Particle Counting by Čerenkov Radiation*

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Čerenkov radiation particle counters of a number of types are in satisfactory use in this laboratory for the counting of high energy mesons, electrons, and protons. The problems associated with Čerenkov counting and some of the uses are discussed together with the design and performance of several types of counters. The uses include high energy electron counting with the exclusion of neutrons and protons, measurement of the velocity of mesons, and the detection of high energy neutrons in the presence of lower energy neutrons by the Čerenkov threshold counting of recoil protons.

(1) INTRODUCTION

THE Čerenkov radiation has been suggested as a means of counting fast charged particles by Getting[1] and by Dicke.[2] Recently, Jelley[3] has succeeded in showing that a photomultiplier can be made to count pulses of light coming from the Čerenkov radiation produced when cosmic-ray particles traverse water. Mather[4] has demonstrated that Čerenkov radiation is emitted when 340-Mev protons pass through suitable materials and has used it to measure accurately the energy of the deflected proton beam of the Berkeley 184-inch synchrocyclotron. The writer[5] has reported the operation of a Čerenkov counter with electrons produced by the 50-Mev bremsstrahlung beam of a betatron. The present paper is a discussion of the problems and techniques of Čerenkov counting with particular emphasis on the types of counters in use at the University of Chicago for the counting of particles produced by the 450-Mev proton synchrocyclotron. For a bibliography of the Čerenkov radiation itself the reader is referred to the papers of Getting, Dicke, and Mather.

(2) THE ČERENKOV RADIATION

The Čerenkov radiation may be considered to be an electromagnetic shock wave and is analogous to the shock wave produced when a projectile travels through air at a speed greater than that of sound. Like the pressure shock wave, the Čerenkov wave is conical with the apex of the cone coincident with the particle producing the disturbance. The radiation is produced when any charged particle traverses a dielectric medium at a speed greater than that of light in the medium. The direction of the radiation is forward at an angle θ from the direction of the particle, where θ is determined by the relation

$$\cos\theta = 1/\beta n; \tag{1}$$

here n is the index of refraction of the dielectric medium. It is easy to derive relation (1) from Huygens' principle as illustrated in Fig. 1.

Since the Čerenkov radiation is a shock wave, it contains components of all frequencies for which the index of refraction is large enough to give a real value of θ in Eq. (1). In ordinary optical materials the radiation includes the visible spectrum with energy flux larger toward the violet end. It may be observed visually when, for example, a strong source of reasonably energetic beta-radiation is immersed in water and has a bluish white appearance to the eye. The intensity and spectral distribution are given by the relation

$$I = \frac{4\pi^2 e^2}{hc^2}\Delta\nu\left\{1-\frac{1}{\beta^2 n^2}\right\} = \frac{2\pi\Delta\nu}{137c}\sin^2\theta. \tag{2}$$

Here I is the number of photons emitted per centimeter of path, $\Delta\nu$ is the frequency interval in cycles per second, and β is the ratio of the speed of the particle to that of light in vacuum. The index of refraction is considered to be constant over the frequency range $\Delta\nu$ of the emitted

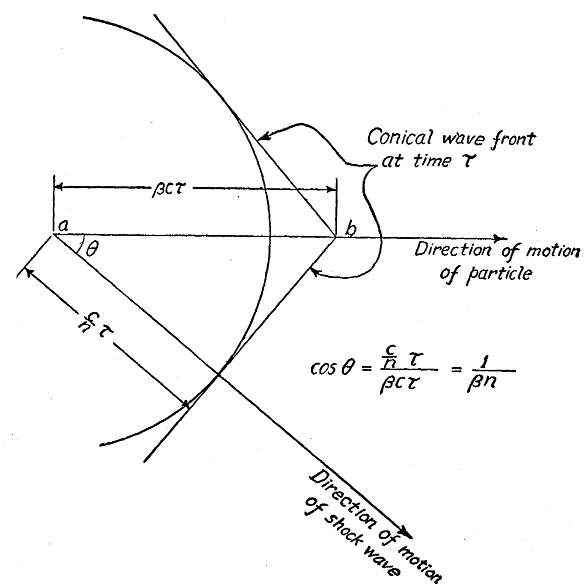

FIG. 1. Construction of Čerenkov wave front by Huygens' principle. Position of particle at time $0 \cdots a$; position of particle at time $\tau \cdots b$; distance traversed by particle in time $\tau \cdots \beta c\tau$; distance traversed by light in time $\tau \cdots (c/n)\tau$.

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[1] I. A. Getting, Phys. Rev. **71**, 123 (1947).
[2] R. H. Dicke, Phys. Rev. **71**, 737 (1947).
[3] J. V. Jelley, Proc. Phys. Soc. (London) **A64**, 82 (1951).
[4] R. L. Mather, Phys. Rev. **84**, 181 (1951).
[5] J. Marshall, Phys. Rev. **81**, 275 (1951).

light. The angle θ is the Čerenkov angle as in Eq. (1). It should be noted that the intensity is a function of the Čerenkov angle alone.

The visible spectrum covers a frequency interval of approximately 3×10^{14} cycles. If one assumes this value for $\Delta\nu$, Eq. (2) gives the intensity of the Čerenkov radiation to be $450\sin^2\theta$-photons per centimeter of particle path. As a numerical example electrons traveling essentially at the speed of light through glass with an index of refraction of 1.50 emit Čerenkov radiation with an intensity of 250 photons per centimeter in the visible spectrum.

The Čerenkov radiation is plane polarized with its electric vector in the radial direction. In other words, the electric vector lies parallel to the shock wave front and points toward or away from the particle responsible for the radiation.

In a dispersionless medium the Čerenkov radiation would remain a mathematically sharp wave front as it moved away from the path of the particle. In all actual mediums it degenerates into a wave packet, the various frequency components of which are transmitted with varying speeds and directions dependent on the values of the index of refraction for those frequencies.

(3) SCINTILLATION COUNTING WITH THE ČERENKOV RADIATION

The Čerenkov radiation can be used as a source of light for scintillation counting. As such it has certain attractive features, but in some ways its properties introduce problems. In this section some of these problems and features will be discussed.

(3a) Choice of Radiator Material

In principle one may use as a radiator any transparent material whose index of refraction is large enough to give a real value of θ in Eq. (1). From the point of view of intensity, it is helpful to use a material of as large an index as possible. However, frequently one wishes to use the Čerenkov radiation to discriminate through intensity or angle between particles of different velocities, and one may not want a high refractive index. For angular discrimination it is advantageous to use a material of low dispersion, and high index is frequently associated with large dispersion. For measurements of angle it is also important that the density and average atomic number of the radiator material be as low as possible. Low density implies a small rate of energy loss through ionization as the particle traverses the radiator. Low atomic number leads to small angular spread through multiple small angle Coulomb scattering. This means that the direction of the particle, and therefore of the Čerenkov radiation, remains well defined as the particle passes through the radiator. The material should have as wide a frequency band of essentially perfect transparency as possible. It should emit no light by phosphorescence or fluorescence, and finally, it should be easily shaped into simple optical parts.

There are liquids that satisfy quite well most of the conditions of the last paragraph, but if one includes the last one, namely that the material be easily fabricated, the best material by far is methyl methacrylate polymer, better known under the trade names of Lucite and Plexiglas. The writer has no reliable information to indicate that there is any difference between the two brands, and they have been used interchangeably in the work reported here. However, there certainly are differences in the light absorption in the violet and ultraviolet from one piece to another supplied by the same manufacturer. Through the kindness of Dr. J. R. Platt of the Physics Department, spectrophotometer transmission curves were obtained for two chance samples of Lucite. These two samples within the errors of the measurement were perfectly transparent in the visible but absorbed light by different amounts in the ultraviolet. The light absorption appears to be a result of the same substance because of the similarity of the two absorption curves, but the substance must be present in the two samples in different concentrations. Since the near ultraviolet is quite an important part of the sensitivity spectrum of most photomultipliers, one should be careful to select good pieces of plastic for radiators. Above all, one should avoid material, such as is sometimes found, which shows a yellow tint to light transmitted through large thicknesses.

Lucite and Plexiglas have been the radiator material used for most of the work reported in this paper. For simplicity of nomenclature both materials will be called Lucite from here on, although in about half the cases the material actually used was Plexiglas.

The rather meager published information and the very rough measurements made in this laboratory indicate a working value for the index of refraction of Lucite of 1.50. The dispersion seems to be slightly smaller than that of most glasses, the index increasing by 0.01, for one sample between about 5850 and 4400 angstrom units. The value of the index varies slightly from one sample to another and sometimes has significant variations inside the same piece. Inhomogeneities of this kind may be found by looking lengthwise through a rod or edgewise through a sheet of the material. Under these conditions image distortion by refraction or total reflection is a sufficiently sensitive test of the optical homogeneity of the piece for most purposes.

(3b) Intensity

Much lower intensities are available from the Čerenkov radiation than from comparable thicknesses of the usual phosphors. A quantitative evaluation of the intensity problem must include the characteristics of the photomultiplier tubes used to detect the light. For this reason there are plotted in Fig. 2 the sensitivity curves for three of the types of multipliers employed for Čerenkov counting by the writer. The data for these curves were taken from material published by the manufacturer.[6] They are replotted in a way which is particularly adapted to the problems of Čerenkov counting. The number of electrons emitted by the cathode per photon incident thereon is plotted for each tube against the frequency of the light. It must be remembered in using these curves that there are violent variations from one tube to another of the same type, but in general shape and approximate magnitude these curves may represent some sort of average tubes.

For each of the three multiplier types there is plotted in Fig. 2 a cathode sensitivity curve for light passed through two inches of Lucite as well as the sensitivity curve for the bare tube. The transmission data used for Lucite (see Sec. 3a) apply to one chance sample, but give a rough idea of the effects involved. The sensitivity curves have been plotted with the assumption that there is no light loss from reflection as the light leaves the Lucite.

Inspection of Fig. 2 shows that, for the light frequency of maximum response, the cathode sensitivity of the 5819 is about 0.065 electron per incident photon, and that the sensitivities of the other tubes are not different from this by a great deal. A rough integration of the 5819 curve for light filtered through Lucite indicates that for the purpose of computing Čerenkov counting efficiency, one can consider the sensitivity to be constant at its maximum value over a band width of 2.9×10^{14} cycles. For the example of Sec. 2 of a highly relativistic particle traversing a medium of refractive index 1.5 (Lucite), one gets from Eq. (2) an effective intensity for this band width of 242 photons per centimeter of particle path. When multiplied by the cathode sensitivity, this figure gives the over-all sensitivity as 15.7 electrons from the cathode per centimeter of path of the particle through the radiator. The assumption is made that all of the light can be brought onto the multiplier cathode. The comparable figures for the 931A and the 1P28 are 12.0 and 12.4 electrons per centimeter.

Although the three photomultiplier types included in Fig. 2 appear to be almost equal in sensitivity, there is one important difference between the 5819 and the other two which makes the 5819 more efficient. The difference is that the 5819 has its photocathode deposited directly on the inside of the glass envelope while those of the others are internal and have only a small area of maximum sensitivity. The result is that the assumption, that all the light in the sense of the data of Fig. 2 can be made to strike the photocathode, can be realized in the 5819 and can even, in effect, be improved upon by making good optical contact between the radiator and the envelope. In the cases of the 931A and the 1P28 the condition can be realized approximately only by rather exact focusing of the light onto the cathode through the Lucite and glass surfaces, each of which reflects some light. There is another advantage to the 5819, which is that more of its sensitive range is in the visible than is the case with the other types. This means that there is not so much effect from variations of ultraviolet absorption in the radiator, so that reasonable efficiencies should be realizable with any material which looks perfectly transparent and colorless to the eye.

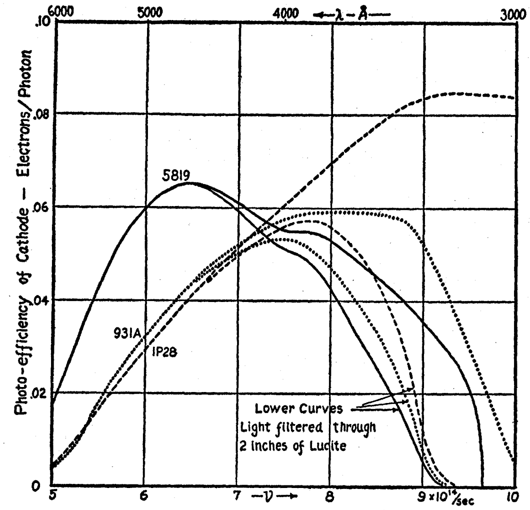

FIG. 2. Cathode efficiency curves for photomultipliers. These curves are computed from data published by the manufacturer. Upper curves represent number of electrons emitted by the photocathode of each tube type for one photon of the indicated frequency incident on its sensitive region. Lower curves are the same thing for light that has been passed through two inches of Lucite. Assumed no light loss from reflection at surface of Lucite.

The intensity and sensitivity figures given above indicate that it should be possible to achieve practically 100 percent counting efficiency with Čerenkov counters if the particles counted are penetrating and of the appropriate velocities. For counting applications it is necessary, of course, to discriminate against noise pulses generated by the photomultiplier. Amplitude discrimination is possible sometimes with large pulses of light and with good tubes, but almost always one runs the risk of losing some of the Čerenkov pulses which happen to be smaller than the ordinary. A more satisfactory method of discrimination is the use of fast coincidences, either between two tubes exposed to light from a common radiator or between a tube exposed to the Čerenkov radiation and one exposed to light from a phosphor or another radiator through which the same particle passes.

(3c) Duration of Pulse

The time of emission of the Čerenkov radiation is vanishingly small for practical purposes. This does not imply, of course, that the light pulse arriving at the photocathode of the multiplier tube in a counter will be instantaneous. Depending on what type of optical

[6] RCA Tube Handbook.

system is used to collect the light and perhaps on the size of the radiator, the duration of the light flash at the cathode may be anything from 10^{-11} sec to about 10^{-9} sec.

For what seem to be reasonable designs of Čerenkov counters, the length of the light pulse is almost always smaller than 10^{-9} sec. This is considerably shorter than the decay time of the fastest phosphors available at the present time. This property of Čerenkov counters gives one immediately the idea of high resolution coincidence counting experiments. The difficulty is that the presently available equipment for detecting and amplifying the light pulses is all slower than the pulse. No commercially available photomultiplier tube is capable of reproducing a pulse shorter than 10^{-9} sec, and unfortunately the fastest multipliers are those with the least gain and the least efficient cathodes. There might be some hope of operating a fast coincidence circuit directly from the output of a 5819 tube responding to a Čerenkov pulse, but the 5819 is a slow tube. The 1P28 is a faster tube, but with its small cathode area it is difficult to concentrate enough light there to produce a big output pulse.

Consider the example of an infinitesimally short 15-electron pulse from the cathode of a 1P28 or 931A. A pulse of this size corresponds to the collection of all the light from a little more than a centimeter of electron path in Lucite and is probably about as large a pulse as can be generated in these tubes by Čerenkov radiation because of cathode geometry. The output pulse size, assuming a current amplification of 10^6, will be 2.4×10^{-12} coulomb. If we assume for simplicity the not unreasonable output pulse length of 2.4×10^{-9} sec, this gives an output current of one milliampere or a pulse height into 200-ohms impedance of 0.2 volt. A pulse of this size is not large enough to operate dependably any fast coincidence circuit familiar to the writer. Ordinarily, then, one must expect to use some sort of amplifier to raise the pulse voltage by about a factor ten. The best amplifiers available at the present time have band widths of about 200 megacycles. With such a band width a pulse of the length mentioned above will be broadened a little bit more. In any case one sees that the limitation on shortness of pulse is in the associated equipment and not in the light source when one is using a Čerenkov counter.

It may be possible to do very high resolution coincidence work, even with comparatively slow photomultipliers and amplifiers, by some sort of artificial pulse sharpening or by a trick in the coincidence circuit like, for instance, the one described by Bay.[7] The limitation in this direction is in the first stages of the photomultiplier where some of the pulse broadening takes place and where the number of electrons is small. Statistical variations in the electron multiplication will cause a certain amount of jitter in the time of the maximum of the output pulse. The writer has not had occasion to try for resolving times any shorter than are required for background reduction, but there seems to be no fundamental reason why resolving times of 5×10^{-10} sec or shorter should not be achieved.

(3d) Direction of Radiation

The directional properties of the Čerenkov radiation can be used to great advantage in achieving efficient collection of the light onto a photomultiplier cathode. The property that is applicable in the simplest way is that the light is all emitted in the forward direction. Thus one can use, as Jelley[3] has done, a cylindrical container with specularly reflecting walls filled with a liquid radiator material. The multiplier cathode is used as one of the ends of the container. Particles are allowed to enter the radiator from the other end, and if the walls are good reflectors, practically all of the Čerenkov light will reach the cathode.

A somewhat more convenient construction is to use as a radiator a transparent solid such as Lucite. Total internal reflection is used to contain the light and to lead it to the cathode. Total reflection is more efficient than metallic reflection, but it must be used with a certain amount of caution since the angle of incidence necessary to insure total reflection must be larger than $\sin^{-1}(1/n)$. For a particle moving at the velocity of light, the cosine of the Čerenkov angle is $1/n$, so that some of the light striking the walls of a cylinder may not be totally reflected. Consider a particle moving at the velocity of light at the axis of a dielectric cylinder but not exactly parallel to it. For this case one-half of the Čerenkov radiation emitted will be above the angle of total internal reflection at the walls and one-half will be below. The efficiency of light collection for particles moving at the velocity of light may be expected to vary somewhat with the position of the particle in the cylindrical radiator, particles near the outside yielding a higher efficiency. This is because a particle moving off the axis produces radiation which strikes the walls, for the most part in planes of incidence not parallel to the axis. This light is more likely to be totally reflected because the angle of incidence at the boundary is larger.

When considering the internal reflection of the Čerenkov radiation, one should remember that the light is radially polarized. The polarization is such that when the radiation strikes a surface normal to the direction of the particle, the electric vector is in the plane of incidence. According to the Fresnel formulas for reflection and refraction at a dielectric boundary, light with this sense of plane polarization is almost totally transmitted over a range of angles of incidence near to the Brewster angle, which for an internal ray in Lucite is about 33°. This makes more efficient the escape of Čerenkov radiation from the end of a radiator and makes particularly attractive a method suggested by Bernardini[8] for discriminating in a nonfocusing

[7] Z. Bay, Phys. Rev. **83**, 242 (1951).

[8] G. Bernardini (private communication).

Čerenkov counter against highly relativistic particles (electrons). The method of Bernardini is to exclude the radiation from electrons by providing a narrow air gap at the end of the radiator. The Čerenkov radiation from electrons being at an angle greater than the critical angle should all be totally reflected at this gap while the light from, say, 100-Mev mesons can be transmitted. In fact, because of the polarization the transmission should be practically perfect.

The polarization properties discussed above are perfect only for a radiator so short that none of the light is reflected at the walls. The difficulty is that the radial plane polarization of the light is destroyed by total internal reflection. The phase shift, with total internal reflection, of the light polarized in the plane of incidence is different from that of the light polarized in the other plane. The result is that the plane polarized Čerenkov light becomes, in general, elliptically polarized, and the considerations of the last paragraph do not apply. The radiation from a particle traveling along the axis of a cylindrical radiator strikes the walls with its electric vector in the plane of incidence. This light retains its plane polarization, but most of the light from particles traveling at other radial positions is modified somewhat, the effect being larger for those particles that are farther from the axis.

For particles of appreciably lower velocity than that of light, total internal reflection is very efficient, and the cylindrical radiator method with a photomultiplier at one end is quite effective. Such a method is being used in this laboratory for the counting of high energy protons.

The directional properties of the radiation make it possible to count particles of some narrow band of velocities in the presence of and to the exclusion of particles of other velocities. This is possible because of the fact that the radiation is all emitted at the Čerenkov angle θ as in Eq. (1). If one is able to focus all or a substantial fraction of the light emitted at a given angle onto the cathode of a photomultiplier tube and to discard light emitted at other angles, one can make a counter having the characteristics described in the first sentence of this paragraph. A focusing method of this sort has been proposed by Getting,[1] and another one has been used by the writer.[5]

(3e) Applicability

Čerenkov counting can be used with high efficiency only for quite energetic particles. First of all, the particle must be moving faster than light in the medium used as a radiator. Not only must the particle have a high velocity, however, but it must be capable of traveling for a considerable distance through the radiator so that it can emit a sufficient number of photons for efficient counting. This distance for electrons in Lucite should be at least one centimeter, and for slow particles of sufficient penetration it is desirable to use longer radiators. If one wishes to make quantitative use of the directional properties of the radiation, it is essential that the multiple small angle Coulomb scattering be small. This calls for high energy particles and, of course, for low density low z radiators.

The Čerenkov counters used so far by the writer have been designed mostly to count particles which are within a few degrees of being parallel in direction as they enter the counter. This condition is not strictly necessary for counting, but it is certainly much easier to do quantitative work if it is met.

Čerenkov counters of one sort or another show promise of being useful tools in the study of phenomena associated with high energy accelerators. For example, let us consider the various particles produced by the Chicago synchrocyclotron from the point of view of Čerenkov counting.

The primary particles accelerated by the cyclotron are protons, deuterons, or alpha-particles. The value of β for the 450-Mev protons is approximately 0.74. With this speed the protons require a radiator with a refractive index larger than 1.35. Lucite makes quite a convenient radiator, since its index is 1.50, provided that one is interested in counting only the most energetic protons. For the study of elastic scattering this is the case and plans are being made here to carry out such a study with Čerenkov counters.

Deuterons and alpha-particles from this cyclotron are of rather inconvenient energy for Čerenkov counting, since their β is about 0.49. This would make necessary a radiator with index greater than 2.04. With a radiator of index less than 2.04, one can count faster particles in the presence of and with the exclusion of the deuterons or alpha-particles. This can be convenient on occasion.

Neutrons are produced by protons striking almost any target. In turn these neutrons can be made to produce protons by allowing them to strike a hydrogenous substance. These protons can be counted with a Čerenkov counter which, if Lucite is the radiator, becomes a detector for the neutrons of only the highest energies produced. In the case of 450-Mev maximum energy protons striking a beryllium target, the effective neutron energy is about 400 Mev.

High energy gamma-rays are produced by the decay of π^0-mesons which in turn are produced by proton-nucleon reactions in the cyclotron beam. They are produced also to some extent by bremsstrahlung of protons decelerated by collisions against nucleons. These high energy gamma-rays can produce electrons which are detected easily by Čerenkov counters. In this case one can discriminate against protons produced by neutron contamination of the gamma-ray beam by using a water radiator. Water with an index of refraction of 1.31 can give Čerenkov radiation only from particles of β greater than 0.76.

Charged mesons of all varieties produced by the cyclotron can be counted with Čerenkov counters, and for these particles the angle of emission of the light is in a convenient range for velocity discrimination. With a Lucite radiator, π-mesons down to about 70 Mev can be counted conveniently, and the counters have the distinction of being equally sensitive for positive and negative mesons which is not the case for phosphors which are sensitive to star particles.

(4) DESIGN AND PERFORMANCE OF FOCUSING CONTOURS

In this section we shall discuss in some detail the various focusing Čerenkov counters that have been proposed or used in other laboratories or in this one. For the purpose of this discussion a focusing counter is defined as one in which the light of one particular Čerenkov angle is focused to a spot where it can be directed onto a photomultiplier cathode, while light of other angles is directed elsewhere. Such a counter is capable, then, of recording particles of a narrow band of velocities and of rejecting particles of other velocities.

Before considering the various types of focusing counters, it may be well to discuss some optical properties which are common to all of them. All types used so far consist basically of a cylindrically symmetrical assembly of radiator and optical parts designed to focus light from a parallel beam of particles of one velocity onto the cathode of a photomultiplier. It is impossible, unfortunately, to focus all the light of one Čerenkov angle from a radiator of larger than infinitesimal size to a sharp point. The best possible focusing can be deduced from the following considerations.

Let us consider a particle which travels parallel to the axis of the system but removed from it by some distance d. Most of the light from such a particle follows paths which are skewed from the axis. Consequently, the photons have angular momentum around it. A very useful concept in the consideration of focusing counters is the angular momentum of the light because it is conserved as the ray moves through the system, and it can never be changed for one ray, no matter how often the light is reflected or refracted at cylindrically symmetrical boundaries. This angular momentum property of the light puts a limit on the sharpness of focus. A skew ray of light has angular momentum, and thus it can never pass through the axis of the system where by symmetry the center of the image must be. The image then has a minimum possible size which is related to the angular momentum of the most skewed light rays.

The application of the angular momentum concept is complicated by the fact that the momentum of a photon is different in a dielectric than in vacuum, the momentum of a photon like that of any other corpuscle being h/λ, which in a dielectric reduces to $h\nu n/c$ rather than $h\nu/c$, its value in vacuum. Since the momentum of a photon is smaller after it leaves the dielectric, and it does leave the dielectric in all focusing counters so far constructed. The factor by which the momentum must be multiplied to obtain the same angular momentum must be larger outside of the dielectric.

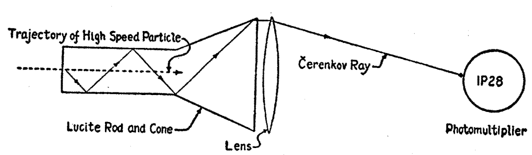

FIG. 3. Getting-Dicke Čerenkov counter.

Let us consider the case of a particle moving at a distance d from the axis of a focusing counter. The photons most highly skewed to the axis have angular momenta equal to $dp_d \sin\theta$, where p_d is the momentum of the photon in the dielectric and θ is the Čerenkov angle. Outside the dielectric the limiting angular momentum is $Dp_v \sin\theta'$, where D is the closest approach of the photon to the axis (ideally in the focal plane), p_v is the momentum of the photon in vacuum, and θ' is the angle between the ray of light outside of the dielectric and the axial direction. From equating these two angular momenta we get the relation

$$D = nd \sin\theta/\sin\theta'. \quad (3)$$

D is under ideal conditions the diameter of the image spot produced by a beam of particles of diameter d. D can never be much smaller than d, and in practical cases it is almost always larger. The effective area of a focusing counter, then, is limited to approximately the sensitive area of the photomultiplier cathode.

(4a) The Getting-Dicke Focusing Counter

In 1946, Getting[1] proposed a design of Čerenkov counter, the optical system of which is shown in Fig. 3. The counter consists of four essential parts: a cylindrical Lucite radiator; a conical Lucite section in optical contact with the radiator, designed to direct light of one Čerenkov angle into a parallel beam; a lens to focus the parallel beam to a point; and finally a diaphragm and a photomultiplier cathode.

Dicke[2] made an attempt to apply such a counter, and although he was unsuccessful in detecting cosmic radiation with it, he did find some pulses above the noise background of the tube when he exposed it to radiation from a 20-Mev betatron. Unfortunately he was unable to demonstrate that the pulses came from Čerenkov radiation rather than from some other source. At the time of Dicke's experiment the distributed type of amplifier was not yet available. The comparatively slow amplifiers he used must certainly have contributed to the failure of the counter as a practical instrument. Even with fast amplifiers and fast coincidence systems, it is difficult to get significant results with a betatron because of its usually very small duty cycle.

Application of Eq. (3) to Dicke's counter, assuming the diagram in his publication to be drawn to scale, indicates that a rather unpleasantly small fraction of the light from an off-axis electron must have been collected by the photomultiplier cathode. The difficulty is that the angle of convergence produced by the lens

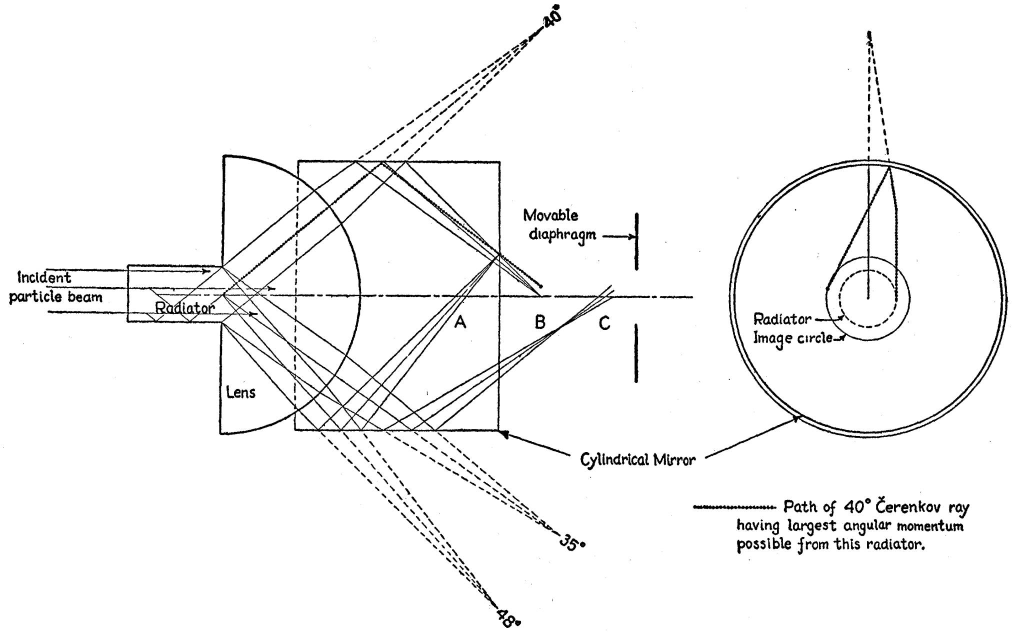

FIG. 4. Optical system of spherical lens, cylindrical mirror focusing counter. Radiator and lens are assumed to be made of Lucite with a refractive index of 1.5.

was quite small compared to the Čerenkov angle. A rough estimate indicates that approximately half of the light from a high speed electron traveling one millimeter off the axis and parallel to it was focused into a circle of 5-mm diameter at the focus. The situation must have been somewhat worse for particles farther from the axis because of imperfect radial focusing.

The considerations of the previous paragraph are based on the assumption that a particle moves through the radiator on a straight line exactly parallel to the axis. In a practical case this is not true, and in particular it was not true in Dicke's experiment, because of the low energy of the electrons available to him. The Getting-Dicke type of counter has not been used by the writer, but there appears to be no fundamental reason why it should not be usable with well-collimated beams of high energy particles.

(4b) Spherical Lens, Cylindrical Mirror Counters

A type of focusing counter that has been used by the writer is shown in Fig. 4. A beam of particles is allowed to enter the radiator from the left. Čerenkov radiation produced there is brought by total internal reflection to the center of the hemispherical lens which, like the radiator, is machined out of Lucite. The Lucite lens, since its refractive index is 1.5, has a focal length equal to twice its radius of curvature. Thus the Čerenkov light for particles of a given speed is focused into a sharp ring at a distance of three radii from the center of curvature of the lens. A cylindrical mirror (glass tubing aluminized on the inside) of half the cylindrical radius of this ring reflects the image back to the center to make a point focus for rays coplanar with the axis of the system.

Rays not coplanar with the axis, as in the case of any cylindrically symmetrical counter, retain their angular momentum and Eq. (3) applies. In this case, however, the angle of convergence is equal to the Čerenkov angle, so that the net result in the absence of spherical aberration is that the image is larger in linear dimension than the end of the radiator by the factor n (1.5 for Lucite). This effect is shown in Fig. 4 in which one skew ray is traced out and marked as a beaded line. It can be seen by reference to both views of the optical system that a skew ray returns, after focusing, to the other side of the axis, approximately 1.5 times as far away from it as its closest approach in the radiator.

A diaphragm placed at the focus will let through light only of Čerenkov angles close to that for which the system is adjusted. As can be seen in Fig. 4, the focusing is reasonably good over quite a range of angles, so that one can vary the velocity setting of such a counter simply by sliding the diaphragm and photomultiplier to different positions on the axis.

Some fraction of the light produced by any particle in the beam that enters the radiator will pass through even a small diaphragm, if it is set to the right angle, although the fraction may be quite small for a particle far from the axis and a small diaphragm. The fraction within a diaphragm circle of radius r, neglecting dispersion and small angle multiple Coulomb scattering, is

$$f=(2/\pi)\sin^{-1}(r/n\rho), \tag{4}$$

where ρ is the radius of the radiator and n is the index of refraction of the lens. The assumption is made here

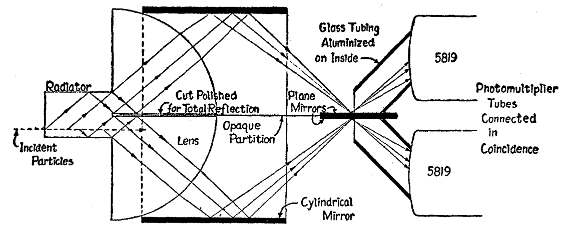

FIG. 5. Split lens focusing counter. Optical system divided beyond end of radiator to reduce background from Čerenkov radiation in the lens.

that one can neglect spherical aberration in the lens. This assumption is justifiable if the lens is made large in radius compared to the radiator. A convenient diaphragm is one equal in size to the radiator. With a Lucite radiator and lens, such a diaphragm accepts about 46 percent of the light from a particle which travels just inside the surface of the radiator.

The Čerenkov light emitted by a radiator on which monoenergetic particles impinge does not all have one definite angle measured from the axis of the system. One can expect a spread of angles of radiation which is a result of a number of causes. First of all, the radiator material always has dispersion. This gives an effective spread of Čerenkov angles which depends on what band width of light is accepted by the photomultiplier. Secondly, the particles are slowed down in passing through the radiator material. This is not serious in the case of electrons since highly relativistic particles essentially do not change their speed when they lose energy. For particles which are not completely relativistic, however, there may be a considerable change in the angle of the Čerenkov radiation as the particles traverse the radiator. Another cause of angular spread in the Čerenkov light is variation in the directions of the particles. This is due to two causes; one is the small angle multiple Coulomb scattering in the radiator, and the other is spread of angle in the incident beam of particles.

In the case of 140-Mev pions the effect of slowing down in Lucite on the Čerenkov angle is about one degree in a five-centimeter length, while in the same length the multiple scattering produces a root mean square angular deviation of about 2.5 degrees. The effect of deviation from parallelism to the axis is to produce a broadening in the radiation angle on each side of the Čerenkov angle by an amount equal to the deviation. These effects seem to account quite satisfactorily in order of magnitude for the observed widths of angular resolution curves.

To use the optical system of Fig. 4 for a counter one can simply insert a photomultiplier so that light passing through the diaphragm strikes its cathode. For the elimination of background counts, however, it is quite convenient to use some sort of coincidence arrangement. The simplest arrangement is one of coincidence between the Čerenkov counter and an external scintillation crystal through which the beam of particles passes. Experimentally it has been found that, while such a counter responds quite satisfactorily to particles of the velocity to which it is adjusted, it is subject to background pulses caused by particles of different velocities which pass through the system. One source of background pulses is the photomultiplier tube itself, which in this arrangement is directly in the beam of particles to be counted. Some of the background pulses appear to be due to Čerenkov light emitted when the particles pass through the glass envelope of the tube. Other background pulses are due to Čerenkov radiation emitted in the lens rather than in the radiator, both by particles which have passed through the radiator and by stray particles.

To obtain a large sensitive area, one likes to have a radiator of large diameter which implies a large lens. For such a counter the above effects may be large, and it may be convenient to avoid recording pulses caused by radiation emitted after the particle has left the radiator. This can be done, as is shown in Fig. 5, by splitting the whole system beyond the radiator into two separate parts, each of which has its own photomultiplier, the two multipliers being connected to a fast coincidence circuit. It should be remarked that such a split counter has not been built although there appears to be no reason why it should not be quite satisfactory.

For smaller counters, having for instance a radiator diameter of one centimeter, quite satisfactory results have been obtained using the arrangement shown in Fig. 6, which is a scale drawing of an existing counter. In this particular case there was no separate radiator, the lens itself performing the function. The lens, for this purpose, was extended $\frac{1}{4}$ inch beyond the center of the sphere. Here the light, before it is allowed to be focused on the axis, is split with a pair of plane mirrors to focus it on the cathodes of two photomultipliers which are connected to a fast coincidence circuit. This arrangement makes a self-contained counter capable of good velocity resolution but has the disadvantage of not having a definite sensitive area, and some of the

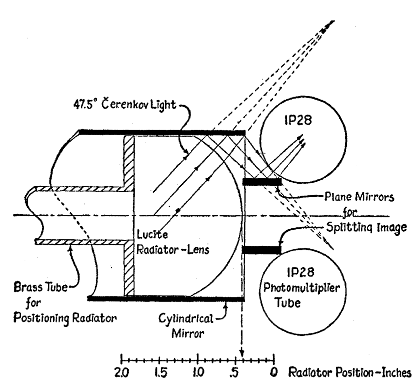

FIG. 6. Focusing counter with small sensitive area used for analysis of meson beams. Radiator in this counter is integral with the lens.

light is wasted, namely, that part which misses the surfaces of the two mirrors.

The counter of Fig. 6 was used to obtain the data shown in Fig. 7. The upper curve is the result of exposing the counter to the 145-Mev negative pion beam from the 170-inch synchrocyclotron and varying the relative position of the radiator to the rest of the optical system. The lower curve is the same thing except that a 12-grams per cm^2 graphite absorber was inserted in front of the counter. This amount of carbon should reduce the energy of the 145-Mev pions to about 121 Mev and should change the Čerenkov angle from 40.4° to 38.1°. The data of Fig. 7 indicate angles before and after the carbon absorber of 39.9° and 38.0°, which considering the accuracy of the measurement, are in tolerable agreement with what one would expect. The counting rate is reduced by the insertion of the absorber because of scattering which removes mesons from the beam. The data for these curves were taken with a rather high bias on the coincidence circuit to obtain better resolution. With the bias reduced to the point where single pulses were just rejected, the resolving angle of the counter was about twice what is shown in Fig. 7.

(4c) Cylindrical Mirror Counter Without Lens

This is a type of counter which has not been used by anyone so far as the writer knows. It would be constructed just as if it were a counter with a lens, but the lens would be left out and the radiator would have a plane end perpendicular to the axis. It would be applicable to particle-radiator combinations leading to small Čerenkov angles and would be considerably better for this purpose than a counter using a lens. Such a counter focuses rays with angular momentum better than does a lens counter. The reason is that the angle of convergence is larger than the Čerenkov angle. For a reasonably large mirror to radiator diameter ratio, the image has the same size as the radiator, and even for a ratio of 2.5 all the light from a one-inch radiator is focused inside a 1.2-inch image. A counter of this type is applicable best to small Čerenkov angles, electrons in Lucite giving, for instance, radiation of such an angle that it is totally reflected at the end of the radiator. Such a counter with a water radiator would be well adapted to protons with kinetic energies of about one Bev. Protons of 950-Mev kinetic energy give, for example, a convergence angle of approximately 38°.

(5) NONFOCUSING ČERENKOV COUNTERS

Counters of this type were first used by Jelley[3] who was able to count cosmic-ray particles with a water radiator in contact with an E. M. I. end-window type photomultiplier.

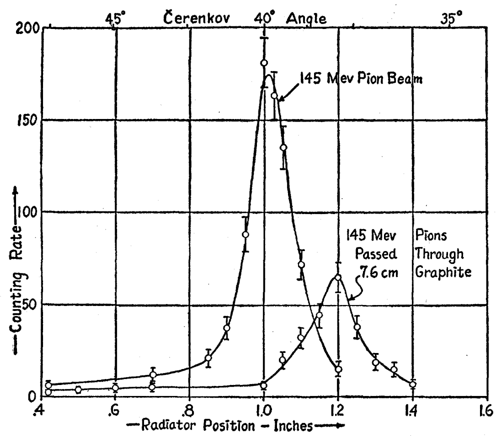

FIG. 7. Velocity resolution of counter of Fig. 6. Radiator position scale corresponds to inch scale of Fig. 6, in which counter is shown in the 0.42-inch position.

(5a) Nonfocusing Lucite Counter

Such a counter has been used in this laboratory[9] as a threshold detector of high energy neutrons by counting proton recoils from a paraffin converter. A 6-in. long Lucite rod, 1.5 in. in diameter, was used as a radiator. It was placed in optical contact with the cathode of a 5819 photomultiplier with the aid of a little silicone vacuum grease. The counter was operated in coincidence with a scintillation crystal placed in front of the radiator. The counting efficiency was quite high for protons of more than 400 Mev but fell off sharply for lower energies, being very little above zero for 350-Mev protons. Used as a detector for neutrons from a beryllium target in the internal 450-Mev maximum energy proton beam of the synchrocyclotron, the converter-counter assembly counted an effective neutron energy distribution having a maximum at 410 Mev and falling to zero at 450 Mev on the upper side and about 360 Mev on the lower side. This is a considerably sharper and also a higher energy spectrum than is obtained with a scintillation counter coincidence arrangement.

(5b) Nonfocusing Water Counter

A counter of this type is in use here for the counting of electrons produced by the decay gamma-rays from π^0 mesons produced by the synchrocyclotron. It operates quite satisfactorily, being superior to the ordinary scintillation pair telescope in that it is much less sensitive to stray radiation background. The water is contained in a glass tube aluminized on the inside and cemented to an aluminum fixture which makes a water tight seal to the 5819 photomultiplier with an O ring gasket. The counter is operated in coincidence either with another Čerenkov counter or with a crystal scintillator.

[9] V. A. Nedzel and J. Marshall, Bull. Am. Phys. Soc. 27, No. 1, 29 (1952).