energy proton bombardments. Thanks also are due G. B. Rossi for assistance in the carbon ion bombardments.

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The Symmetrical Pseudoscalar Meson Theory of Nuclear Forces

MAURICE M. LÉVY
Institute for Advanced Study, Princeton, New Jersey
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An analysis has been made of the neutron-proton interaction yielded by the symmetrical pseudoscalar meson theory, with pseudoscalar coupling, using the Tamm-Dancoff[1] nonadiabatic treatment, which has been extended in order to include pair formation and higher order effects in the exchange of mesons.[2]

By eliminating all the probability amplitudes $a_\lambda^{(m,n)}$ of states where m mesons and n nucleon pairs are present, an equation can be obtained for $a_\lambda^{(0,0)}$, which, in the nonrelativistic region, reduces to the Schrödinger equation.[2] In this region, this amplitude is identical with the "large" Fourier component of the Bethe and Salpeter wave function,[3] for equal times of the interacting nucleons.

In the following treatment, the exact pseudoscalar interaction has been calculated in the nonrelativistic region. For distances of the order of the nucleon Compton wavelength, the interaction has been replaced by an infinitely repulsive potential of radius r_0, since the pseudoscalar meson theory cannot be expected to give reliable results in this region, on account of the existence of heavier mesons than π-mesons, isobaric states of nucleons, etc. Besides, there are some indications that the pseudoscalar interaction becomes indeed repulsive at short distances, the lowest order terms of the potential being dominated by the so-called "contact" terms which actually have a range of order $\hbar/Mc$. For example, the second-order interaction behaves in the relativistic region like a "nonlocal" operator equivalent to a potential $V_0^{(2)}(r)$, defined by

$$V_0^{(2)}(r)\psi^{(0,0)}(\mathbf{r})=-\frac{1}{3}\frac{f^2}{4\pi}M^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\frac{K_1(Mr)}{r}\times\int\frac{K_1(Mr')}{r'}\psi^{(0,0)}(\mathbf{r}')d\mathbf{r}',\quad(1)$$

where $\psi^{(0,0)}(\mathbf{r})$ is the Fourier transform of $a^{(0,0)}(\mathbf{p})$, in the center-of-mass system, and $K_n(x)$ the nth order Hankel function of imaginary argument. $V_0^{(2)}$ is repulsive for singlet and triplet even states.

For $r>r_0$, the second-order interaction reduces to the well-known potential (which can also be obtained from the pseudovector coupling)

$$V_2(r)=+\frac{f^2}{4\pi}\left(\frac{\mu}{2M}\right)^2\frac{(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{3}\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\left[1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right]S_{12}\right\}\frac{e^{-\mu r}}{r},\quad(2)$$

where $S_{12}=3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})/r^2-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$. In the fourth-order in f, the main contribution to the interaction comes from processes which involve the creation of two pairs of nucleons. It is spin and charge independent and can be written

$$V_4(r)=-3\left(\frac{f^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^2\frac{1}{\mu r^2}\left\{\frac{2}{\pi}K_1(2\mu r)+\left[\frac{2}{\pi}K_0(\mu r)\right]^2\right\}.\quad(3)$$

The first term of (3) comes from virtual processes involving the simultaneous exchange of 2 mesons; the second from effects in which only one meson is present at the same time in the intermediate states.

Higher order terms of the interaction which do not involve radiative effects have been found to be small for $r\geqslant r_0$, for two reasons:

(a) Their strength becomes smaller and smaller even if $f^2/4\pi$ is of the order of 10, because of higher and higher powers of $(Mr)^{-1}$ multiplying the interaction. For example, the sixth-order interaction is of the order of $(f^2/4\pi)(Mr)^{-2}$ times the fourth-order interaction.

(b) Their range also becomes smaller and smaller, since a process involving the exchange of m mesons yields an interaction range of order $(1/m)(\hbar/\mu c)$.

An analysis has also been made of the contribution of the radiative effects to the interaction, starting from the equation of Bethe and Salpeter, and transforming it into a one-time equation, by means of a method which has been given previously.[2] The vertex parts of the irreducible diagrams are taken into account by modifying γ_5, which becomes, to the second order in f, between two states of four-momenta p and p':

$$\Gamma_5(p,p')=\gamma_5[1-(f^2/4\pi^2)U(p'-p)],\quad(4)$$

where

$$U(p)=\int_0^1\frac{x(1-2x)p^2}{x(1-x)p^2+M^2}dx.\quad(5)$$

Similarly, the Δ_F function is replaced by Δ_F', in order to account for closed loops insertion in the meson lines:

$$\Delta_F'(x)=-\frac{2i}{(2\pi)^4}\int e^{ikx}\frac{d^4k}{k^2+\mu^2}\left[1-\frac{f^2}{2\pi^2}U(k)\right].\quad(6)$$

These radiative terms give contributions to the interaction which are of order $(f^2/4\pi)(Mr)^{-2}$. In Eq. (4), we have omitted terms which arise from the fact that, in the initial and final states, the nucleons are bound. A careful examination of these terms shows that they give corrections of the order of $(f^2/4\pi)(Mr)^{-3}\log Mr$.

A study of the low energy properties of the neutron-proton system has been carried out using, for $r\geqslant r_0$, the potential $V_2(r)+V_4(r)$ defined by (2) and (3), and a repulsive core for $r<r_0$. The range of the forces has been chosen equal to the Compton wavelength of the π-mesons: $\hbar/\mu c=(1.40\pm0.03)10^{-13}$ cm. The two constants $f^2/4\pi$ and r_0 have been determined by fitting exactly the deuteron binding energy, (-2.227 ± 0.003) Mev, and the neutron-proton zero energy scattering length $(-23.68\pm0.06)10^{-13}$ cm. These values are as follows:

$$f^2/4\pi=9.7\pm1.3,\quad r_0=(0.38\pm0.03)(\hbar/\mu c).\quad(7)$$

This value of r_0 is in agreement with the value of r at which the interaction (1) becomes important.

The following derived quantities have been obtained using these constants:

Singlet effective range: $1.78(\hbar/\mu c)$,
Triplet effective range: $1.185(\hbar/\mu c)$,
Electric quadrupole moment: 2.08×10^{-27} cm^2,
Proportion of D state: 0.051,
Mean value of the momentum: $\langle p^2/(\mu c)^2\rangle=1.76$.

These results are in good agreement with experiment, except the quadrupole moment, which is too small by about 20 percent. This quantity is, however, sensitive to an increase of the tensor force and can probably be improved by including the sixth-order corrections.[4]

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