The Average Energy of Beta-Ray Spectra*

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The average beta-energy per disintegration has been measured for P^{32}, Y^{90}, I^{131}, and Ca^{45}. An air-filled water-electrode extrapolation ionization chamber was used for measurement of the rate of emission of energy. The absolute disintegration rate was determined by 4π-counting. Good agreement was found between the measured average energy values and values predicted by the Fermi theory using maximum energy data from the literature. Experimental results obtained were P^{32}, 0.696±0.03 Mev; Y^{90}, 0.895±0.035 Mev; I^{131}, 0.189±0.008 Mev (includes conversion electrons); and Ca^{45}, 0.0746±0.003 Mev.

I. INTRODUCTION

THE ability of a beta-ray emitting radioactive isotope to produce biological effect is determined by the average energy emitted per disintegration. For most isotopes, however, accurate values of only the maximum beta-energies are available. This paper describes a new method of measuring average energies.

If E_m is the rate of energy production per gram by some radioactive material and n is the disintegration rate per gram of the same material, the average energy per disintegration is

$$\bar{E}=E_m/n. \quad (1)$$

The average beta-ray energy per disintegration was first measured by Ellis and Wooster[1] for radium E. The heat produced by the radium E (and therefore E_m) was measured in a very sensitive calorimeter. The disintegration rate n was determined by alpha-counting the growth of the daughter nuclide, radium F. Other calorimeter measurements of average energy have been made for RaE,[2–4] P^{32},[5] and H^3.[6] Disintegration rates were determined for P^{32} with an end-window beta-counter of known efficiency and by H^3 by measuring the growth of the helium daughter.

The calorimetric method and the present method are the only ones which allow a direct determination of the average energy of a beta-ray spectrum without further knowledge of the maximum energy, the type of transition, or its complexity. The spectral energy distribution for a large number of beta-ray emitters has been measured by means of magnetic beta-ray spectrometers. From the experimentally determined energy spectra, Marinelli, Brinckerhoff, and Hine[7] have computed the average energies of many beta-ray emitters. The agreement with the predicted values from the Fermi theory using experimental values for the maximum energy is satisfactory. The accuracy of their average energy determinations is of the order of 10 percent. For many complex spectra, average energies of the individual branches only were reported.

The present work was undertaken to investigate a relatively simple means of determination of average energy for both simple and complex beta-spectra. The ionization produced by a radioisotope is more easily measured than the very small amount of heat produced. For this reason an extrapolation ionization chamber is used in place of the calorimeter. Disintegration rate was determined by the most accurate, generally applicable method, 4π-counting.

II. THE EXTRAPOLATION IONIZATION CHAMBER

A. Theory of Operation

The extrapolation ionization chamber was invented by Failla[8] and has been used in various problems of energy absorption measurement.[9–12] The theory of operation of the extrapolation chamber as used in this experiment is now presented.

In an infinite homogeneous medium with radioactive material distributed uniformly throughout, the energy absorbed per gram of medium just equals the energy produced per gram and is given by $E_m=n\bar{E}$. This energy absorption will likewise hold at the plane interface between two semi-infinite volumes of this same medium. If the radioactivity is removed from one of the volumes, but the material medium is kept the same throughout, the energy absorbed in an infinitesimally thin layer of either volume at the interface is $n\bar{E}/2$. An infinitesimally thin air layer at the interface may be approximated by extrapolating to zero plate separation the curve of ionization per unit volume in a parallel

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plate, variable plate separation, ionization chamber (extrapolation chamber). To correspond to semi-infinite volumes, the electrodes must be "infinitely thick" to the radiation being studied. The rate of energy absorption may be determined by the Bragg-Gray cavity theorem:[13,14]

$$E_m = J_m \langle W_{\text{air}} \cdot \rho_m \rangle_{\text{Av}}, \tag{2}$$

where J_m is the number of ion pairs formed per gram of air per second, and E_m is the energy dissipated in the medium Z per gram per second. $\langle W_{\text{air}} \cdot \rho_m \rangle_{\text{Av}}$ is the mean value of the product of ρ_m, the mass stopping power of the medium relative to air, and W_{air}, the mean energy expended in the production of a pair of ions in the air, averaged over all speeds of particles traversing the cavity. Gray gives

$$\rho_m = \frac{(dE/dx)_{\text{medium}}}{(dE/dx)_{\text{air}}} = \frac{(n_m)_{\text{medium}}}{(n_m)_{\text{air}}} f_{\bar{Z}}, \tag{3}$$

where (dE/dx) is the rate of energy loss per g/cm^2 of path length, n_m is the number of electrons per unit mass, and $f_{\bar{Z}}$ is a function of the mean atomic number of medium Z. The function $f_{\bar{Z}}$ has a value of unity for air, and is taken from the Bethe-Williams theory of the ionization loss by charged particles.[15,16] For the case of beta-rays of about 1-Mev initial energy, $f_{\bar{Z}}$ increases by approximately 1.5 percent per unit increase in Z for Z near that of air.

For the particular case of water source electrode and air cavity (which were used in the average energy measurement), the relative stopping power has been evaluated as a function of energy by Wang,[17] using the theoretical calculations of Bethe[18] for air and of Halpern and Hall[19] for water.

W_{air} may be taken as 32.5±1.0 ev/ion pair for air for all energies above about 8 kev.

After combining Eq. (2) with Eq. (1), and inserting a factor of two because only one medium is radioactive, we have for media of identical composition

$$\bar{E} = 2J_m \langle W_{\text{air}} \cdot \rho_m \rangle_{\text{Av}}/n. \tag{4}$$

For a backscattering electrode (nonradioactive) not of the same material as the source electrode, the equation may be written

$$\bar{E} = 2J_m \langle W_{\text{air}} \cdot \rho_m \rangle_{\text{Av}}/nB, \tag{5}$$

where B is called the "backscattering correction factor." Relative values of B have been determined by Baily[20] for a large number of backscatterers, using an extrapolation chamber with P^{32}, Tl^{204}, Sr^{90}, and S^{35} sources in water solution. B is found to be remarkably independent of the beta-energy. For a water source electrode and aluminum backscattering electrode, B is about 1.14 for isotopes above 0.5-Mev maximum energy, decreasing at S^{35} (0.169 Mev) to 1.12.

The ionization per unit volume, J_v, in the extrapolation chamber is observed to decrease linearly with increasing electrode separation. The increased absorption in the air of the cavity and an obliquity effect due to the non-infinite width of the source are believed to account for this. The activity A of beta-radiation observed at an angle θ with the normal to the plane of a source is approximately[21]

$$A = A_0(\cos\theta/\mu x)[1-\exp(-\mu x/\cos\theta)], \tag{6}$$

where μ is the mass absorption coefficient, x is the source thickness, and A_0 is a constant. For an infinitely thick source, A is approximately proportional to $\cos\theta$. From this it may be shown that if the source radius b is large compared to plate separation d, then the ionization per unit volume will fall off linearly with d because of the obliquity effect. More detailed calculations including air absorption also yield a linear decrease in J_v with d. For an ideal infinitely thin source, J_v will become infinite as d approaches zero.

A gamma-ray correction has been calculated, and amounts to 2–3 percent for an isotope emitting equal gamma-ray and beta-ray energy. The expression for the ratio of the gamma-ray energy to the beta-ray energy absorbed in the air volume is

$$\frac{E_{\text{air}\gamma}}{E_{\text{air}\beta}} = \sum_{\gamma} \frac{n_\gamma E_\gamma}{n\bar{E}} \left(\frac{\mu_{\text{air}\gamma}\mu_{1\beta}}{\mu_{1\gamma}\mu_{\text{air}\beta}} \right) \{1-\exp[-\mu_{1\gamma}Y] + \mu_{1\gamma}Y[[-\text{Ei}(-\mu_{1\gamma}Y)]-[-\text{Ei}(-\mu_{1\gamma}b)]]\}, \tag{7}$$

where the summation is over all the gamma-rays in the disintegration scheme of the isotope. $\mu_{1\beta}$ and $\mu_{i\gamma}$ are the beta- and gamma-ray absorption coefficients in the source medium, and $\mu_{\text{air}\beta}$ and $\mu_{\text{air}\gamma}$ the corresponding coefficients in the air cavity. The thickness of the source is Y, its radius is b, n_γ is the disintegration rate for each gamma-ray, $-\text{Ei}(-y)$ is the exponential integral.

A bremsstrahlung correction of up to 0.5 percent has also been made.[22]

B. Design of the Chamber

An extrapolation chamber was built and modified so that either solid or liquid radioactive materials could be used for one of its plane parallel electrodes. A water solution of a radioactive salt was used for one electrode in the average energy measurements (Failla *et al.*[12]). For the other electrode, the backscattering electrode, alu-

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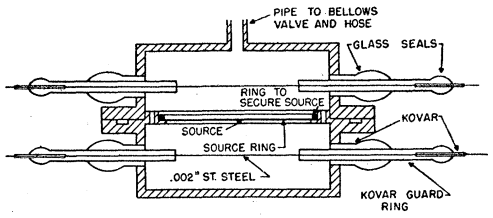

FIG. 1. Schematic diagram of the 4π proportional counter.

minum and water solidified with agar were used. The water-agar electrode was used so that both electrodes could have as nearly identical composition as possible. Spacing between the plane parallel electrodes was adjustable with three micrometers. The absolute value of plate separation was determined by an electrical measurement of the capacity between the guard-ringed collecting electrode and the high voltage electrode. By comparison between the mechanical distance measurement and the electrical capacity measurement, the ionization per unit volume could be measured independent of the area of the collecting electrode. This was important when an agar electrode was used. Ionization current was measured with a vibrating reed electrometer.

III. THE 4π-COUNTER

To determine the disintegration rate n for the solution used as the radioactive electrode, absolute disintegration rate measurements were made with a 4π-counter.

A proportional counter was built in the "pillbox" shape[23] (see Fig. 1). The interior of the counter is $3\frac{1}{4}$ inches in diameter by $2\frac{5}{8}$ inches high. The source is at the center of a collodion film 10–20 μg/cm^2 thick which is $1\frac{3}{8}$ inches in diameter. To make the source conducting, aluminum may be evaporated on the film yielding a layer about 5 μg/cm^2 thick. For isotopes with energetic beta-rays, aluminum foil 0.14 mg/cm^2 thick was used. The counter was usually filled to 45-cm pressure with n-butane. Normal operation voltage is about 4100 volts.

The center wires are guard-ringed, with the guard rings at the same high positive potential as the center wire. The brass case of the counter (coated with Aquadag) serves as the cathode and is maintained at a negative high voltage with respect to ground. This connection tends to prevent corona discharge troubles.

The counting rate was independent of pressure between 30 and 60 cm, and the plateau slope against voltage was less than $\frac{1}{2}$ percent in 500 volts. Location of the plateau was taken from the curve of counting rate *vs* pulse-height selector setting.

Disintegration rate comparisons with a P^{32} standard calibrated with a 4π-counter at the National Bureau of Standards, and a coincidence-counted Co^{60} standard of the National Bureau of Standards, checked within the accuracy of those measurements (5 percent and 3 percent). Checks against Na^{24} and Co^{60} standards coincidence counted in this laboratory agreed to within 1 percent, although the expected accuracy of the coincidence counting was taken as ± 5 percent. For Co^{60}, close agreement between 4π-counting and coincidence counting was found for an evaporated source 1 μg/cm^2 thick, indicating that the intrinsic efficiency of the counter is nearly 100 percent.

The chief loss in counting efficiency is caused by self-absorption in the source. A Ca^{45} sample simply allowed to dry counted 36 percent lower than a properly precipitated thin sample of the same series. A lower limit is placed on the energy of the beta-particles to be counted in a 4π-counter by the source thickness problem.

IV. EXPERIMENTAL RESULTS

Estimated errors for quantities inserted in Eq. (5) were W_{air}, ± 3 percent; B, ± 1 percent (aluminum backscattering electrode); n, ± 2 percent; $J_m \pm 1$ percent (aluminum backscattering electrode), ± 2 percent (agar backscattering electrode). This leads to a probable error in E of about ± 4 percent. W_{air} is taken as 32.5 ev/ion pair.

Wang[17] points out the possibility of large errors in the value of ρ_m. The results for P^{32} below would seem to indicate that no large error of this type is present in these measurements. If improved values of ρ_m are available in the future, then the results below should be corrected accordingly.

A. P^{32}

Results for P^{32} were as follows:

Backscattering electrode	ρ_m	B	$\bar{E}$
Water (agar)	1.11	1.00	0.700 Mev
Aluminum	1.11	1.14[24]	0.692 Mev

We may take the mean as our best value for average energy $\bar{E}=0.696\pm 0.03$ Mev. This may be compared with other values reported in the literature[5,7,14,25,26] and collected in Table I.

The beta-spectrum of P^{32} is well known. Since the average energy obtained by the present method is in good agreement with values obtained by independent methods, the present method may be assumed to yield good values for other isotopes.

B. Y^{90}

For Y^{90}, ρ_m was taken as 1.10, a compromise between the values of Wang[17] and Gray[14] for electrons of about 0.9-Mev average energy. B was estimated as 1.14 from the data of Baily.[20] The final result is $\bar{E}=0.895\pm 0.035$ Mev. Using $E_{max}=2.18$ Mev,[27] and the Coulomb correction factor of Nordheim and Yost,[28] numerical

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integration of the Fermi spectrum using the forbiddenness factor $[(W_0-W)^2+(W^2-1)]$ yields $\bar{E}=0.90$ Mev. For Y^{90}, one cannot distinguish between the allowed and forbidden shape on the basis of average energy because the theoretically predicted average energies are almost equal ($\bar{E}=0.89$ Mev for the allowed spectrum).

It should be noted here that the relatively low average energy found for RaE is because the correct forbiddenness factor is approximately proportional to $(W_0-W)^2$. For the other isotopes found thus far to have forbidden-shape spectra, the average energy would be little affected by the degree of forbiddenness of the transition.

C. I^{131}

Taking $B=1.14$ and $\rho_m=1.11$ and making a gamma-ray correction of 5.4 percent, we find $\bar{E}=0.189\pm0.008$ Mev. The total electronic energy per disintegration calculated from the decay scheme of Metzger and Deutsch[29] and using the Coulomb correction factor of Longmire and Brown[30] is 0.180 Mev (including 7.3 kev/disintegration of energy because of conversion electrons).

D. Ca^{45}

Ca^{45} of the highest specific cativity obtainable from Oak Ridge was used. A solution of Bentonite, a colloidal mud, was used to precipitate 4π-counter samples of 0.1 mg/cm^2 average thickness. A correction of 1.5 percent for self-absorption in the counter samples was made. Taking $\rho_m=1.11$ and $B=1.13$, four runs with aluminum backscatterer yielded $\bar{E}=0.0748$ Mev. One run with water (agar) backscatterer yielded 0.0738 Mev. Our mean value of $\bar{E}$ is 0.0746 ± 0.003 Mev. The theoretical value using the maximum beta-energy of Macklin *et al.*[31] and the Coulomb correction factor of Longmire and Brown[30] is 0.0761 Mev.

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TABLE I. Average energy values for P^{32}.

Method	Value (Mev)	Reference
Spectrometer	0.695 ± 0.02	Marinelli *et al.*[a] using data of Siegbahn[b]
Spectrometer	0.700	Zumwalt *et al.*[c]
Calorimeter	0.68 ± 0.02	Zumwalt *et al.*[c] and Jenks[d]
Ionization chamber	0.63 ± 0.035	Gray[e]
Extrapolation chamber	0.696 ± 0.03	Present measurements

[a] See reference 7. [b] See reference 25. [c] See reference 5. [d] See reference 26. [e] See reference 14.

V. CONCLUSIONS

The average energy results of this research agree within the accuracy of the experiment with average energy values based on the Fermi theory using the best experimental data in the literature for maximum beta-energy. The method provides a relatively simple means of determining average energy for both simple and complex beta-spectra, provided there is not an overwhelming amount of gamma-radiation. With new tables of the Coulomb correction factor to be published,[32] it should be possible to compute large numbers of very accurate average energy values using published values of maximum beta-energy in the literature.

For isotopes of known average energy, the extrapolation chamber may be used to determine absolute disintegration rate.[12] This should be particularly useful for beta-emitting isotopes of low energy.

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