

tering amplitudes used were

$$f_N = 0.94,⁹ f_{Cl} = 0.99,¹⁰ \text{ and } f_H = 0.38.¹⁶$$

The three (*hkl*) reflections (111), (211), and (221) are decisive in eliminating the ordered structure, for which calculated *F* values are presented in column 4, based on the same set of parameters. The disagreements are so great that no reasonable variation of the parameters can resolve them.

A by-product of the work is evidence that the earlier value of f_N , 0.85,^{2,10} is incorrect and that a scattering amplitude for hydrogen of 0.38¹⁶ is to be preferred to the earlier value of 0.40.¹⁰ The (100) reflection which is quite sensitive to the hydrogen amplitude and little affected by the temperature and structure parameters is most significant in this respect.

DISCUSSION

The data presented establish clearly that the room temperature structure of NH_4Cl involves a disorder in the orientation of the ammonium ions. This is in agree-

¹⁶ Hughes, Burgy, and Ringo, *Phys. Rev.* **77**, 291 (1950).

ment with less direct evidence.¹ Since the low temperature structure of ammonium chloride is clearly an ordered one, the lambda-transition is of the order-disorder type. The N-H distance of $1.03 \pm 0.02 \text{ \AA}$ found for both the hydrogen and deuterium compound is closely equal to that found in ND_4Br ,^{4,17} $1.03 \pm 0.02 \text{ \AA}$ by neutron diffraction, and in good agreement with values obtained by nuclear magnetic resonance¹⁸ 1.025 and 1.039 \text{ \AA}, respectively. The thermal motion of hydrogen as indicated by both B and α appears to be somewhat greater than that of deuterium, in accord with expectation.

Since ND_4Cl patterns at -180°C and -78°C showed no essential differences, the line width transition at about -140°C in NH_4Cl observed in nuclear magnetic resonance experiments¹⁹ may be reasonably presumed to involve no appreciable change in atomic positions.

¹⁷ The smaller value quoted in reference 3 is revised upward when the new value of f_N is used.

¹⁸ Gutowsky, Kistiakowsky, Pake, and Purcell, *J. Chem. Phys.* **17**, 972 (1949).

¹⁹ H. S. Gutowsky and G. E. Pake, *J. Chem. Phys.* **16**, 1164 (1948).

Scattering of Electrons in Metals by Dislocations*

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The problem of the change in electrical conductivity in metals upon cold-working is treated by the method of Koehler in which the scattering of electrons by pairs of parallel edge-type dislocations is assumed to be the major effect. A comparison is made between the electronic shielding assumed in the scattering potential used by Koehler and the shielding assumed by Landauer in a somewhat different treatment. The large effect of the discontinuity in the ionic displacement across the plane connecting the dislocation axes is shown to cancel that of a previously neglected term in the scattering potential. The scattering matrix element is evaluated, and the change in resistivity is computed by the perturbation method of Mackenzie and Sondheimer. Finally, the anisotropy in the resistivity is discussed, and it is shown that the slip direction is the direction of low resistivity.

I. INTRODUCTION

SEVERAL recent publications¹⁻⁴ have treated theoretically the problem of the change in electrical conductivity of metals upon cold-working. In all of this work it is assumed that the change is primarily because of the scattering of electrons from edge-type dislocations, and the possible effects of associated clusters of vacancies, or of screw-type dislocations, are ignored. The calculations differ mainly in the type of

scattering potential employed, Koehler¹ (K) and Mackenzie and Sondheimer² (MS) using in their potentials the ionic displacements around a pair of dislocations, and Landauer³ (L) and Dexter⁴ (D) making use of only the density change. These two potentials take account of the shielding of the positive charge by the electrons in somewhat different ways. In Sec. II a general comparison is made among these methods and Bardeen's self-consistent method⁵; it is shown that all three approaches agree in the limit of slowly varying potentials, i.e., perfect shielding.

One of the characteristic features of an edge-type dislocation is that there is a discontinuity in the ionic displacement across the missing plane of atoms. In the

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¹ J. S. Koehler, *Phys. Rev.* **75**, 106 (1949), to be referred to as K.

² J. K. Mackenzie and E. H. Sondheimer, *Phys. Rev.* **77**, 264 (1950), to be referred to as MS.

³ Rolf Landauer, *Phys. Rev.* **82**, 520 (1951), to be referred to as L.

⁴ D. L. Dexter, *Phys. Rev.* **85**, 936 (1952), to be referred to as D.

⁵ J. Bardeen, *Phys. Rev.* **52**, 688 (1937).

treatments of K and MS, in which the scattering potential depended on the displacements, this discontinuity gave rise to a large amount of fictitious scattering; attempts were made to get rid of this effect by changing the expression for the ionic displacements as obtained from elastic theory. Section III contains an alternative procedure for taking account of the discontinuity, and examines the results of the approximations in K and MS, particularly as regards the anisotropy in resistivity.

II. ELECTRONIC SHIELDING

It is well known that the "free" electrons in a metal distribute themselves around any charge in such a way as to shield the potential from that charge; the shielded potential caused by a point charge, Ze , for example, can easily be found⁶ by the Thomas-Fermi method to be

$$v(r) = (Ze/r) \exp(-qr), \quad (1)$$

where

$$q^2 = (4m^*e^2/\hbar^2)(3n_0/\pi)^{1/3}. \quad (2)$$

Here n_0 is the density of free electrons, and the other symbols have their usual meanings. For most good metals q turns out to be $\approx 10^8$ cm⁻¹.

One possible method⁷ for taking account of the shielding of extended charge distributions is by the deformation potential method, sketched below in simplified form. The width of the filled portion of the conduction band can easily be found by free-electron theory to be

$$E(\mathbf{r}) = (\hbar^2/2m^*)[3\pi^2n(\mathbf{r})]^{2/3}, \quad (3)$$

where $n(\mathbf{r})$ is the density of electrons at position \mathbf{r} . Since in equilibrium the Fermi level is constant, the bottom of the conduction band varies as $-E$. Assuming that the scattering potential V follows the bottom of the conduction band and that the electron density closely follows the positive charge density, we obtain, for small density changes,

$$V(\mathbf{r}) = \frac{2}{3}(E_0/e^2n_0)\delta\rho^+(\mathbf{r}), \quad (4)$$

where $\delta\rho^+(\mathbf{r})$ is the change in positive charge density at the position \mathbf{r} , and E_0 is the width of the filled part of the conduction band in the perfect crystal. This is the scattering potential used in L and D, where $\delta\rho^+(\mathbf{r})$ was calculated⁸ for a single edge-type dislocation from the elastic theory of a continuous, isotropic medium.

Another method^{1,2} is to use as a potential the difference between the sums of the shielded ionic potentials [Eq. (1)] for the distorted and perfect lattices,

$$V(\mathbf{r}) = \sum_j v(|\mathbf{r} - (\mathbf{R}_j + \mathbf{u}_j)|) - \sum_j v(|\mathbf{r} - \mathbf{R}_j|), \quad (5)$$

where the \mathbf{R}_j are ionic positions in the perfect lattice, and the \mathbf{u}_j are the ionic displacements associated with

the distorted lattice; this is the procedure followed in K and MS. Landauer³ has criticized this approach as not properly taking into account the redistribution of electrons around a distortion, and has suggested instead the use of the potential in Eq. (4). It will be seen that Eq. (5) leads to results equivalent to those obtained by use of Eq. (4) in the region where the latter is valid (perfect shielding) and is a better approximation than Eq. (4) in the region of imperfect shielding.

In order to compare Eqs. (4) and (5), it will be convenient to make use of the operator $\nabla^2 - q^2$. It can be seen from Eq. (1) that $(\nabla^2 - q^2)v(\mathbf{r}) = -4\pi Ze\delta(\mathbf{r})$, where $\delta(\mathbf{r})$ is the Dirac delta-function; by use of this operator we obtain from Eq. (5)

$$(\nabla^2 - q^2)V(\mathbf{r}) = -4\pi Ze \sum_j \{\delta(\mathbf{r} - \mathbf{R}_j - \mathbf{u}_j) - \delta(\mathbf{r} - \mathbf{R}_j)\}, \quad (6)$$

or

$$(\nabla^2 - q^2)V(\mathbf{r}) = -4\pi\delta\rho^+(\mathbf{r}). \quad (7)$$

We shall later make use of the fact that this equation has the solution,

$$V(\mathbf{r}) = (1/Ze) \int v(|\mathbf{r} - \mathbf{r}'|)\delta\rho^+(\mathbf{r}')d\tau'. \quad (8)$$

Let us now expand $V(\mathbf{r})$ and $\delta\rho^+(\mathbf{r})$ in Eq. (7) into lattice waves, and obtain

$$V(\mathbf{r}) = 4\pi \sum_{\mathbf{K}} [\rho_{\mathbf{K}} e^{i\mathbf{K} \cdot \mathbf{r}} / (K^2 + q^2)]. \quad (9)$$

Thus when the charge density may be expressed in terms of waves of wavelength large compared with 10^{-8} cm ($q^2 \gg K^2$), the potential becomes

$$V(\mathbf{r}) = (4\pi/q^2)\delta\rho^+(\mathbf{r}). \quad (10)$$

By using Eqs. (2) and (3) we see that this is equal to the expression in Eq. (4); thus the two methods are equivalent for slowly varying potentials.

Equation (9) can be compared with the results of Bardeen's self-consistent solution⁵ for the potential by making use of Eqs. (30) and (32) in reference 5, from which we obtain, changing the notation in reference 5 to agree with ours,

$$V(\mathbf{r}) = 4\pi \sum_{\mathbf{K}} \frac{\rho_{\mathbf{K}} e^{i\mathbf{K} \cdot \mathbf{r}}}{K^2 + q^2(1 - \xi^2/3 - \xi^4/15 - \dots)}, \quad (11)$$

if $\xi \equiv K/2k_0$ is less than unity, where k_0 is the wave number of an electron at the surface of the Fermi distribution. If $\xi^2 \ll 1$, this agrees with Eq. (9), but a more significant comparison can be made in the form

$$V(\mathbf{r}) \cong 4\pi \sum_{\mathbf{K}} \frac{\rho_{\mathbf{K}} e^{i\mathbf{K} \cdot \mathbf{r}}}{q^2 + K^2(1 - q^2/12k_0^2)}. \quad (12)$$

If we take $q \sim 2.2 \times 10^8$ cm⁻¹ and $k_0 \sim 1.4 \times 10^8$ cm⁻¹ for copper, the denominator becomes $q^2 + K^2(1 - 0.21)$.

⁶ N. F. Mott and H. Jones, *The Theory of the Properties of Metals and Alloys* (Oxford University Press, London, England, 1936), pp. 88, 294.

⁷ J. Bardeen and W. Shockley, Phys. Rev. **80**, 72 (1950), and references contained therein.

⁸ J. S. Koehler, Phys. Rev. **60**, 398 (1941).

Since q^2/k_0^2 varies as $1/n_0^{\frac{1}{2}}$, the correction term in the coefficient of K^2 is larger in absolute magnitude for metals with a lower electron density, as is to be expected; similarly, the shielding of the positive charge is less effective in the poorer metals, and the potential is greater.

III. EVALUATION OF THE SCATTERING MATRIX ELEMENT

The scattering of an electron between two plane wave states \mathbf{k} and \mathbf{k}' is governed by the matrix element

$$\langle \mathbf{k}' | -eV | \mathbf{k} \rangle = \frac{1}{\tau} \int \exp(-i\boldsymbol{\kappa} \cdot \mathbf{r}') V(\mathbf{r}') d\tau', \quad (13)$$

where $\boldsymbol{\kappa} \equiv \mathbf{k}' - \mathbf{k}$, and τ is the volume of the metal; thus it is this matrix element that will be required for the calculation of the change in resistivity associated with the presence of scattering centers. Let us examine in detail the potential in a metal lattice containing one or more edge-type dislocations, i.e., a lattice from which part of the material has been removed. Quite generally, we may write the perturbing potential in the lattice at the point \mathbf{r}' as

$$V(\mathbf{r}') = \sum_j' v(\mathbf{r}' - \mathbf{R}_j - \mathbf{u}_j) - \sum_j v(\mathbf{r}' - \mathbf{R}_j) \\ \cong \sum_j' \mathbf{u}_j \cdot \nabla_j v(\mathbf{r}' - \mathbf{R}_j) - \sum_i'' v(\mathbf{r}' - \mathbf{R}_i), \quad (14)$$

where the prime after the summation symbol reminds us that we are not to sum over the missing atoms, and the double prime indicates summation over only the missing material. The subscript j on the nabla operator indicates differentiation with respect to the coordinates of \mathbf{R}_j . Replacing the summations by integrations, we obtain

$$V(\mathbf{r}') = \Delta^{-1} \int \mathbf{u}(\mathbf{r}) \cdot \nabla v(\mathbf{r}' - \mathbf{r}) d\tau - \Delta^{-1} \int v(\mathbf{r}' - \mathbf{r}'') d\tau'' \\ = V_1(\mathbf{r}') + V_2(\mathbf{r}'), \quad (14')$$

where Δ is the atomic volume. Because the rearranged lattice is approximately in register over the region of the missing material, the first term $V_1(\mathbf{r}')$ is evaluated over the entire volume of the metal; it is this term $V_1(\mathbf{r}')$ that was used as a scattering potential in K and MS. In the second term $V_2(\mathbf{r}')$ the integration extends over just the region of missing atoms; in the same approximation that we have made in connection with V_1 , the term V_2 may be replaced by the surface integral $-(\lambda/\Delta) \int v(\mathbf{r}' - \mathbf{s}'') dS''$, where λ is the unit crystallographic slip distance.

Now the displacement \mathbf{u} has a discontinuity across the missing material, corresponding to putting the lattice back in register; this discontinuity makes a large contribution to the term V_1 . It will now be shown that $V_2(\mathbf{r}')$ cancels the contribution to V_1 from the discontinuity. By use of the vector identity for the divergence

of a product $\mathbf{v}\mathbf{u}$, the first term in Eq. (14') becomes

$$V_1(\mathbf{r}') = (Ze)^{-1} \int \delta\rho^+(\mathbf{r}) v(\mathbf{r}' - \mathbf{r}) d\tau \\ + \Delta^{-1} \int \nabla \cdot (\mathbf{v}\mathbf{u}) d\tau, \quad (15)$$

where we have made use of the relation

$$\delta\rho^+ = -(Ze/\Delta) \nabla \cdot \mathbf{u}.$$

Now let us remove an infinitesimal slice along the plane of missing atoms; with the exception of this deleted volume, the function $\mathbf{v}\mathbf{u}$ is continuous, and the divergence theorem may be applied to the second term in Eq. (15). The exponential dependence in v prevents any contribution from the outer surface, and we are left with the result

$$V_1(\mathbf{r}') = (Ze)^{-1} \int v(\mathbf{r}' - \mathbf{r}) \delta\rho^+(\mathbf{r}) d\tau \\ + \Delta^{-1} \int v(\mathbf{r}' - \mathbf{s}'') \mathbf{u}(\mathbf{s}'') \cdot \mathbf{n} dS'', \quad (16)$$

where the second integral is over the surface of the deleted section, and \mathbf{n} is the outward normal to the surface. Since $\mathbf{u}(\mathbf{s}'') \cdot \mathbf{n}$ is equal to $\lambda/2$, the second term on the right of Eq. (16) just cancels $V_2(\mathbf{r}')$, and Eq. (14') becomes

$$V(\mathbf{r}') = (1/Ze) \int v(\mathbf{r}' - \mathbf{r}) \delta\rho^+(\mathbf{r}) d\tau,$$

as found [Eq. (8)] and discussed in the last section.

Following K and MS, let us consider the scattering of electrons from a positive-negative dislocation pair, the positive edge-type dislocation at $x=0$, $y=R/2$, and the negative at $x=0$, $y=-R/2$. The corresponding displacements have been evaluated by Koehler,⁸

$$u_x = A \left\{ \tan^{-1} \frac{y-R/2}{x} - \tan^{-1} \frac{y+R/2}{x} \right\} \\ + 2Bx \left\{ \frac{y-R/2}{x^2+(y-R/2)^2} - \frac{y+R/2}{x^2+(y+R/2)^2} \right\}, \\ u_y = -C \log \left\{ \frac{x^2+(y-R/2)^2}{x^2+(y+R/2)^2} \right\}^{\frac{1}{2}} \\ - 2Bx^2 \left\{ \frac{1}{x^2+(y-R/2)^2} - \frac{1}{x^2+(y+R/2)^2} \right\}, \quad (17)$$

$$A = \lambda/2\pi, \quad B = (\lambda/8\pi)/(1-\nu),$$

$$C = (\lambda/4\pi)(1-2\nu)/(1-\nu),$$

where ν is Poisson's ratio. The matrix element Eq. (13)

can be evaluated by elementary methods, and we find change in resistivity in the x (slip) direction,

$$\langle \mathbf{k}' | -eV | \mathbf{k} \rangle = - \left[\frac{32\pi^2 Z e^2 \sin(\kappa_z \tau^{\frac{1}{2}}/2) \sin(\kappa_y R/2)}{\tau \Delta (\kappa^2 + q^2) \kappa_z \kappa_s^2} \right] \times \{ A \kappa_x^2 / \kappa_y + (2B - C) \kappa_y - A \kappa_s^2 / \kappa_y \}, \quad (18)$$

$$\Delta \rho_x = \frac{3\pi^2 Z^2 e^2 m^{*2} N \lambda^2}{\hbar^3 k_0^2 q^4 \Delta^2 n_0} \left(\frac{1-2\nu}{1-\nu} \right)^2 \times \{ a \cot^{-1} a + a^2 / (a^2 + 1) \}, \quad (20)$$

where we have made use of Eqs. (14') and (17). In this expression $\boldsymbol{\kappa} = \mathbf{k}' - \mathbf{k}$, $\kappa_s^2 = \kappa_x^2 + \kappa_y^2$ and $\tau^{\frac{1}{2}}$ is the length of the cubic metal specimen. The last term in the curly brackets is the contribution from $V_2(\mathbf{r})$. Squaring, and setting the interference factor $\sin^2(\kappa_y R/2)$ equal to $\frac{1}{2}$ for the reasons given in K, we obtain

$$|\langle \mathbf{k}' | -eV | \mathbf{k} \rangle|^2 = \left[\frac{512\pi^4 Z^2 e^4 \sin^2(\kappa_z \tau^{\frac{1}{2}}/2) (2B - C - A)^2 \kappa_y^2}{\Delta^2 \tau^2 q^4 (1 + \kappa^2/q^2)^2 \kappa_z^2 \kappa_s^4} \right]. \quad (19)$$

Note that except for the factor $(1 + \kappa^2/q^2)^{-2}$, which is associated with the difference in electronic shielding in the two potentials, this is just twice the square of the matrix element found in D, as corresponds to the scattering from two dislocations instead of one. Also note that the scattering is small for $\boldsymbol{\kappa}_s$ primarily along the slip direction. This can best be understood from a consideration of the variation in positive charge (which around a single dislocation goes as $y/[x^2 + y^2]$).

This expression differs from the corresponding results in K [Eq. (K26)] and MS [Eq. (MS19)] primarily in the angular dependence. Mackenzie and Sondheimer neglected the potential term V_2 and attempted to take account of the discontinuity in displacement by replacing y by its absolute value in the arctangent terms in the expression for u_x [Eq. (16)] between $y = -R/2$ and $y = R/2$. The resulting displacement is, of course, not in accord with the equations of elasticity, and in fact corresponds to replacing and smearing out the atoms previously removed to create the dislocation. The result is an increase in the resistance in the slip direction and a decrease in the y direction (since the positive charge density, and hence, scattering potential are smoothed out for an electron traveling in the y direction). Similarly omitting the contribution from V_2 , Koehler replaced x by its absolute value in the arctangent terms in Eq. (17) for all values of y . This procedure is also not in accord with the elastic equations, and corresponds to the introduction of a discontinuity in charge density along the missing plane of atoms, and thus to a large increase in resistance along the slip direction.

Making use of Eq. (19), we now evaluate the resistivity by the method⁹ described in MS, and find for the

where $a \equiv q/2k_0$ and N is $\frac{1}{2}$ the number of dislocations per cm^2 . $\Delta \rho_x$ is of course zero, and the ratio $\Delta \rho_x / \Delta \rho_y$ is equal to $\frac{1}{3}$. Values for this ratio of about 8 and 2 were obtained in K and MS, respectively. That the slip direction is the direction of low resistance can be seen qualitatively in the following ways. First, the potential changes are less abrupt for an electron traveling in the slip direction, and so the scattering will be less, than for an electron traveling in the y direction. Secondly, although the (approximately) $1/\kappa_s^2$ dependence in the square of the matrix element indicates that small angle scattering is of importance (which would strongly weight the resistance in the x direction), the additional dependence on $\boldsymbol{\kappa}_s$ in the transport equation overcomes this factor, and in fact it is large angle scattering that contributes most to the resistance, particularly in the y direction. Now for a current traveling in the x direction the possible electronic momentum changes available in phase space are also, on the average, primarily in the x direction. But we have already seen that the matrix element is small for the case in which the momentum change is primarily in the x direction. Hence, the electrical resistance in this direction is relatively small.

This expression for the resistivity change can be obtained from Eq. (MS34) by setting equal to zero in MS in term in A^2 , and replacing the quantity $(2B - C)^2$ by $\frac{1}{2}(2B - C - A)^2$ [see Eqs. (19) and (MS19)]. The average value of the resistivity change for a polycrystalline specimen, equal to $4/3$ times $\Delta \rho_x$ above, differs by a factor $(2B - C - A)^2 / 2[A^2 + (2B - C)^2]$ from the average change calculated in MS. For copper ($\nu = 0.34$) this factor has the value 0.092.

By use of Eqs. (2) and (3), we may write the average change in resistivity in the form

$$\langle \Delta \rho \rangle_{av} = \frac{m^2 \lambda^2 k_0 E_0^2}{27 \pi^2 \hbar^3 e^2 n_0^2 (n_0 \Delta)^2} \left(\frac{1-2\nu}{1-\nu} \right)^2 \times \{ a \cot^{-1} a + a^2 / (a^2 + 1) \} \times (m^*/m)^2 Z^2 N. \quad (21)$$

Making use of specific heat data¹⁰ for copper, we set $E_0 = 4.78$ eV, $m^*/m = 1.47$, $k_0 = 1.37 \times 10^8$ cm^{-1} , $q = 2.19 \times 10^8$ cm^{-1} , and $a = 0.80$; taking $\lambda = 2.55 \times 10^{-8}$ cm, $\nu = 0.34$, and assuming one free electron per atom (so that $n_0 \Delta = 1$ and $n_0 = 0.85 \times 10^{23}$ cm^{-3}), we obtain

$$\langle \Delta \rho / \rho \rangle_{av} = 2.46 \times 10^{-15} Z^2 N,$$

⁹ Equation (4) for the collision operator in reference 2, and the resulting expressions, should be multiplied by the factor $\frac{1}{2}$.

¹⁰ F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), Chapter IV.

where we have used the thermal resistivity at 20°C, $\rho = 1.89 \times 10^{-18}$ Gaussian units, i.e., 1.69×10^{-6} ohm cm. The observed fractional change in heavily cold-worked copper is about 2 percent, which corresponds to $N \sim 8 \times 10^{12}$ pairs of dislocations per cm², if the effective atomic number Z is unity. Koehler⁸ has estimated that $N \sim 3 \times 10^{11}$ pairs per cm² on the basis of energy storage measurements. With a different set of values for k_0 and m_0 , MS obtained $N \sim 5 \times 10^{12}$ dislocation pairs/cm². Correcting this result by the factor $2[A^2 + (2B - C)^2] / (2B - C - A)^2$, we would obtain $N \sim 6 \times 10^{13}$ /cm². However, this value is probably too high, since it has assumed the normal electronic mass and a density of 1.4 electrons per atom in the evaluation [see Eq. (MS40)]. Koehler quotes a result $N \sim 7 \times 10^{11}$ /cm². (See, however, MS, p. 269.)

We may also compare Eq. (21) with the results obtained by the deformation potential method, Eq. (D3). The resistivity change per dislocation calculated here is equal to that in D times a factor $\frac{1}{2}[a \cot^{-1} a + a^2 / (a^2 + 1)]$. For a large in comparison with unity this factor reduces to 1, but in this case neither approximation is a good one [see Eq. (12)]. For copper $a = 0.80$ and the factor becomes 0.55. Since the deformation potential is somewhat larger than the potential used here [compare Eqs. (4), (9), and (10)], one would indeed expect to calculate a larger resistivity by the use of the deformation potential, and according to Eq. (12), in fact, one would expect to bracket the correct answer by the two methods. The effect of the fictitious singularity in the assumed density change on the dislocation axis was estimated in D to give too large a resistance change by a factor of about 2, and since it is only in the region close to the singularity that the two potentials differ appreciably, this factor is in qualitative agreement with the result obtained just above.

The estimate made here for the number of dislocation pairs, 8×10^{12} per cm², should perhaps be modified in the following way, making use of a calculation similar to Bardeen's⁵ for the thermal part of the resistance in

monovalent metals. The electrical resistivity ρ arising from collisions of the electrons with lattice vibrations can be calculated by the model used here, i.e., a model which neglects effects associated with the particular ionic potentials near the nuclei and considers just the effects resulting from changes in density; this value for ρ can then be compared with the experimental value, thus giving some idea of the accuracy of the results of this model. From reference 5 we find ρ to be proportional to $(C/\zeta)^2$, which for the present model is given by Eqs. (51), (49), and (47) of reference 5 with $V(r_s) + E_c = 0$ and $f(u) = 1$. Inserting the experimental values given above for copper (so that Bardeen's parameter β becomes 1.31), we find that numerically $(C/\zeta)^2$ is equal to 0.460. The experimental value is 1.74, so that the resistance calculated according to this model is about $\frac{1}{4}$ that observed. Now the calculation for phonons makes use of the same matrix elements as were used for the dislocation calculation, and both calculations required summations over the distribution of electron propagation vectors and the Fourier expansions of the lattice irregularities. Although these expansions are not the same in the two problems, the error incurred by use of this general approach should be of the same order of magnitude in the two cases; we thus correct N by the above factor, and obtain $N \sim 2 \times 10^{12}$ per cm².

The conclusion of the writer is that the best estimate, based on this simple theory, for the density of dislocations is about an order of magnitude higher than the estimate obtained from the energy storage measurements. If the latter is fairly accurate, closer agreement must be sought in an interpretation of the effects of clustered vacancies, screwtype dislocations, half-dislocations, and anisotropy in the elastic constants.

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