Half-Life and Mass Assignment of Argon 39†

H. ZELDES, B. H. KETELLE, AND A. R. BROSI
Chemistry Division, Oak Ridge National Laboratory,
AND
C. R. FULTZ AND R. F. HIBBS
Assay Laboratory, Y-12 Plant, Carbide and Carbon Chemicals Corporation, Oak Ridge, Tennessee
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AN argon isotope which decays with an "α" type forbidden beta-spectrum and a half-life greater than 15 yr was tentatively assigned to mass number 39 in an earlier report.[1] This tentative mass assignment has been confirmed, and a definite half-life has been established for A^{39} in the work presented here.

The argon produced in four samples of KCl by bombardment in a nuclear reactor for about one year was released by dissolving the KCl in water. After purification with hot calcium vapor the volume of inert gas in three of the samples was measured. The bremsstrahlung radiation of these three samples was compared to that of the fourth sample in a high pressure ionization chamber. The isotopic abundance ratios in the three samples with known volumes, measured in a 60° sector type mass spectrometer, are given in Table I.

TABLE I. Isotopic abundance ratios of argon from bombarded KCl

Mass No.	Percent abundance Sample 1	Sample 2	Sample 3
36	0.24	0.16	0.092
38	85.96	85.76	84.65
39	5.35	5.43	5.32
40	8.38	8.57	9.84
(41)	0.077	0.066	0.10

The disintegration rate of the fourth sample was determined by mounting it, in a thin-walled bulb, as a source in a magnetic lens beta-ray spectrometer. The transmission of the spectrometer was determined by measuring the beta spectra of Na^{24}, P^{32}, Co^{60} and Au^{198} sources with known disintegration rates. From the spectrometer transmission and the integral of the "α"-type beta-energy distribution, the fourth sample was found to have 6.8×10^6 disintegrations per second.

Since the disintegration rate and the number of atoms with masses 36, 38, 39, 40, and 41 are known for each of the three samples, a decay constant can be computed provided that the activity can be assigned to a definite mass number. The argon isotopes with mass numbers 36, 38, and 40 are known to be stable with respect to beta-decay and therefore can be excluded from consideration. From a study of Cl^{39} Haslam and co-workers[2] have concluded that A^{39} has a long half-life. However, Zucker and Watson[3] have produced a 2-min activity by a deuteron bombardment of enriched A^{38} which they assign to A^{39}.

In the present experiments the amount of radioactivity correlates within 10 percent with the number of mass 39 atoms but only within a factor of two with the number of mass 41 atoms. In addition, if the observed activity is attributed to mass 41 instead of mass 39, the data yield a half-life of about 4 yr, whereas direct decay measurements now show that the half-life is greater than 50 yr. It is concluded, therefore, that the long-lived beta-emitter observed in this work is associated with mass number 39, and the computations summarized in Table II have been made on this basis.

The data in Table II have been corrected for two sources of error. When air containing approximately the same volume of argon as the A^{39} samples was treated with hot calcium vapor, the volume percent of inert gas was reproducible, but it averaged 5 percent higher than the accepted value for the abundance of argon in air. Therefore, the volumes recorded in Table II are 5 percent less than the measured volumes. It seems probable that hydrogen was not completely removed by the hot calcium treatment. The disintegration rate of the beta-ray spectrometer source was corrected for absorption losses in the source container. This correction, which amounted to 6.5 percent, was determined by measuring a Au^{198} source with and without absorbers.

TABLE II. A^{39} decay constant data.

Sample	1	2	3
Vol in mm³ at 0°C and 760 mm	32.6	62.3	57.5
Percent abundance of mass 39	5.35	5.43	5.32
Atoms of mass 39	4.7×10^{16}	9.1×10^{16}	8.2×10^{16}
Activity relative to sample 4	0.628	1.11	0.903
Disintegrations per second	4.3×10^6	7.6×10^6	6.2×10^6
Decay constant sec^{-1}	9.2×10^{-11}	8.3×10^{-11}	7.5×10^{-11}
Half-life (years)	240	265	290

On the basis of the measurements reported here it is concluded that A^{39} has a half-life of 265 ± 30 yr. The $\log(ft)$ value is 9.9, and the $\log[(W_0^2-1)ft]$ value is 10.4. The latter falls within the range of values given by Mayer, Moszkowski, and Nordheim[4] for beta-transitions where $\Delta l=1$ and $\Delta I=2$.

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Broadening of Microwave Absorption Lines by Collisions with the Cell Walls*

R. H. JOHNSON AND M. W. P. STRANDBERG
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts
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THE literature regarding line broadening by collisions with the walls of the absorbing cell presents various formulas. They all assume a Lorentz line shape in which the line breadth parameter is of the form

$$\Delta\nu=(A/\Lambda)(2kT/m)^{\frac{1}{2}}. \tag{1}$$

Here $\Delta\nu$ is half the width of the line between points of half-intensity; Λ is a length characteristic of the cell, or the volume to surface ratio of the cell; $(2kT/m)^{\frac{1}{2}}$ is the mean molecular speed; A is a numerical factor. We consider only the case of a cell consisting of the space between parallel infinite planes separated by a distance a. Taking $\Lambda=a$,

$$\Delta\nu=(A/a)(2kT/m)^{\frac{1}{2}}. \tag{2}$$

Gordy's formulas[1] specialized to the parallel plane case have $A=1/3\pi$ and $A=1/3\pi^{\frac{1}{2}}$; these were later corrected in a mimeographed errata sheet of limited circulation to $A=1/2\pi\sqrt{6}$ and $A=1/\pi\sqrt{6}$. No explanation precedes the statement of the formulas except "It is easy to show that . . ." Bleaney and Penrose[2] give $A=1/4\pi$.

We have experimental and theoretical reason to indicate that these values for A are too small. Suppose that a molecule with an electric dipole moment is exposed to a pulse of electric field

$$E_Z=E_0\cos\omega t',\quad 0<t'<t. \tag{3}$$

Proceeding in a way similar to that of Rabi[3] we can show that

$$|a_j|^2-|a_j^0|^2=\frac{\Omega^2(|a_i^0|^2-|a_j^0|^2)}{(\delta\omega)^2+\Omega^2}\sin^2\left\{\frac{t}{2}[(\delta\omega)^2+\Omega^2]^{\frac{1}{2}}\right\}, \tag{4}$$

where $|a_i^0|^2$, $|a_j^0|^2$ are the initial probabilities that a molecule be in molecular state i and j, respectively; $|a_j|^2$ is the final probability that the molecule be in state j; $\Omega=E_0\mu_Z^{ij}/\hbar$; μ_Z^{ij} is the matrix element of the Z-component of electric dipole moment between states i and j; $\delta\omega=\omega_{ij}-\omega$; $\hbar\omega_{ij}=E_i-E_j$; E_i and E_j are the energies of states i and j, respectively. It was assumed that $\omega_{ij}\gg1/t$, and (4) was averaged over initial phase angles of a_i^0/a_j^0 since this phase is initially unknown.

In collision broadening, each molecule is exposed to pulses of radiation of different lengths t. We suppose that if there are n molecules, the number dn having their last collision between times t and $t+dt$ in the past is

$$dn=nf(t)dt. \quad (5)$$

We also assume that the collision returns the molecule to a random Boltzmann distribution. The time derivative of (4) is the instantaneous rate at which a molecule which had its last collision a time t in the past is making a transition. Now multiply by the number of such molecules, given in (5), and integrate over all molecules. This gives for the net number of absorbing transitions per second,

$$\frac{n\Omega^2(|a_i^0|^2-|a_j^0|^2)}{2(\delta\omega)}\int_0^\infty f(t)\,\sin t(\delta\omega)dt, \quad (6)$$

in the limit of small electric fields (no saturation). Thus, the power attenuation constant per centimeter is

$$\alpha_{ij}(\delta\omega)=\frac{D_{ij}}{\pi(\delta\omega)}\int_0^\infty f(t)\,\sin t(\delta\omega)dt, \quad (7)$$

where

$$D_{ij}=\frac{4\pi^2N}{3hc}\frac{e^{-E_i/kT}}{\Sigma_n e^{-E_n/kT}}\frac{\nu_{ij}^2|\mu_{ij}|^2}{kT}, \quad (8)$$

and N is the number of molecules per cm^3; h is Planck's constant; c is the velocity of light; $\Sigma_n \exp(-E_n/kT)$ is the partition sum; $\nu_{ij}=\omega_{ij}/2\pi$; k is Boltzmann's constant; T is the absolute temperature; $|\mu_{ij}|^2=(\mu_Z{}^{ij})^2+(\mu_Y{}^{ij})^2+(\mu_X{}^{ij})^2$. Equations (7) and (8) reduce to the Van Vleck-Weisskopf[4] formula for $f(t)=\exp(-t/\tau)/\tau$.

For the case of broadening solely by collisions with the walls of the parallel-plane cell,

$$f(t)=\frac{2}{at^2}\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}\int_{z=0}^a z\exp(-mz^2/2kTt^2)dz. \quad (9)$$

Substitution of (9) in (7) results in a difficult integral, but one which can be evaluated approximately for small $(\delta\omega)$. The result is

$$\alpha_{ij}{}^N=\frac{D_{ij}a}{\pi^{\frac{3}{2}}}\left(\frac{m}{2kT}\right)^{\frac{1}{2}}\left\{\log_e\left[\frac{1}{a(m/2kT)^{\frac{1}{2}}|\delta\omega|}\right]-0.4\right\}. \quad (10)$$

There is less than 30 percent error for

$$\delta\omega\leqslant(0.1/a)(2kT/m)^{\frac{1}{2}}, \quad (11)$$

and the percent error approaches zero as $\delta\omega\rightarrow 0$. Figure 1 shows the results of the present work and a Lorentz shape using $A=1/2\pi\sqrt{6}$ (Gordy's third value). The very considerable discrepancy is evident. Our work shows a logarithmic singularity at $\delta\omega=0$ which is

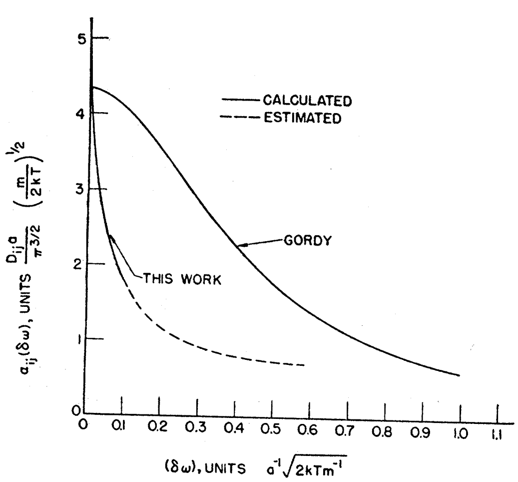

FIG. 1. Line shapes in wall broadening; infinite planes separated by distance a; area under both curves is same.

TABLE I. OCS half-width at half-intensity.

Cell	Calculated Doppler half-width	Calculated lower limit for wall-collision half-width (from Eq. (13))	Observed minimum half-width
X-band Stark	19 kc/sec	26 kc/sec	50 kc/sec
K-band Stark	19 kc/sec	65 kc/sec	100 kc/sec

of little experimental significance; for example, any finite Doppler broadening would eliminate it. Since

$$\int_{-\infty}^{\infty}\alpha_{ij}d(\delta\omega)=D_{ij}, \quad (12)$$

so that the total area under both curves is the same, an estimated effective $\Delta\nu$ for experimental purposes is about six times Gordy's third value and corresponds to $A\cong 0.4$. An exact solution for this parallel-plane case would require a numerical integration, but appears feasible. More complicated cases require more difficult integrations.

The value $A\cong 0.4$ substituted in Eq. (2) gives

$$\Delta\nu\cong 0.4(1/a)(2kT/m)^{\frac{1}{2}}. \quad (13)$$

Although Eq. (13) was derived for the parallel-plane case, it represents a lower limit for $\Delta\nu$ if a is the shortest dimension of the cell.

Unpublished experimental results (at room temperature) obtained by J. R. Eshbach of this laboratory a year ago showed that the limiting line half-width for OCS, as pressure and microwave power level are reduced, was $\Delta\nu\cong 50$ kc/sec in X-band Stark guide ($a=0.48$ cm) and $\Delta\nu\cong 100$ kc/sec in K-band Stark guide ($a=0.19$ cm). For these cells the shortest dimension is several times less than the other dimensions, so the wall-collision half-width is expected to be only slightly greater than that given by Eq. (13). That half-width is combined with the Doppler half-width to get the observed half-width. As Table I indicates, Eshbach's experiments agree quite well with the present theory.

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The Stability of Plane Poiseuille Flow

L. H. THOMAS
Watson Scientific Computing Laboratory, Columbia University, New York, New York
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FOR small disturbances to the plane parallel steady flow of incompressible fluid between parallel planes at $y=\pm 1$, the change in the stream function may be analyzed into components of the form

$$e^{i\alpha x}e^{ict}\varphi(y),$$

where $\varphi(y)$ must satisfy the Orr-Sommerfeld equation

$$\varphi''''-2\alpha^2\varphi''+\alpha^4\varphi+i\alpha R[(1-c-y^2)(\varphi''-\alpha^2\varphi)+2\varphi]=0,$$

with boundary conditions

$$\varphi=0,\quad \varphi'=0,\quad \text{for}\quad y=\pm 1,$$

where R is the Reynolds number of the flow. For given α and R this has solutions only for characteristic values of c.

Work by many authors on this equation, culminating in that of Lin,[1] using asymptotic formulas, leads to the conclusion that for a certain region in the $\alpha-R$ plane, c has a negative imaginary part, corresponding to a disturbance increasing in amplitude exponentially with the time, so that for $R>5300$ (about), the steady flow should be unstable. This conclusion has been criticized, particularly by Pekeris,[2] who made computations indicating stability for