and (4) was averaged over initial phase angles of a_i^0/a_j^0 since this phase is initially unknown.

In collision broadening, each molecule is exposed to pulses of radiation of different lengths t. We suppose that if there are n molecules, the number dn having their last collision between times t and $t+dt$ in the past is

$$dn=nf(t)dt. \tag{5}$$

We also assume that the collision returns the molecule to a random Boltzmann distribution. The time derivative of (4) is the instantaneous rate at which a molecule which had its last collision a time t in the past is making a transition. Now multiply by the number of such molecules, given in (5), and integrate over all molecules. This gives for the net number of absorbing transitions per second,

$$\frac{n\Omega^2(|a_i^0|^2-|a_j^0|^2)}{2(\delta\omega)}\int_0^\infty f(t)\,\sin t(\delta\omega)dt, \tag{6}$$

in the limit of small electric fields (no saturation). Thus, the power attenuation constant per centimeter is

$$\alpha_{ij}(\delta\omega)=\frac{D_{ij}}{\pi(\delta\omega)}\int_0^\infty f(t)\,\sin t(\delta\omega)dt, \tag{7}$$

where

$$D_{ij}=\frac{4\pi^2N}{3hc}\frac{e^{-E_i/kT}}{\Sigma_n e^{-E_n/kT}}\frac{\nu_{ij}^2|\mu_{ij}|^2}{kT}, \tag{8}$$

and N is the number of molecules per cm^3; h is Planck's constant; c is the velocity of light; $\Sigma_n \exp(-E_n/kT)$ is the partition sum; $\nu_{ij}=\omega_{ij}/2\pi$; k is Boltzmann's constant; T is the absolute temperature; $|\mu_{ij}|^2=(\mu_Z{}^{ij})^2+(\mu_Y{}^{ij})^2+(\mu_X{}^{ij})^2$. Equations (7) and (8) reduce to the Van Vleck-Weisskopf[4] formula for $f(t)=\exp(-t/\tau)/\tau$.

For the case of broadening solely by collisions with the walls of the parallel-plane cell,

$$f(t)=\frac{2}{at^2}\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}\int_{z=0}^{a} z\exp(-mz^2/2kTt^2)dz. \tag{9}$$

Substitution of (9) in (7) results in a difficult integral, but one which can be evaluated approximately for small $(\delta\omega)$. The result is

$$\alpha_{ij}{}^N=\frac{D_{ij}a}{\pi^{\frac{3}{2}}}\left(\frac{m}{2kT}\right)^{\frac{1}{2}}\left\{\log_e\left[\frac{1}{a(m/2kT)^{\frac{1}{2}}|\delta\omega|}\right]-0.4\right\}. \tag{10}$$

There is less than 30 percent error for

$$\delta\omega\leqslant(0.1/a)(2kT/m)^{\frac{1}{2}}, \tag{11}$$

and the percent error approaches zero as $\delta\omega\rightarrow 0$. Figure 1 shows the results of the present work and a Lorentz shape using $A=1/2\pi\sqrt{6}$ (Gordy's third value). The very considerable discrepancy is evident. Our work shows a logarithmic singularity at $\delta\omega=0$ which is

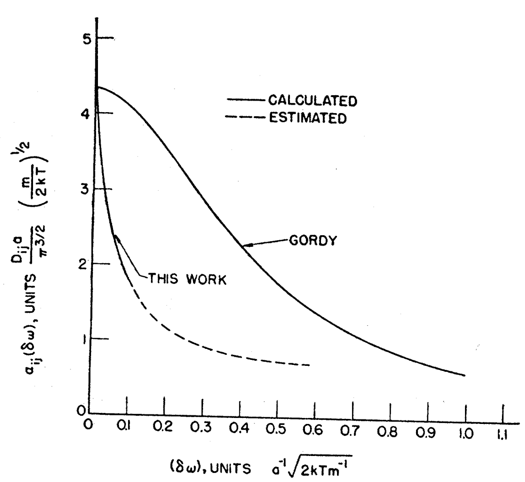

FIG. 1. Line shapes in wall broadening; infinite planes separated by distance a; area under both curves is same.

TABLE I. OCS half-width at half-intensity.

Cell	Calculated Doppler half-width	Calculated lower limit for wall-collision half-width (from Eq. (13))	Observed minimum half-width
X-band Stark	19 kc/sec	26 kc/sec	50 kc/sec
K-band Stark	19 kc/sec	65 kc/sec	100 kc/sec

of little experimental significance; for example, any finite Doppler broadening would eliminate it. Since

$$\int_{-\infty}^{\infty}\alpha_{ij}d(\delta\omega)=D_{ij}, \tag{12}$$

so that the total area under both curves is the same, an estimated effective $\Delta\nu$ for experimental purposes is about six times Gordy's third value and corresponds to $A\cong 0.4$. An exact solution for this parallel-plane case would require a numerical integration, but appears feasible. More complicated cases require more difficult integrations.

The value $A\cong 0.4$ substituted in Eq. (2) gives

$$\Delta\nu\cong 0.4(1/a)(2kT/m)^{\frac{1}{2}}. \tag{13}$$

Although Eq. (13) was derived for the parallel-plane case, it represents a lower limit for $\Delta\nu$ if a is the shortest dimension of the cell.

Unpublished experimental results (at room temperature) obtained by J. R. Eshbach of this laboratory a year ago showed that the limiting line half-width for OCS, as pressure and microwave power level are reduced, was $\Delta\nu\cong 50$ kc/sec in X-band Stark guide ($a=0.48$ cm) and $\Delta\nu\cong 100$ kc/sec in K-band Stark guide ($a=0.19$ cm). For these cells the shortest dimension is several times less than the other dimensions, so the wall-collision half-width is expected to be only slightly greater than that given by Eq. (13). That half-width is combined with the Doppler half-width to get the observed half-width. As Table I indicates, Eshbach's experiments agree quite well with the present theory.

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The Stability of Plane Poiseuille Flow

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FOR small disturbances to the plane parallel steady flow of incompressible fluid between parallel planes at $y=\pm 1$, the change in the stream function may be analyzed into components of the form

$$e^{i\alpha x}e^{ict}\varphi(y),$$

where $\varphi(y)$ must satisfy the Orr-Sommerfeld equation

$$\varphi''''-2\alpha^2\varphi''+\alpha^4\varphi+i\alpha R[(1-c-y^2)(\varphi''-\alpha^2\varphi)+2\varphi]=0,$$

with boundary conditions

$$\varphi=0,\quad \varphi'=0,\quad \text{for}\quad y=\pm 1,$$

where R is the Reynolds number of the flow. For given α and R this has solutions only for characteristic values of c.

Work by many authors on this equation, culminating in that of Lin,[1] using asymptotic formulas, leads to the conclusion that for a certain region in the $\alpha-R$ plane, c has a negative imaginary part, corresponding to a disturbance increasing in amplitude exponentially with the time, so that for $R>5300$ (about), the steady flow should be unstable. This conclusion has been criticized, particularly by Pekeris,[2] who made computations indicating stability for values of α and R for which, according to Lin, the flow should be unstable.

It was suggested three or four years ago by J. von Neumann, C. C. Lin, and C. L. Pekeris, that this question could be settled by numerical work and an attempt was then made which was successful in finding c only up to $R=1600$. We have now found it possible to integrate the equation successfully for larger values of R and have obtained the results for c given in Table I, which

TABLE I. Characteristic values of c.

α \ R	1600	2500	6400	10,000	35,000
0.9		0.2857+0.0211i	0.2444+0.0012i	0.2261−0.0040i	
1.0	0.3231+0.0262i	0.3011+0.0142i	0.2569 −0.0009i	0.2375−0.0037i	0.1886+0.0009i
1.1	0.3384+0.0206i	0.3148+0.0108i	0.2677+0.0007i	0.2470+0.0003i	0.1911+0.0116i
1.2		0.3267+0.0107i	0.2763+0.0056i	0.2535+0.0075i	

For $\alpha=1.05$, $R=8000$, $c=0.2524-0.0017i$

are believed accurate to 0.5 in the last place. Interpolation gives a critical Reynolds number $R=5780$ for $\alpha=1.02$, which has been checked by integrations. These numbers confirm Lin's results closely, and it may now be regarded as proved that plane Poiseuille flow becomes unstable at about $R=5800$. It may be noted that for a given value of α the flow is unstable only for a finite range of Reynolds numbers as was also found by Lin.

This latest work was done on International Business Machines Corporation's Selective Sequence Electronic Calculator by Donald A. Quarles, Jr. and Phyllis K. Brown. The numerical work was done to 13 digits using a step of 0.01 in y with an integration formula having an error per step proportional to the 8th derivative. The problem took about 150 hours of operating time, equivalent to about 100 years of hand computing.

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Radiations from Nb^{97}†

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THE properties of the 17-hr Zr^{97} and of its daughter element, the 70-minute Nb^{97}, have been the subject of considerable investigation.[1-5] Spectrometric measurements[5] have yielded beta-ray energies of 1.91±0.02 Mev and 1.267±0.02 Mev, and gamma-ray energies of 0.747±0.005 Mev for Zr^{97} and 0.665±0.005 Mev for Nb^{97}. The gamma-ray at 0.747±0.005 Mev was shown to be emitted from an isomeric level in Nb^{97} of half-period 60 sec.

In the present investigation $Zr^{96}O_2$ (isotopic concentration 90 percent in Zr^{96}), obtained from the Y-12 plant, Carbide and Carbon Chemicals Division, Union Carbide and Carbon Corporation, Oak Ridge, Tennessee, was irradiated by slow neutrons in the Oak Ridge pile. The radioactive materials were received within twenty-four hours after cessation of irradiation and chemical separations were immediately commenced. The slow neutron irradiated zirconium dioxide was dissolved by potassium pyrosulfate fusion, and the separation of the niobium daughter activity from zirconium was effected by the use of Steinberg's "oxalate" procedure.[6]

The decay of Nb^{97}, freshly separated from its parent element, was followed for ten half-periods, and the half-period, taken from the slope of the decay curve was found to be 72.1±0.7 minutes. This value is to be compared with previously reported values of 68 minutes[7] and 75 minutes.[2] The decay of Zr^{97} was followed for 200 hours, and the resulting half-period was calculated to be 17.0±0.2 hours, in agreement with the earlier measurements.

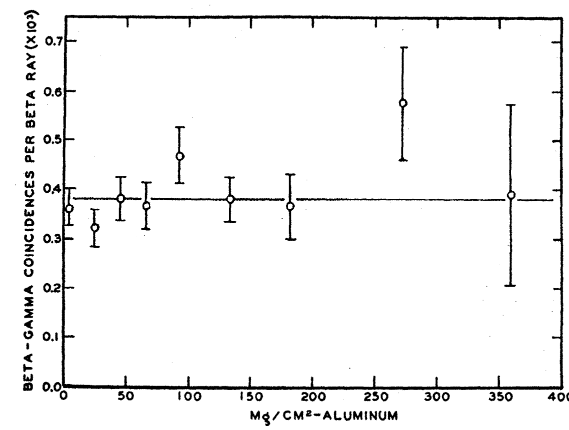

FIG. 1. Beta-gamma coincidence rate of Nb^{97} as a function of the surface density of aluminum placed before the beta-ray counter.

The beta-rays of Nb^{97}, freshly separated from its parent element, were absorbed in aluminum, and a Feather[8] plot of the data gave a maximum beta-ray energy of 1.40 Mev.

The beta-gamma coincidence rate of the 72-minute Nb^{97} is shown as a function of absorber thickness before the beta-ray counter in Fig. 1. It is seen to be constant, independent of the beta-ray energy, suggesting that the beta-ray spectrum of Nb^{97} is simple. Calibration of the beta-gamma coincidence counting arrangement by the beta-gamma coincidence rate of Sc^{46} showed that each beta-ray of Nb^{97} is followed, on the average, by 0.7 Mev of gamma-ray energy. Each point of Fig. 1 was, of course, properly corrected for decay of the source.

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Microwave Spectroscopy at High Temperature-Spectra of CsCl and NaCl*

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A SPECTROMETER for measurement of microwave absorption by gases at high temperature has been constructed,[1,2] and with it spectra of gaseous NaCl, KCl, CsCl, and TlCl have been obtained. Microwaves pass through a 5-foot absorption cell which can be held at temperatures at least as high as 875°C. Absorption lines are modulated by Stark effect to give sensitive detection.

At approximately 775°C the pure rotational transition $J=1\rightarrow2$ of NaCl was observed. Frequencies for the two Cl isotopes and various vibrational states are listed in Table I. These give $B_e(Cl^{35})=6536.9\pm0.3$ Mc, $\alpha_e(Cl^{35})=48.1\pm0.1$ Mc, and the internuclear distance $r_e=2.3606\pm0.0003$A. Frequency measurements of the absorption lines were made with a frequency standard. However, experimental conditions gave lines several megacycles broad, which limited the precision of measurements which could be easily obtained to that indicated in Table I.