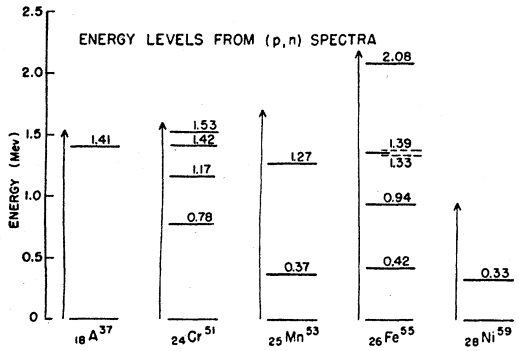

FIG. 2. Energy levels obtained from five (p, n) investigations. The vertical arrows indicate the excitation region studied.

The (p, n) reactions were produced by allowing monoenergetic protons from the Rockefeller electrostatic generator to strike thin targets (25 to 50 kev) prepared by evaporation onto backings of 10-mil tantalum. Target materials of pure cobalt metal, enriched Cr^{53} metal,[1] and NaCl of high purity were used. The (p, n) thresholds of Cl^{35} and Na^{23} are estimated from disintegration data to be above 4 Mev.

The neutron energies were measured by means of proton-recoil tracks in Eastman NTB emulsions of 200 microns thickness. Plates were placed a mean distance of 18 cm from the target at both 0 and 90 degrees with respect to the incident proton beam. Recoil tracks within 10 degrees of the neutron direction were accepted for measurement. The range-energy relation for the emulsion was determined by measuring neutrons of known energies from the $Li(p, n)$ reaction. Standard corrections were applied to convert the track length distributions to relative neutron intensities.[4]

The energy distribution of neutrons emitted in the forward direction for the three reactions is shown in Fig. 1. It is seen that

TABLE I. Measured Q-values and relative intensities for the neutron groups shown in Fig. 1. The last column lists the positions of the excited states in the residual nuclei.

Reaction	Neutron group	Q-value (Mev)	Relative intensities	Excitation energy (ΔQ in Mev)
$Cr^{53}(p, n)Mn^{53}$	A	-1.37	0.35 ± 0.04	Ground state
	B	-1.74	0.43 ± 0.04	0.370 ± 0.05
	C	-2.64	0.22 ± 0.03	1.27 ± 0.05
$Cl^{37}(p, n)A^{37}$	D	-1.58	0.78 ± 0.08	Ground state
	E	-2.99	0.22 ± 0.03	1.41 ± 0.05
$Co^{59}(p, n)Ni^{59}$	F	-1.84	0.28 ± 0.04	Ground state
	G	-2.17	0.72 ± 0.07	0.330 ± 0.05

well-defined groups are present which result from transitions to the ground state and low excited states of the residual nuclei. The Q-values and relative intensities of these groups are listed in Table I. Levels in A^{37} have been observed by Zucker and Watson[5] using the $A^{36}(d, p)$ reaction. Their value of 1.44 Mev for the first excited state agrees with our result. Grosskreutz[6] has also observed this level from the (p, n) reaction. No additional information is available on the excited states of Mn^{53} and Ni^{59}.

It is recognized that the selection rules of beta- and gamma-ray emission which result in large differences in transition probabilities will mask the existence of excited states with unfavorable spin and parity. On the other hand, it is relatively easy for the fast protons and neutrons of the (p, n) reaction to carry several units of orbital angular momentum[4] and, therefore, the intensity of transitions to the residual states is not strongly influenced by the spin and parity of these states. Hence, one might expect to find more levels from the (p, n) investigation. This is the case for Fe^{55} where a direct comparison is possible.[4] It should be borne in mind, however, that the limited resolution of our (p, n) measurements prevents recognition as individual levels when the separation is ~ 50 kev or less. In Fig. 2 we have plotted the levels obtained for the five nuclei we have studied.[7]

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Numerical Work on the Fluctuation Problem in Cascade Shower Theory

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THE analytical development of the theory of fluctuations in the cascade showers of cosmic radiation has been presented in a series of recent publications. These cover the nucleon cascade in homogeneous nuclear matter,[1,2] the nucleon cascade in a finite absorber,[3] and the electron-photon cascade neglecting ionization loss (approximation A).[4] In each case the diffusion equations for the distribution function have been solved formally, and also general formulas have been obtained for its kth moment (the (k, l)th moment in the electron-photon cascade). The problem of obtaining explicit exact expressions for the distribution function by carrying out the integrations in the formal solutions looks so forbidding as to make one consider alternative, approximate methods, and such a method has been developed by Green and Messel,[5] who expand the distribution function in terms of its moments and the moments of some "trial" function. If the trial function is a reasonable approximation to the actual distribution function the expansion converges rapidly so that only the first few moments are required. (In the present case it turns out that the Pólya distribution is an excellent trial function.) Apart from the problem of finding the distribution functions themselves, a quantitative knowledge of their first few moments would be of great value in interpreting the experimentally observed fluctuations in cosmic-ray cascade showers, their behavior with height, latitude, etc.

The purpose of this note is therefore to announce that we are planning a program of numerical work on the Ferranti electronic computer at the University of Toronto Computation Center. It is proposed first to concentrate on the simplest case, namely the nucleon cascade in homogeneous nuclear matter. Such cascades occur, for example, in the nuclei of photographic emulsions, giving rise to the familiar "stars," so this work should make possible quantitative predictions about the numbers of particles occurring in the stars from various different kinds of nuclei. Moreover, experience gained in this work should pave the way to more difficult calculations of the nucleon cascade in a finite absorber, and the electron-photon cascade. We shall compute the first three moments of $\Phi(\epsilon, n_1, n_2; x)$, which expresses the probability of finding n_1 nucleons with energies $>\epsilon E_0$ and n_2 nucleons with energies $<\epsilon E_0$ at depth x in homogeneous nuclear matter as a result of a single primary nucleon of energy E_0; $\log_{10}\epsilon$ will be taken from -1.0 to -4.0 and x from 0 to 35 collision units, in appropriate intervals. With the Ferranti computer it will be possible to integrate the expressions for the moments directly, thus avoiding the somewhat dubious saddle point integrations of earlier, approximate evaluations of the first two moments, which were carried out on desk machines.[6]

This work will be carried out with the collaboration of Dr. C. C. Gotlieb and his staff at the Computation Center; we are greatly indebted to Dr. Gotlieb for according these facilities and for a discussion of the detailed programming.

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A Method of Synthesis of the Statistical and Impact Theories of Pressure Broadening

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TWO types of theories of pressure broadening have been current in the literature: the "statistical" theory of Kuhn[1] and Margenau,[2] and the "impact" theory of Weisskopf,[3] Lenz,[4] and others. Spitzer[5] and Holstein[6] have indicated the limits of validity of the two types of theory: At low pressures the statistical theory is certainly valid on the wings of the line, the impact theory in the center, while at high pressures the statistical theory is valid if its fundamental assumption that the frequency perturbation for a given atom is the sum of that due to all others in the gas,

$$\nu=\sum_{i=1}^{N}\nu(R_i), \tag{1}$$

is not too seriously incorrect. Most of these discussions and this note are confined to the "adiabatic" or nondegenerate case in which the perturbations due to the atoms in the gas may be expressed simply as frequency perturbations.

Lindholm[7] carried out a complete study of the transition between the two theories for a frequency perturbation proportional to $1/R^6$. Unfortunately, he used a simplified model of the collision in which the frequency was constant during the collision and thus the perturbation started and stopped abruptly; some of his results seem to be in error because of this fact.

This note presents a more general and more workable technique for synthesizing the two theories and studying the transition between them. The spectral intensity which is desired is given by the Fourier integral

$$I(\omega)=\left|\int_{-\infty}^{\infty}\exp\left[i\left(\omega t-\int_{-\infty}^{t}\nu(t')dt'\right)\right]dt\right|^2, \tag{2}$$

where $\nu(t')$ is the frequency at the time t', given by (1) with the R_j assuming the correct values for this time. Now we transform (2) by the standard correlation-function transformation[8] and obtain

$$I(\omega)=\int_{-\infty}^{\infty}e^{i\omega\tau}\varphi(\tau); \tag{3a}$$

$$\varphi(\tau)=\int_{-\infty}^{\infty}dt\,\exp\left[i\int_{t}^{t+\tau}\nu(t')dt'\right]=\left\langle\exp\left[i\int_{0}^{\tau}\nu(t')dt'\right]\right\rangle_{\mathrm{Av}}. \tag{3b}$$

We now use (1) and obtain

$$\varphi(\tau)=\left\langle\prod_j\exp\left[i\int_0^\tau\nu(R_j)dt\right]\right\rangle_{\mathrm{Av}}.$$

To an approximation more than adequate for our purposes the motions of the molecules in the gas are independent, so that

$$\varphi(\tau)=\left\{\left\langle\exp\left[i\int_0^\tau\nu(R)dt\right]\right\rangle_{\mathrm{Av}}\right\}^N. \tag{4}$$

More general methods of averaging are possible than the following, in case the situation is more complicated, but this should suffice to show the method: Suppose that the paths of atoms are straight relative to each other; then the parameters to be averaged over are b, the impact parameter, and x_0, the position of the atom at the start of the time-interval 0 to τ along the straight line of impact parameter b. Now

$$2\pi\int bdb\int dx_0=V, \tag{5}$$

where V is the volume available to the N atoms of the gas. Then

$$\varphi(\tau)=\left(1-\frac{2\pi}{V}\int_0^\infty bdb\int_{-\infty}^{\infty}dx_0\times\left\{1-\exp\left[i\int_0^\tau\nu([(x_0+vt)^2+b^2]^{\frac{1}{2}})dt\right]\right\}\right)^N=\exp[-nV'(\tau)]. \tag{6}$$

Here $n=N/V=$ density of atoms per cc, and

$$V'(\tau)=2\pi\int_0^\infty bdb\int_{-\infty}^{\infty}dx_0\times\left\{1-\exp\left[i\int_0^\tau\nu([(x_0+vt)^2+b^2]^{\frac{1}{2}})dt\right]\right\}. \tag{7}$$

It is an easy matter, for which we have no space here, to show that (6) and (7), with (3a), lead to the impact and statistical theories in the proper limits. The criteria for validity of the two theories are easily derived, being the same as those given in references 5 and 6; in particular, these hold even for $\nu\sim1/R^3$, a case in which there had been some question as to the validity of the impact theory in the low-pressure limit. Analytical methods can be applied to give first corrections to the two types of theories, while V', which is a function only of one variable for a given form of $\nu(R)$, can be approximated numerically for any given $\nu(R)$ without much difficulty. The numerical calculation of the line shape for various densities, from (3a), should then be fairly easy.

Further details, and some computations using the above method, will be given in a later publication.

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Quadrupole Coupling Ratio of the Chlorine Isotopes*

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THE ratio of the nuclear quadrupole coupling constants, eqQ, for the chlorine isotopes has been found to depend to about 0.02 percent on the molecular environment. Radio frequency transitions were measured for several chlorine compounds in the solid state.

A variation in the ratio $(eqQ)_{35}/(eqQ)_{37}$ with molecular species is to be expected because of nuclear polarization and certain kinds of molecular vibrations.[1] A change as large as 0.1 percent has previously been reported in microwave spectra[2], but measurements of direct quadrupole transitions[3] with an accuracy of the order of 0.01 percent had failed to show conclusively any such variation.

The lines are narrow enough to permit considerably better accuracy, and were hence measured to an accuracy of about 0.001 percent. At Columbia University this degree of precision was obtained by using a Signal Corps type BC221 frequency meter to measure differences between the line frequency and a nearby harmonic of a standard crystal oscillator. At Bell Telephone