posed, or measurements must be made in which the effects are avoided; to check existing theories with dc experiments is useless.

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Recent Studies of the Isotopes of Emanation, Francium, and Radium*

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An earlier study[1] of the alpha-decay characteristics of the low mass isotopes of francium and emanation produced by bombardment of thorium with 340-Mev protons for the purpose of correlating these characteristics with the 126-neutron shell has been continued and expanded to include the element radium.

Greatest progress has been made in the case of emanation where the properties listed in Table I have been measured.

Table I. Radioactive properties of low mass emanation isotopes.

Isotope	Observed half-life	Alpha-particle energy ±0.02 Mev	EC/α branching ratio
Em^{209}	31 min	6.02	4–6
Em^{210}[a]	2.7 hr	6.02	~0.1
Em^{211}	16 hr	5.82	2.8
Em^{212}	23 min	6.23[b]	<0.01

[a] This isotope is to be identified with the 2.1-hr activity reported by Ghiorso, Meinke, and Seaborg, Phys. Rev. **76**, 1414 (1949).
[b] This value supersedes that given by Hyde *et al.* (see reference 1); also the Fr^{212} alpha-particle energy should be raised to 6.36 Mev.

This work was greatly facilitated by the development of a method for the preparation of platinum plates with the emanation atoms so firmly affixed that counting techniques typical for nongaseous radioactive samples could be employed. In brief, this method consisted of ionization of the gaseous atoms in a glow discharge and acceleration of these ions into a platinum plate at a potential of a few hundred volts. The method is being applied successfully to krypton and xenon as well as emanation and should be widely applicable in nuclear studies of the nuclides of these elements. This technique resembles that reported by Bergström *et al.*[2] in the study of mass-spectrographically separated radioactive isotopes of rare gas elements.

It is interesting to note that a plot of the alpha-decay energies for the emanation isotopes against neutron number is strikingly similar to the corresponding plot for the isotopes of polonium and astatine as shown in Fig. 1. The alpha-decay energy of At^{211} is given as 5.96 Mev (alpha-particle energy, 5.85 Mev) to correspond with a recent redetermination by us.

Many experiments were performed to obtain information on francium isotopes other than Fr^{212} in this mass region. In this work carrier free fracium fractions were isolated from thorium target solutions by an improved method developed by Hyde.[3] Any isotopes of half-life greater than 5 minutes would have been identified easily. It can be stated that the apparent half-lives of Fr^{213}, Fr^{211}, and isotopes of mass less than 211 are all shorter than 5 minutes. Incomplete results indicate a half-life of 2–5 minutes for Fr^{211} with electron capture (EC) prominent.

Mass assignments in the genetically-linked $Fr^{212}-Em^{212}-At^{208}-Po^{208}$ system were made certain by a mass spectrographic assignment of the key nuclide Fr^{212}. This was done with the time of flight mass spectrometer developed by Glenn.[4]

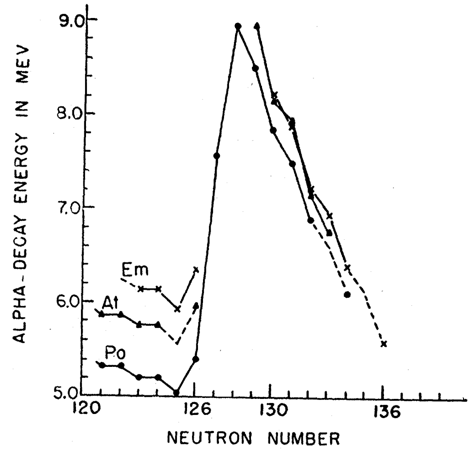

Fig. 1. Similarity of shell effect in elements 84–86 shown by plot of alpha-decay energy *versus* neutron number.

Attempts were made to isolate chemically radium isotopes of mass 214 or less and thus prove that the shell effect extended to this element. However, the stabilization was expected to lengthen the alpha-decay half-lives to the order of only a few minutes. Evidence was found for the following sequence:

$$\begin{array}{ccccc} Ra^{213} & \xrightarrow{\alpha} & Em^{209} & \xrightarrow{EC} & At^{209}. \\ \sim 2\text{ min} & & 31\text{ min} & & \\ & & \downarrow \alpha & & \\ & & Po^{205} & & \end{array}$$

The Ra^{213} was not observed directly because of its short half-life and because of the interference from heavier radium isotopes.

Cross checks are being carried out using a quite distinct method of preparation of the nuclides, namely the bombardment of lead foils with carbon ions. Miller *et al.*[5] have recently reported the attainment of a sizable beam of energetic (>100 Mev) C^{+6} ions in the Crocker Laboratory 60-inch cyclotron and have effected such reactions as $Au^{197}(C^{12}, 4n)At^{205}$. For our purposes bombardment of lead foils produces radium isotopes of mass 216 or less by such reactions as $Pb^{208}(C^{12}, 4n)Ra^{216}$. These directly produced radium isotopes decay quickly by alpha-particle emission or electron capture to the emanation and francium isotopes in which we are interested. An outstanding advantage of this method, particularly for Ra^{213}, is that none of the higher mass isotopes of these elements can possibly be produced, and hence the interference from them is not present.

As a by-product of the studies of the emanation fraction from the thorium plus proton bombardments, some properties of the previously unreported Em^{221} were observed. The gaseous fraction from the dissolution of a thorium foil target bombarded with 100-Mev protons was purified and placed on a platinum plate using the glow discharge collection technique. This plate when examined in the alpha-ray pulse analyzer showed the alpha-particle peaks corresponding to 4.8-minute Fr^{221} and its 0.020 second daughter At^{217}. Later the expected growth and decay of the Po^{213} alpha-peak were observed. The $Fr^{221}-At^{217}$ double peak decayed with a half-life of 24 minutes. These facts can be interpreted only as meaning that Em^{221} is a beta-emitter of 24 minutes half-life. The alpha-branching is appreciable (of the order of 25 percent) and is currently under investigation.

This work is continuing and a complete report will be issued later.

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The Symmetrical Pseudoscalar Meson Theory of Nuclear Forces

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An analysis has been made of the neutron-proton interaction yielded by the symmetrical pseudoscalar meson theory, with pseudoscalar coupling, using the Tamm-Dancoff[1] nonadiabatic treatment, which has been extended in order to include pair formation and higher order effects in the exchange of mesons.[2]

By eliminating all the probability amplitudes $a_\lambda^{(m,n)}$ of states where m mesons and n nucleon pairs are present, an equation can be obtained for $a_\lambda^{(0,0)}$, which, in the nonrelativistic region, reduces to the Schrödinger equation.[2] In this region, this amplitude is identical with the "large" Fourier component of the Bethe and Salpeter wave function,[3] for equal times of the interacting nucleons.

In the following treatment, the exact pseudoscalar interaction has been calculated in the nonrelativistic region. For distances of the order of the nucleon Compton wavelength, the interaction has been replaced by an infinitely repulsive potential of radius r_0, since the pseudoscalar meson theory cannot be expected to give reliable results in this region, on account of the existence of heavier mesons than π-mesons, isobaric states of nucleons, etc. Besides, there are some indications that the pseudoscalar interaction becomes indeed repulsive at short distances, the lowest order terms of the potential being dominated by the so-called "contact" terms which actually have a range of order $\hbar/Mc$. For example, the second-order interaction behaves in the relativistic region like a "nonlocal" operator equivalent to a potential $V_0^{(2)}(r)$, defined by

$$V_0^{(2)}(r)\psi^{(0,0)}(\mathbf{r})=-\frac{1}{3}\frac{f^2}{4\pi}M^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\frac{K_1(Mr)}{r}\times\int\frac{K_1(Mr')}{r'}\psi^{(0,0)}(\mathbf{r}')d\mathbf{r}', \quad (1)$$

where $\psi^{(0,0)}(\mathbf{r})$ is the Fourier transform of $a^{(0,0)}(\mathbf{p})$, in the center-of-mass system, and $K_n(x)$ the nth order Hankel function of imaginary argument. $V_0^{(2)}$ is repulsive for singlet and triplet even states.

For $r>r_0$, the second-order interaction reduces to the well-known potential (which can also be obtained from the pseudovector coupling)

$$V_2(r)=+\frac{f^2}{4\pi}\left(\frac{\mu}{2M}\right)^2\frac{(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{3}\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\left[1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right]S_{12}\right\}\frac{e^{-\mu r}}{r}, \quad (2)$$

where $S_{12}=3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})/r^2-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$. In the fourth-order in f, the main contribution to the interaction comes from processes which involve the creation of two pairs of nucleons. It is spin and charge independent and can be written

$$V_4(r)=-3\left(\frac{f^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^2\frac{1}{\mu r^2}\left\{\frac{2}{\pi}K_1(2\mu r)+\left[\frac{2}{\pi}K_0(\mu r)\right]^2\right\}. \quad (3)$$

The first term of (3) comes from virtual processes involving the simultaneous exchange of 2 mesons; the second from effects in which only one meson is present at the same time in the intermediate states.

Higher order terms of the interaction which do not involve radiative effects have been found to be small for $r\geqslant r_0$, for two reasons:

(a) Their strength becomes smaller and smaller even if $f^2/4\pi$ is of the order of 10, because of higher and higher powers of $(Mr)^{-1}$ multiplying the interaction. For example, the sixth-order interaction is of the order of $(f^2/4\pi)(Mr)^{-2}$ times the fourth-order interaction.

(b) Their range also becomes smaller and smaller, since a process involving the exchange of m mesons yields an interaction range of order $(1/m)(\hbar/\mu c)$.

An analysis has also been made of the contribution of the radiative effects to the interaction, starting from the equation of Bethe and Salpeter, and transforming it into a one-time equation, by means of a method which has been given previously.[2] The vertex parts of the irreducible diagrams are taken into account by modifying γ_5, which becomes, to the second order in f, between two states of four-momenta p and p':

$$\Gamma_5(p,p')=\gamma_5[1-(f^2/4\pi^2)U(p'-p)], \quad (4)$$

where

$$U(p)=\int_0^1\frac{x(1-2x)p^2}{x(1-x)p^2+M^2}dx. \quad (5)$$

Similarly, the Δ_F function is replaced by Δ_F', in order to account for closed loops insertion in the meson lines:

$$\Delta_F'(x)=-\frac{2i}{(2\pi)^4}\int e^{ikx}\frac{d^4k}{k^2+\mu^2}\left[1-\frac{f^2}{2\pi^2}U(k)\right]. \quad (6)$$

These radiative terms give contributions to the interaction which are of order $(f^2/4\pi)(Mr)^{-2}$. In Eq. (4), we have omitted terms which arise from the fact that, in the initial and final states, the nucleons are bound. A careful examination of these terms shows that they give corrections of the order of $(f^2/4\pi)(Mr)^{-3}\log Mr$.

A study of the low energy properties of the neutron-proton system has been carried out using, for $r\geqslant r_0$, the potential $V_2(r)+V_4(r)$ defined by (2) and (3), and a repulsive core for $r<r_0$. The range of the forces has been chosen equal to the Compton wavelength of the π-mesons: $\hbar/\mu c=(1.40\pm0.03)10^{-13}$ cm. The two constants $f^2/4\pi$ and r_0 have been determined by fitting exactly the deuteron binding energy, (-2.227 ± 0.003) Mev, and the neutron-proton zero energy scattering length $(-23.68\pm0.06)10^{-13}$ cm. These values are as follows:

$$f^2/4\pi=9.7\pm1.3,\quad r_0=(0.38\pm0.03)(\hbar/\mu c). \quad (7)$$

This value of r_0 is in agreement with the value of r at which the interaction (1) becomes important.

The following derived quantities have been obtained using these constants:

Singlet effective range: $1.78(\hbar/\mu c)$,
Triplet effective range: $1.185(\hbar/\mu c)$,
Electric quadrupole moment: 2.08×10^{-27} cm^2,
Proportion of D state: 0.051,
Mean value of the momentum: $\langle p^2/(\mu c)^2\rangle=1.76$.

These results are in good agreement with experiment, except the quadrupole moment, which is too small by about 20 percent. This quantity is, however, sensitive to an increase of the tensor force and can probably be improved by including the sixth-order corrections.[4]

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