements of the star-producing radiation by means of nuclear photoplates, George and Jason[1] and Page[2] have plotted the logarithm of the frequency N of stars with k or more prongs against the prong number k. Their data were obtained on the Jungfraujoch[1,2] at 3450 m, and at sea level.[1] They showed that the integral star-size distribution at mountain altitudes, plotted semilogarithmically, is well represented by two straight lines A and B (similar to those in Fig. 1), which intersect at a prong number of about 7. Subsequently, Barton, George, and Jason considered that this property of the size-frequency distribution must be regarded as fortuitous.[1] Page, on the other hand, had suggested that the break near $k=7$ may be due to the composition of the emulsion which consists of two distinct groups of target nuclei: the light nuclei of C, N, O and the heavy nuclei of Ag and Br.[3] This suggestion implied a simple explanation for the change in slope of the size-frequency curve: large stars ($k>7$ or 8) can be generated only in the heavy nuclei; at prong numbers of ≈ 7 down to 5, a small but increasing fraction of the stars are produced in the light nuclei as well; finally, at 3 and 4 prongs, this fraction becomes considerable.

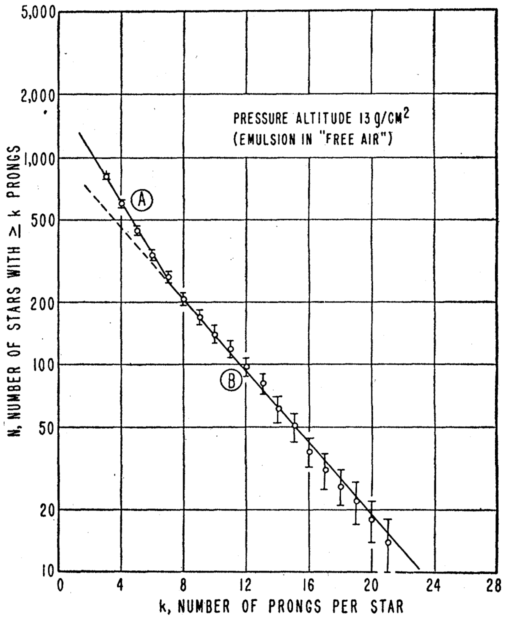

FIG. 1. Integral star-size distribution obtained from flight I with emulsions suspended far from other materials. The slopes of lines A and B are given in Table I. From the extrapolation of B, the frequency of stars produced in Ag and Br nuclei is estimated.

If this idea is correct, one might expect the break to appear not only in the lower atmosphere but also in the stratosphere, despite the change in the energy spectrum of the N-radiation with altitude. We have accumulated several star collections, in the course of a number of stratosphere investigations,[4] both in air and Pb, which could be used to test this hypothesis. Our integral distributions of star size in the stratosphere show the same break near $k=7$ as those at lower elevations. We have also analyzed similar data from other laboratories which likewise display the same effect. The presence of the break in such varied exposures strongly supports the contention that it reflects a property of the emulsion.

EXPERIMENTAL DETAILS AND RESULTS

Our star data were obtained from three Skyhook balloon flights[5] in Minnesota, at geomagnetic latitude 55°N. Ilford G-5 emulsions, 400μ thick, were used throughout. Flight I attained a maximum altitude of 99,000 ft, and stayed above 90,000 ft for 4 hours, the average pressure altitude during this time being 13 g/cm^2. The "free air" plates exposed in this flight were small (3×2 in.), few in number (6), and surrounded only by a thin aluminum can suspended far from other materials. Figure 1 shows the integral size-frequency

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[*] This work was reported at the Schenectady meeting of the American Physical Society, June 15, 1951 [Phys. Rev. 83, 874 (1951)].

[1] E. P. George and A. C. Jason, Proc. Phys. Soc. (London) **A62**, 243 (1949); Barton, George, and Jason, Proc. Phys. Soc. (London) **A64**, 175 (1951).

[2] Nora Page, Proc. Phys. Soc. (London) **A63**, 250 (1950).

[3] H is omitted from this list, since the type of stars considered here are not produced in hydrogen.

[4] Shapiro, Stiller, Birnbaum, and O'Dell, Phys. Rev. **83**, 455 (1951).

[5] We appreciate the cooperation of Dr. U. Liddell and Mr. J. Holloway, of ONR, and C. B. Moore, A. T. Bauman, and their staff of the General Mills Aeronautical Research Laboratories in these flights.

TABLE I. Slopes of star-size distributions at various altitudes in air and Pb.

Data	Atmos. depth (g/cm^2)	Slope α_A	Slope α_B	k_t
Balloon flights				
Naval Research Laboratory (NRL) (flight I)	13 ("free air")	0.286±0.012	0.199±0.016	6.2
NRL (flight I)	13 (in Pb)	0.286±0.055	0.188±0.046	7.5
Brookhaven National Laboratory (BNL)[a]	15	0.301±0.044	0.195±0.017	6.9
Chicago[b]	15	0.180	0.137	10
NRL (flight II)	22	0.327±0.018	0.197±0.010	7.3
Bristol[c]	50	0.267±0.004	0.213±0.005	8.0
NRL (flight III)	300	0.412±0.079	0.248±0.059	8.0
Mountain exposures				
London (Birkbeck College)[d]	680	0.48	0.33	8.9
Manchester[e]	680	0.521	0.331	6.3
Bristol[f]	680	0.365±0.085	0.280±0.010	8.2
Sea level exposure				
London[d]	1030	0.48	0.33	8.8

[a] See reference 8.
[b] See reference 11.
[c] See reference 10.
[d] See reference 1.
[e] See reference 2.
[f] See reference 9.

distribution obtained from the free-air emulsions. As an index of star size we adopted the number of "heavy" prongs, i.e., tracks with specific ionization > 1.4 times the minimum. It is evident that the data are well represented by two lines which intersect at $6<k<7$. The absolute values α_A and α_B of their least-squares slopes are given in the first line of Table I.[6] Another exposure, of plates buried vertically in a block of Pb 5 cm deep, was made in the same flight. This yielded the same kind of star-size graph, with slopes which agree closely with those for the free-air exposure (see Table I). That the slopes in Pb differ inappreciably from those in air had previously been observed at mountain altitudes.[1]

Plates inside another Pb absorber, 15 cm in height (described more fully elsewhere[4]), were exposed at a somewhat lower altitude in flight II. The balloon reached a maximum altitude of 94,000 ft, and during the 6.5 hours spent above the release point, the mean pressure altitude was 22 g/cm^2. The integral size distribution appears in Fig. 2, and the slopes α_A and α_B are given in Table I. The value 0.327±0.018 of α_A (the slope for $k<7$) is somewhat larger here than either value of the same slope at 13 g/cm^2. Comparing the two Pb exposures, this difference does not lie outside experimental error.[7]

Flight III consisted of an ascent to 92,000 ft, followed directly by descent, the total duration being 2.4 hours. The average pressure altitude (which is, of course, less meaningful in this type of exposure than in a sustained flight at "ceiling") was 300 g/cm^2. The exposure was intended merely to provide a correction for up-and-down accumulation; for this purpose, the collection of a very limited sample of stars (190) sufficed. The meager data have been included because they represent an exposure at a mean atmospheric depth intermediate between 50 and 680 g/cm^2. Again the frequency-size data conform to the 2-slope pattern with a break near $k=7$. The slopes α_A and α_B are intermediate in magnitude between the average values in the upper stratosphere and those at the mountain altitude.

Altogether, the four star distributions from our flights at three different altitudes are alike in shape and in the location of their break. Their slopes agree at a given altitude, and increase with atmospheric depth between the upper stratosphere and 300 g/cm^2.

In Table I our results are compared with those of other stratosphere investigations, three mountain exposures, and one sea-level exposure. For the experiments at the Brookhaven National Laboratory[8] and the H. H. Wills Laboratory in Bristol,[9,10] as well as our own, we had access to the tabulated star-size data and were

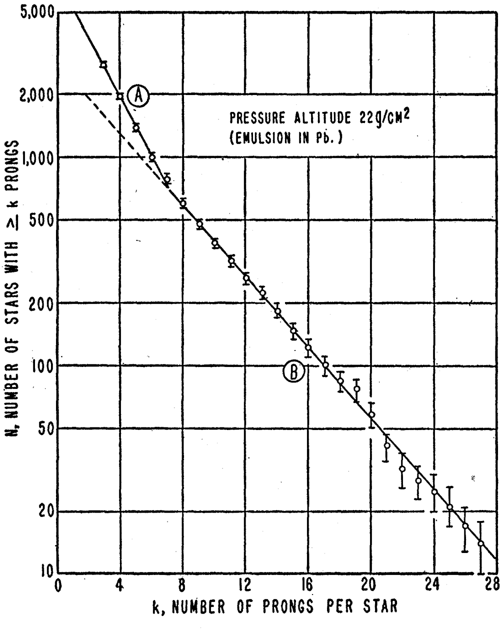

FIG. 2. Integral star-size distribution obtained from emulsions embedded in a Pb block sent aloft in flight II. The break near prong number 7 is characteristic of such distributions regardless of altitude.

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[6] All slopes mentioned in this paper are negative. For convenience, the symbols α_A, α_B will be used throughout to denote their absolute values.

[7] We believe, however, that the difference is real, not only because of the lower altitude in flight II, but also because the Pb block used in it was more massive than the other by a factor $\simeq 4$. Both conditions would result in a degraded nucleon spectrum, which would increase the relative number of small stars.

[8] We wish to thank Drs. John Hornbostel and Edward O. Salant for permission to use their unpublished data.

[9] Mountain data: Brown, Camerini, Fowler, Heitler, King, and Powell, Phil. Mag. **40**, 862 (1949).

[10] Stratosphere exposure: Camerini, Davies, Fowler, Franzinetti, Lock, Perkins, and Yekutieli, to be published in Phil. Mag. A preprint of this work was forwarded to us by Dr. F. Singer of ONR, London. We are grateful to Professor C. F. Powell's group for making this paper available before publication.

therefore able to compute the slopes by the method of least squares. In these instances, the standard error in each slope, estimated from the differential data as described in the Appendix, is given in the table. For the remaining four experiments, the slopes were deduced graphically (Chicago[11]) or their values were given explicitly by the authors (London[1] and Manchester[2]). The last column in Table I lists the values of k_i, the prong number at the point of intersection of lines A and B, obtained from the integral graphs. All the stratosphere stars were registered in ultrasensitive emulsions, Ilford G-5, Eastman NTB3, or (British) Kodak NT4.[12]

The various star-size distributions show the same basic feature: their semilogarithmic plots are reasonably well represented by two straight lines[13] which intersect near the prong number seven.[14] (The graphs, in fact, resemble Figs. 1 and 2 so closely that we have not reproduced them here.) They strongly support the hypothesis that the break is a consequence of the emulsion composition.

STAR-PRODUCTION RATIO IN HEAVY AND LIGHT NUCLEI

This result can be used for an approximate calculation of the ratio of star production in the heavy and light nuclei of the emulsion. It has been pointed out that line B must be due to stars (with $k \geq 8$) generated in the Ag and Br nuclei alone. We may therefore estimate, as a first approximation, that the total contribution of the Ag and Br disintegrations to the star frequency is given by the extrapolation of line B, shown as a dashed line in Figs. 1 and 2. The ordinate of this line at $k=3$ is then the frequency ν_h of stars with 3 or more prongs produced in the heavy nuclei alone. From the data we obtain directly the frequency ν of all stars with $k \geq 3$ in normal emulsion.

We have computed the ratio ν/ν_h for four of the star collections listed in Table I. The results, given in Table II, can now be compared with values of the same ratio obtained in other ways.

One method is that of Harding,[15] who used the gelatin-sandwich technique. Stars formed in the gelatin[16] can arise only from the disintegration of the light nuclei, C, N, and O. From Harding's results, $\nu/\nu_h = 1.56 \pm 0.2$.[17] Well within experimental error, this value is the same as three of those in Table II, and it is compatible with the fourth (1.3±0.1).

The ratio ν/ν_h can also be calculated from the known composition of the emulsion, if we assume that the cross section for star production is proportional to the geometric one, for all Z, and for all energies of the incident particle. Normalizing the macroscopic cross section per cm³ of emulsion in the Ag and Br nuclei alone to unity,[18] the cross section for the light elements is found to be 0.36, and for the entire emulsion, 1.36. Thus we expect the ratio ν/ν_h to be 1.36. The results in Table II agree somewhat better with the ratio obtained from Harding's data than with that computed from considerations of geometric cross section. However, the large errors in the experimental values of the ratios preclude a definite choice between the two.[19]

VARIATION OF SLOPES WITH ALTITUDE

Examination of the trend in the slopes as we progress from mountain to stratosphere altitudes shows that α_A and α_B become smaller, indicating an increase in the relative number of large stars. This is expected, for with increasing altitude the spectrum of the star-producing radiation becomes richer in high energy nucleons.

TABLE II. Ratio of stars with ≥3 prongs in emulsion to stars in Ag and Br.

Data	Atmos depth g/cm²	Ratio
Naval Research Laboratory	13	1.5±0.3[a]
Brookhaven National Laboratory	15	1.5±0.4
Bristol	50	1.3±0.1
Bristol	680	1.5±0.2

[a] Errors are standard errors.

It can be seen that the Brookhaven (BNL) values obtained near the top of the atmosphere are the same as ours within experimental error. Lord's slopes for the same altitude are distinctly lower, the difference being more pronounced for α_A. The following may be the explanation: part A of the curve contains the small stars, mainly of 3 and 4 prongs, many of which are readily missed in scanning the emulsions if special precautions are not taken. An underestimate of the fraction missed would result in too low a slope for line

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[11] J. J. Lord, Phys. Rev. **81**, 901 (1951).

[12] Additional data on star-size distributions have been reported by M. Addario and S. Tamburino, Phys. Rev. **76**, 983 (1949), and H. L. Bradt and M. F. Kaplon, Phys. Rev. **78**, 680 (1950). Their results, based on 488 and 1000 stars, respectively, were obtained in whole or in part with Ilford C-2 emulsions. The latter are insensitive compared with the electron-sensitive emulsions (in which the tracks of all charged particles are registered) used in our stratosphere studies and those upon which we have drawn for comparison (see references 8, 10, 11, and Table I). Since the total data collected from these balloon flights were quite adequate (>20,000 stars), and since they were gathered independently by four different laboratories, we felt justified in basing our analysis of the stratosphere stars on the studies made with ultrasensitive emulsions exclusively.

[13] There is some indication in the graphs that the distributions might be even better represented by three straight lines, with the second break occurring in the neighborhood of $k=16$. The third line C is steeper than B. However, the statistical data for C, excepting Bristol's at 50 g/cm², are too meager for accurate analysis. Therefore, for simplicity, the data for $k>7$ were fitted to a single best line B.

[14] The value $k_i=7$ is an average, rounded off to the nearest integer, of the k_i for all the data treated by least squares, i.e., those for which slope errors appear in Table I (excepting the data at 300 g/cm²).

[15] J. B. Harding, Nature **163**, 440 (1949).

[16] The breakdown of these stars according to multiplicity helps explain why, contrary to what might be expected from the emulsion composition, there is no abrupt jump in N_k (with decreasing k) at $k \approx 7$, but only a change in slope. Of 26 stars with $k \geq 3$ produced in the C, N, O, Harding found only two with 5 prongs, eight with 4 prongs, and sixteen with 3 prongs. Thus, despite the theoretical possibility of releasing 6 to 8 charged particles from a light nucleus, no stars of this size were observed. Actually, it seems likely that if G-5 emulsions, rather than C-2, had been used in this study, the visibility of more tracks would have enhanced the average multiplicity and yielded one or two stars with 6 or 7 prongs.

[17] The standard error is given here.

[18] The following partial densities of G-5 emulsion components, in g/cm³, were used in this calculation: Ag, 1.85; Br, 1.36; C, 0.27; N, 0.067; O, 0.27. Minor constituents, and H, were omitted, since they do not affect the present result.

[19] The fact that the dispersion in the ratios is much less than what one would expect from the errors associated with them lends support to our statement in the Appendix that the errors assigned to the slopes (from which the precision of these ratios is deduced), tend to be overestimates.

A. This suggests one of the uses of an integral size-frequency distribution, namely, as an easily obtainable, rough check on the scanning efficiency.

Bristol's slope α_A at 50 g/cm^2 is somewhat lower than those of both NRL and BNL at greater altitudes, whereas one would expect it to be higher. The reason may be that the results were uncorrected for scanning efficiency.[20] Such corrections, based on extensive cross-checks between the various observers,[21] were applied to our data and to Brookhaven's.

Turning to the mountain experiments, it should first be noted that George and Jason, as well as Page, used Ilford C-2 emulsions; the Bristol group used Kodak NT4 plates. This difference appears to be reflected in the respective results: those of London and Manchester show close agreement, whereas the absolute slopes obtained from the Bristol data are lower. Some of the tracks counted as "gray" prongs in the ultrasensitive emulsions are not visible in the less sensitive C-2 plates. Thus, the average Bristol star size (observed in the NT4 emulsions) must be larger than the others and hence the slopes in the size-distribution curve are smaller. In addition, the relatively low value of α_A could result in part from an underestimate of the small-star frequency, as suggested above for the stratosphere data.

The values assigned to the sea-level slopes in Table I, based on a smaller number of stars, are the same as those obtained from the mountain data of George and Jason. These authors indicate that, within their limits of error, there is no appreciable difference between the two size distributions.

From our discussion of the results in Table I it appears that, although the shape of the size distribution is strongly influenced by the properties of the emulsion, the increasing mean energy with altitude of the star-producing nucleons is reflected in the progressive change in the slopes of the distribution curves. This is especially evident in the slopes α_B.

CONCLUSIONS

It has been shown that integral star-size distributions at altitudes ranging from 13 g/cm^2 down to sea level (over 98 percent of the atmosphere) show a characteristic break near the prong number 7. We conclude that this break is due mainly to the emulsion used to record the stars. Using this result to calculate the ratio of star production in heavy and light nuclei gives fair agreement with values obtained by other methods. Finally, the change with altitude of the energy spectrum of the nucleonic component is reflected in the slopes of the distribution curves.

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APPENDIX. ERRORS IN THE SLOPES

The values of the slopes of the integral star-size graphs given in Table I were obtained by the method of least squares in those cases in which we had access to the original data in tabular form. α_A was calculated from the 5 points $3 \leq k \leq 7$ and α_B from those with $k \geq 8$.

The statistical errors in the slopes cannot be obtained from the integral data because, contrary to the requirements of least squares theory, these data are not completely independent.[22] Therefore, the errors were calculated from the differential data. If N_k be the number of stars with k or more prongs, and n_k the number with just k prongs, then, according to the graphs,

$$N_k = Ae^{-\alpha k},$$

where A and α are constant for a portion (such as A or B, Fig. 1) of a given star distribution. It follows that

$$n_k = N_k - N_{k+1} = Ae^{-\alpha k}(1 - e^{-\alpha}).$$

This function cannot be conveniently treated by the method of least squares. But the approximate function $n_k = Be^{-\alpha k}$ can be so treated. The errors given in Table I are proportional to those obtained by the least squares analysis of the approximate n_k function, the relative error in an integral slope being equated to that in the corresponding differential slope. This approximation for n_k tends to give a conservative estimate of the errors.

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[20] The recent Bristol investigations have been primarily concerned with meson production processes, and only incidentally with star frequencies.

[21] The actual values are of interest as an indication of the considerable fraction of small stars which may be missed. Our average efficiency in systematic scanning at a magnification of ≈150× is 78 percent for 3-prong stars, 88 percent for 4-prong stars, and >99 percent for stars with at least 5 prongs. These figures are of course unlikely to be the same for all emulsions or all observers, but they apply to the data at hand. Moreover, they are probably representative of what can be expected from surveys of stratosphere-exposed plates, as judged from the following: Brookhaven kindly informs us that their over-all efficiency for 3- to 5-prong stars is 85 percent. When our own three values above are weighted by the relative frequencies, we obtain the same efficiency for this group of small stars.

[22] The authors are grateful to Mr. John Mandel for an interesting discussion on this point.