Nonlinear Meson Theory of Nuclear Forces. III. Quantization of the Neutral Scalar Case with Nonlinear Coupling

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The problem of a number of infinitely massive point nucleons interacting according to the neutral scalar meson theory with a nonlinear coupling of the power-law type was considered classically in an earlier paper. It was found there that the interaction is the same as with the usual linear coupling. This system is now quantized, and it is shown that the same conclusion is valid in quantum theory. The case of exponential coupling, considered by Glauber, lies outside the scope of the present investigation.

IN two earlier papers, a classical nonlinear meson theory of nuclear forces was presented.[1] The neutral scalar theory with nonlinearity in the field was discussed in I, and the similar theory with nonlinearity in the coupling was considered in II. The quantization of the latter case with infinitely massive point nucleons can easily be carried through with the help of a simple canonical transformation. When this is done it is found that the earlier conclusion, that the interaction of a number of nucleons is the sum of Yukawa terms regardless of whether the coupling is linear or nonlinear, still holds in quantum theory provided the coupling is of the power-law form.

The field Hamiltonian is

$$H=\int[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\nabla\phi)^2+\tfrac{1}{2}\phi^2-f(\mathbf{r})F(\phi)]d\tau,$$

where ϕ and π are the meson field amplitude and canonically conjugate momentum, f is the nucleon source density, F is the nonlinear coupling function, and units are chosen such that $\hbar$, c, and the meson mass are equal to unity. The quantum condition on the field is

$$[\phi(\mathbf{r},t),\ \pi(\mathbf{r}',t)]=i\delta(\mathbf{r}-\mathbf{r}').$$

The canonical transformation[2]

$$S=\exp\left[i\int\phi_0(\mathbf{r})\pi(\mathbf{r})d\tau\right],$$

where ϕ_0 is a c-number function that commutes with ϕ and π, has the following effects:

$$S\phi S^*=\phi+\phi_0,\quad S\pi S^*=\pi,$$

$$\begin{aligned}SHS^*=&\int[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\nabla\phi+\nabla\phi_0)^2+\tfrac{1}{2}(\phi+\phi_0)^2\\&\qquad-f(\mathbf{r})F(\phi+\phi_0)]d\tau\\=&\int[\tfrac{1}{2}(\nabla\phi_0)^2+\tfrac{1}{2}\phi_0^2-f(\mathbf{r})F(\phi_0)]d\tau\\&+\int[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\nabla\phi)^2+\tfrac{1}{2}\phi^2]d\tau\\&+\int\phi[-\nabla^2\phi_0+\phi_0-f(\mathbf{r})F'(\phi_0)]d\tau\\&-\int f(\mathbf{r})[\tfrac{1}{2}\phi^2F''(\phi_0)+\cdots]d\tau.\end{aligned}$$

If now ϕ_0 is chosen to be the solution of the classical field equation given in II, Eq. (1), the third line of the last expression for the transformed Hamiltonian given above is identically zero, and the first line is just the classical Hamiltonian treated in II. The second line is the quantum Hamiltonian for free mesons and need not be discussed further. We now show that the fourth line, which contains the rest of the terms in the Taylor's series expansion of $F(\phi+\phi_0)$, vanishes if f represents a number of point nucleons and if F has the power-law form $F(\phi)=b\phi^m$.

Consider the contribution to the nth derivative term that arises from one of the nucleons. The factor ϕ^n can be evaluated at the nucleon and taken outside the integral, and the rest is proportional to the limit as a approaches zero of $g(d^nF/d\phi_0^n)$, where g is the single nucleon source strength and a is the radius of the source. Now the nth derivative of F is proportional to ϕ_0^{m-n}; from II, Eq. (5), ϕ_0 is proportional to $1/a$, and from II, Eq. (7), g is proportional to a^{m-1}. Thus, the term in question is proportional to a^{n-1} and approaches zero in the limit $a\to 0$ if $n\geqq 2$, as it is for all the terms in the fourth line of the expression for the transformed Hamiltonian.

The foregoing development shows that the quantum interaction energy is the same as that calculated classically in II and, hence, does not lead to saturation. The case in which the nonlinear function F has an exponential dependence on ϕ, considered recently by Glauber,[3] cannot be handled as simply as the power-law case, since the F derivative terms do not then vanish in the point source limit.

It should be noted, as pointed out in II, that the conclusions reached here are valid only for point nucleons and, hence, will not be expected to hold, for example, when nucleon recoil is taken into account.

[1] L. I. Schiff, Phys. Rev. 84, 1, 10 (1951); referred to here as I and II, respectively. The notation of the present paper is the same as that used in these references.

[2] This transformation was also applied independently to the nonlinear field case considered classically in I, by D. R. Yennie and M. Gell-Mann (private communication).

[3] R. J. Glauber, Phys. Rev. 84, 395 (1951).