frequency of ω^{-2}. The curve labeled B uses a variation of the Q factor with frequency of $\omega^{-3/2}$. It is apparent that the 24,000 Mc/sec data more nearly fit the curve that uses the $\omega^{-3/2}$ variation than that predicted by the London theory.

We wish to acknowledge the assistance given by Dr. Rufus G. Fellers, especially for his treatment of the radiation loss problem in the resonant cavities. We also wish to thank Mr. A. Szwed for his assistance in some of the temperature measurements.

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Low Energy n-d Scattering: Comparison of Experiment with Theory

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D-wave effects are shown to be significant, though small, in the experimental n-d scattering data at 4.5 and 5.5 Mev. Consequently, since the calculations of Buckingham and Massey include only S and P wave phases, a phase shift analysis of the experimental data is carried out to determine these phases so that a suitable comparison can be effected between experiment and theory. The resulting "experimental" P-wave phases agree only qualitatively with those of the symmetric-interaction calculations, but they completely disagree with those of the neutral-interaction calculations. The lack of quantitative agreement in the former case could be attributed to the range, depth, and possibly shape of the inter-nucleon potential chosen by Buckingham and Massey especially since their S-wave phases, which they found to be insensitive to force-type, are not in satisfactory agreement with the experimental results of slow neutron scattering.

INTRODUCTION

A PAPER by Wantuch appeared recently in which experimental results were reported for the absolute values of the differential cross sections for 4.5- and 5.5-Mev neutrons scattered from deuterons.[1] A comparison was made therein of these experimental data and the theoretical calculations of Buckingham and Massey,[2] with the following conclusions. The values of the total cross section, obtained by extrapolation, definitely preferred the symmetric force theory. The actual angular distributions, however, did not agree well with either the symmetric or neutral force theory, but, on the other hand, the ratio $\sigma(\pi)/\sigma(\theta)$ definitely preferred the neutral force theory. Very much the same sort of contradictory conclusions have been drawn by others from the results of n-d scattering experiments at low energies.[3–5]

Rosenfeld has stated that the agreement between the variation of the experimental n-d total cross sections with energy and that predicted by the symmetric force theory calculations of Buckingham and Massey provides the "weightiest argument" in favor of this type of interaction.[6] Since Buckingham and Massey have considered only the S- and P-wave phase shifts in their calculations, the above mentioned discrepancies might possibly be accounted for by the presence of small D-wave phases which would deform the angular distributions to an appreciable extent only near $\theta=\pi$ or 0 while the total cross-section values would be effectively unchanged. The present paper is the result of an investigation into this possibility, carried out in the hope of clarifying the situation.

A more exhaustive treatment has been given to Wantuch's experimental data in order that an effective comparison could be made with the theoretical calculations of Buckingham and Massey. First, a least-square fit of the data to expansions in $\cos\theta$ revealed the presence of small D-wave phases and provided a reliable extrapolation of these data for calculating the corresponding total cross sections. Second, an attempt has been made to find the actual values of the experimental S- and P-phase shifts from the coefficients in these expansions in order that a more direct comparison with the theoretical calculations could thereby be effected. Finally, a more reliable method has been employed for interpolating the calculated values given by Buckingham and Massey for these phase shifts to the energies involved in the experiment.

The following notation was adopted.

$\sigma(\theta)$: differential scattering cross section as a function of θ, the neutron scattering angle in the center-of-mass system.

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[1] E. Wantuch, Phys. Rev. **84**, 169 (1951). The data used here were obtained from his thesis, New York University, 1950.

[2] R. A. Buckingham and H. S. W. Massey, Proc. Roy. Soc. (London) **A179**, 123 (1941). Their force-types I and III are referred to here as neutral (or "0") and symmetric (or "X"), respectively (see reference 3).

[3] L. Rosenfeld, *Nuclear Forces* (Interscience Publications, New York, 1948), Sec. 14.12.

[4] Martin, Burhop, Alcock, and Boyd, Proc. Phys. Soc. (London) **A63**, 884 (1940). See also discussion by R. A. Buckingham, which follows thereafter.

[5] Hamouda, Halter, and Scherrer, Phys. Rev. **79**, 539 (1950); and Helv. Phys. Acta **24**, 217 (1951); also, I. Hamouda and G. de Montmollin, Phys. Rev. **83**, 1277 (1951).

[6] See reference 3, p. 298.

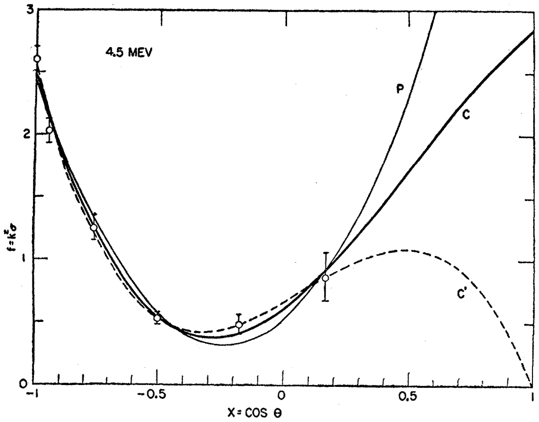

FIG. 1. Angular distribution ($f=k^2\sigma$ *versus* $x=\cos\theta$) for n-d scattering at 4.5 Mev. The small circles represent Wantuch's experimental points with which the associated vertical lines indicate the quoted limits of error. Curve P represents the best fit of a quadratic in x; curve C' represents the best fit of a cubic in x; and curve C represents the best fit of a cubic in x subject to the restriction that the resulting total cross section agree with an independently measured value.

E_{lab}: energy of incident neutron in laboratory system.
k: angular wave number in center-of-mass system; $k^2/E_{\text{lab}}=2.144$ Mev$^{-1}\cdot$barns^{-1}.
$f(x)=k^2\sigma(\theta)$: dimensionless form of the differential scattering cross section; $x=\cos\theta$.
$f_0=2\pi\int_{-1}^{+1}f(x)dx$: dimensionless form of corresponding total cross section.
δ_{ml}: scattering phase shift with $l=0, 1, 2, \cdots$ corresponding to $S, P, D, \cdots$ waves, respectively, and $m=2$ or 4 corresponding to doublet or quartet spin states, respectively.
$P_l(x)$: Legendre polynomial of degree l.

EXPANSION OF DIFFERENTIAL CROSS SECTIONS: D-WAVE EFFECTS

The differential cross section for n-d scattering in the center-of-mass system can be expressed in terms of the characteristic phases as follows:[7]

$$f(x)= \Sigma_m(m/6)\,|\,\Sigma_l(2l+1)P_l(x)(\sin\delta_{ml})\exp(i\delta_{ml})\,|^2. \quad (1)$$

Since H^3 exists in a ^2S ground state, in the central-force approximation, and has no other stable bound states, all the phases except δ_{20} (which approaches π) must approach zero as $k\rightarrow 0$. In fact, for $\delta_{ml}\neq\delta_{20}$,

$$\delta_{ml}\rightarrow C_{ml}k^{2l+1} \text{ as } k\rightarrow 0, \quad (2)$$

where C_{ml} is a constant. As k increases from zero, a particular phase shift δ_{ml} does not become significant until that value of k is reached for which

$$ka\approx l, \quad (3)$$

where a is the effective range of the n-d interaction.

[7] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), second edition, Chapter II.

Taking[8]

$$a\approx 5\times 10^{-13} \text{ cm}, \quad (4)$$

one then finds for $E_{\text{lab}}=5.5$ Mev,

$$ka\approx 1.72. \quad (5)$$

Hence, in accord with (3), we felt justified in taking $\delta_{ml}=0$ for $l\geqq 3$ at $E_{\text{lab}}=4.5$ and 5.5 Mev. Also, we anticipated that if D-wave effects are present, the δ_{m2} phases are small.

We first of all settled the question of whether D-wave effects are present at all in Wantuch's experimental data by investigating the consequence of assuming that they are not. Under this assumption $f(x)$, as given by Eq. (1), reduces to a second degree polynomial in x. The curves in Figs. 1 and 2 labeled P show, in each case, the best (least-square) fit to the given experimental points obtainable with a second degree polynomial. These curves pass reasonably close to the experimental points, but must be ruled out nevertheless, since the inordinately high values they give for $f(x>0.5)$ are completely incompatible with independently measured total cross-section values (see below). Hence, we felt assured that D-wave effects, though small, are indeed significantly present.

Next we considered the possibility that D-wave effects are present only to such an extent that all but first-order dependence on δ_{m2} can be neglected in Eq. (1). In this case, Eq. (1) reduces to a cubic in x:

$$f(x)=b_0+b_1x+b_2x^2+b_3x^3. \quad (6)$$

The curve in Fig. 1 (4.5 Mev) labeled C' and that in Fig. 2 (5.5 Mev) labeled C show, in each case, the best (least-square) fit of the $f(x)$ given by (6) to the experimental points. These curves pass within or close to the limits of error quoted for the experimental points—adjacent high and low values compensating for each other. In the 4.5-Mev case, however, this straight least-square fit with the cubic polynomial gives a spurious result: Although the curve C' gives an excellent fit to the experimental points, it turns over in the range $(\theta<80°)$ in which no experimental readings were obtained and falls rapidly to slightly below zero at $\theta=0$.

A check on the curves determined in this manner was made by comparing the value of the total cross section, as computed from the polynomial coefficients in each case, with an independent experimental determination of this quantity. It follows from Eq. (6) that

$$f_0=4\pi(b_0+b_2/3). \quad (7)$$

Upon substituting the values of b_0 and b_2 determined by the above curve-fitting procedure, we obtained

$$f_0=10.8, \text{ for } E_{\text{lab}}=4.5 \text{ Mev}; \quad (8a)$$

$$f_0=17.9, \text{ for } E_{\text{lab}}=5.5 \text{ Mev}. \quad (8b)$$

From experimental data on total n-d scattering cross

[8] H. A. Bethe, Revs. Modern Phys. 9 (1937), Sec. 73A.

sections obtained by Nuckolls *et al.*,[9] we found that

$$f_0=16.3\pm0.5,\ \text{for}\ E_{\text{lab}}=4.5\ \text{Mev}; \qquad (9a)$$

$$f_0=17.9\pm0.9,\ \text{for}\ E_{\text{lab}}=5.5\ \text{Mev}. \qquad (9b)$$

Although the agreement between (8a) and (9a) is poor, the excellent agreement between (8b) and (9b) indicates that a cubic polynomial in x gives an adequate representation of the differential cross section for all θ at $E_{\text{lab}}=5.5$ Mev. Moreover, since D-wave effects should increase here with increasing energy, we concluded that Eq. (6) must also provide an adequate representation of the experimental $f(x)$ at $E_{\text{lab}}=4.5$ Mev, i.e., the assumption that higher order terms in δ_{m2} can be neglected at these energies is justified so far as the given experimental data are concerned.

In view of this conclusion and the anomalous behavior of the curve C' in Fig. 1 for $E_{\text{lab}}=4.5$ Mev, $f(x)$ given by (6) was next fitted to the data at this energy by means of a modified least-square procedure which gave the best fit subject to the restriction that the total cross section calculated from the resulting coefficients via (7) must agree exactly with the independently determined value given by (9a) above (i.e., $f_0=16.3$)—such a procedure being consistent with the results obtained above for 5.5 Mev. The resulting curve is C of Fig. 1. This curve gives a good fit to the experimental points and, accordingly, it was adopted to represent the experimental data at $E_{\text{lab}}=4.5$ Mev.

The values of the b coefficients in Eq. (6) determined, in each case, by the above procedures are

$$b_0=0.608;\ b_1=1.503;\ b_2=2.067;\ b_3=-1.337; \qquad (10a)$$

$$b_0=0.746;\ b_1=1.752;\ b_2=2.024;\ b_3=-1.701; \qquad (10b)$$

at $E_{\text{lab}}=4.5$ and 5.5 Mev, respectively. These coefficients have the following significance in terms of the phase shifts δ_{20}, δ_{40}, δ_{21}, δ_{41}, δ_{22}, δ_{42} as derived from Eq. (1).

$$b_0=\tfrac{1}{3}(\sin^2\delta_{20}+2\sin^2\delta_{40})-(5/3)(\delta_{22}\sin\delta_{20}\cos\delta_{20}+2\delta_{42}\sin\delta_{40}\cos\delta_{40}); \qquad (11a)$$

$$b_1=2[\sin\delta_{20}\sin\delta_{21}\cos(\delta_{20}-\delta_{21})+2\sin\delta_{40}\sin\delta_{41}\cos(\delta_{40}-\delta_{41})]-5(\delta_{22}\sin\delta_{21}\cos\delta_{21}+2\delta_{42}\sin\delta_{41}\cos\delta_{41}); \qquad (11b)$$

$$b_2=3(\sin^2\delta_{21}+2\sin^2\delta_{41})+5(\delta_{22}\sin\delta_{20}\cos\delta_{20}+2\delta_{42}\sin\delta_{40}\cos\delta_{40}); \qquad (11c)$$

$$b_3=15(\delta_{22}\sin\delta_{21}\cos\delta_{21}+2\delta_{42}\sin\delta_{41}\cos\delta_{41}). \qquad (11d)$$

Since we have found D-wave effects to be significant, though small, in the experimental data considered here, a comparison cannot be made between these data and the calculations of Buckingham and Massey, which include only S- and P-phase shifts, unless some procedure is developed which will lead to an evaluation of

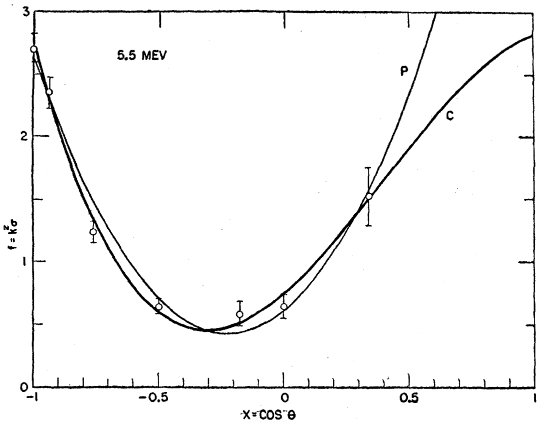

FIG. 2. Same as Fig. 1, for n-d scattering at 5.5 Mev. Curves C and C' coincide in this case.

the distortion of the angular distributions produced by these D-wave effects. It should be noted that D-wave effects (and perhaps those of higher angular momentum) may be more significant at these energies than the foregoing results indicate, but that the experimental data, as presented, reveal no more.

EVALUATION OF PHASE SHIFTS

Since the results of the Buckingham and Massey calculations are expressed in terms of the phases they obtained (for the various types of interactions), the most decisive way in which a comparison could be made between their calculations and the experimental data would be to deduce a corresponding set of these phases directly from the data. It is obvious, however, that this is not possible by any straightforward procedure in our situation since, all other difficulties aside, there are six unknown phases altogether at each energy and Eqs. (10) and (11) provide for only four experimentally determined relationships between them.

We were interested at the time only in making a valid comparison of the experimental data with the theoretical work of Buckingham and Massey such that it might be possible to determine whether the data favors their calculations for a neutral or symmetric type of interaction. In the course of their calculations, Buckingham and Massey found that, independent of the force-type considered, the following equation is approximately valid:

$$\delta_{20}=\pi+\delta_{40}. \qquad (12)$$

We made use of this equation to provide a fifth relationship between the six phases since it did not predispose our results in favor of either force-type. The question of a sixth relationship was left open at this point.

We chose δ_{20} as the independent variable and expressed the remaining phases in terms of it. Upon eliminating δ_{22} and δ_{42} from Eqs. (11) and making use of Eq. (12), the following simple relationships were

[9] Nuckolls, Bailey, Bennett, Bergstralh, Richards, and Williams, Phys. Rev. 70, 805 (1946).

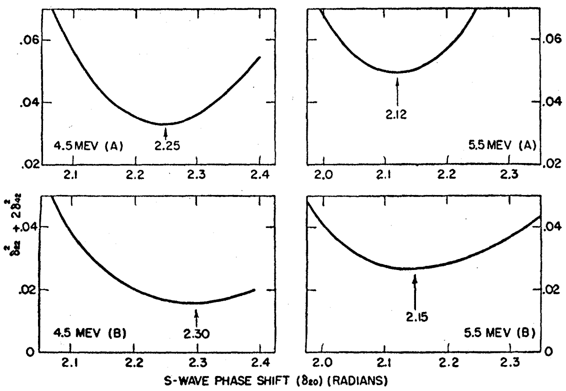

FIG. 3. Curves used in determining (from their minima) a unique value for δ_{20}. $(\delta_{22}^2+2\delta_{42}^2)$ is shown as a function of δ_{20}: the left-hand and right-hand curves correspond to 4.5 and 5.5 Mev, respectively; the upper and lower curves correspond to sets A and B of phases, respectively.

found between δ_{21}, δ_{41}, δ_{20}:

$$\sin 2\delta_{21}+2\sin 2\delta_{41}=C(\delta_{20}); \qquad (13a)$$

$$\cos 2\delta_{21}+2\cos 2\delta_{41}=S(\delta_{20}); \qquad (13b)$$

where

$$S(\delta_{20})=2(2-b_0-b_2/3)-\cos 2\delta_{20};$$

$$C(\delta_{20})=2(1-b_0+b_1-b_2/3-b_3/3)\csc 2\delta_{20} -2(1-b_0-b_2/3)\cot 2\delta_{20}-\sin 2\delta_{20}.$$

The D-wave phases were expressed as functions of δ_{20} by means of the following relationships:

$$(\sin 2\delta_{20})\delta_{22}+2(\sin 2\delta_{20})\delta_{42}=(6/5)(\sin^2\delta_{20}-b_0); \qquad (14a)$$

$$(\sin 2\delta_{21})\delta_{22}+2(\sin 2\delta_{41})\delta_{42}=(2/15)b_3; \qquad (14b)$$

with δ_{21} and δ_{41} replaced by their values obtained from Eqs. (13). The above equations do not produce a one-to-one correspondence between a given value of δ_{20} and the resulting values of δ_{21}, δ_{41}, δ_{22}, δ_{42}. This multiple-valuedness is partially restricted, however, by the requirement that the absolute value of these latter phases must lie in the first quadrant. As a result, we obtained only two different sets of values for δ_{21}, δ_{41}, δ_{22}, δ_{42} which were consistent with a given value of δ_{20} in the range of δ_{20} values considered. These two sets of values were designated as set A and set B.

The two sets of P and D phases were calculated for values of δ_{20} between 1.8 and 2.4 radians (at intervals of 0.05 radian). Outside these limits, one or more of the resulting D-wave phases became too large to be considered possible. The values of δ_{20} were chosen to lie in the second quadrant since δ_{20} must approach π as $k\rightarrow 0$ and decrease as k increases.

Now the coefficient of the term in x^4 of Eq. (1) is

$$b_4=(75/4)(\sin^2\delta_{22}+2\sin^2\delta_{42})\rightarrow(75/4)(\delta_{22}^2+2\delta_{42}^2). \qquad (15)$$

This term cannot be zero, of course, unless $\delta_{22}=\delta_{42}=0$. The results of the previous section, however, lead to the conclusion that the contribution of this term to the given differential and total cross sections must be negligible in comparison to that of the other term considered. As a result, it was decided to minimize the contribution of this term—thus providing a sixth relationship between the "experimental" phases. In Fig. 3 are shown the graphs of $(\delta_{22}^2+2\delta_{42}^2)$ as a function of δ_{20}, as deduced from the calculations described above, and the value of δ_{20} selected from the minimum in each case.

This minimization procedure can be justified in our case by a consideration of the magnitude of $(\delta_{22}^2+2\delta_{42}^2)$ over the range of δ_{20} values (Fig. 3). The contribution to the total cross section of the term in x^4 is

$$\Delta f_0=15\pi(\delta_{22}^2+2\delta_{42}^2). \qquad (16)$$

At the minima of the curves in Fig. 3, this was found to have the following values:

$$4.5\text{ Mev: } (A),\ \Delta f_0=1.6;\ (B),\ \Delta f_0=0.8; \qquad (17a)$$

$$5.5\text{ Mev: } (A)\ \Delta f_0=2.3;\ (B),\ \Delta f_0=1.3. \qquad (17b)$$

In neither case are these values less than the experimental uncertainties in f_0 given in Eqs. (9a), (9b)—indicating, therefore, the validity of the minimization procedure. As a further check, the coefficient (15) of the term in x^4 was determined by a perturbation method in which the difference between the nominal values of the experimental points and the corresponding points of the curves C of Figs. 1 and 2 was fitted by a least-square procedure to the function b_4x^4. This gave the following values of $(\delta_{22}^2+2\delta_{42}^2)$:

$$4.5\text{ Mev: } 0.0002;\ 5.5\text{ Mev; } 0.0002. \qquad (18)$$

Both values are much lower than the corresponding minima of Fig. 3—hence, the minimization procedure is again indicated.

The sets A and B of phase shifts values are shown in Table I as they were determined from the foregoing analysis, in accordance with the given experimental data and the theoretical relationship (12). It should be emphasized that these results have a limited applicability, inasmuch as they depend on Eq. (12), except in a comparison of the experimental data with the calculations of Buckingham and Massey.

As indicated above, neither of the two sets A and B of phases can be ruled out on the basis of the given information, so that there is a resulting lack of uniqueness in this phase shift analysis. A possible procedure for removing this ambiguity is to rule out set B on the basis of the results of Buckingham and Massey which show that δ_{41} is positive regardless of the force-type

TABLE I. The two possible sets (A and B) of phase shift values as derived from the experimental data and Eq. (12).

Set	E_{lab}	δ_{20}	δ_{21}	δ_{22}	δ_{40}	δ_{41}	δ_{42}
A	4.5 Mev	2.25	−0.95	0.15	−0.89	0.12	−0.08
A	5.5 Mev	2.12	−0.94	0.19	−1.02	0.15	−0.08
B	4.5 Mev	2.30	0.71	−0.08	−0.84	−0.41	0.07
B	5.5 Mev	2.15	0.73	−0.11	−0.99	−0.38	0.08

considered. This procedure was used by Critchfield in his phase shift analysis of low energy p-d scattering.[10]

COMPARISON WITH THEORY AND CONCLUSIONS

Buckingham and Massey have calculated the values of the S and P phases for n-d scattering at energies corresponding to $k=2$, 4, and $5\times(10^{-12}\text{ cm})^{-1}$ for both a neutral and symmetric type of inter-nucleon interaction. In order to interpolate their results to the experimental energy values, we made use of the following expansions for these phases:[11]

$$k\cot\delta_{20}=-1/a_2+\beta_2k^2; \tag{19}$$

$$k^3\cot\delta_{21}=A_2+B_2k^2+C_2k^4; \tag{20a}$$

$$k^3\cot\delta_{41}=A_4+B_4k^2+C_4k^4. \tag{20b}$$

The best (least-square) fit of their calculated S phases

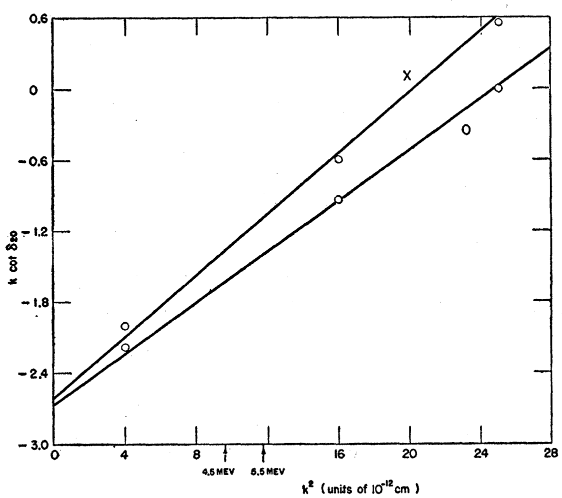

FIG. 4. Variation of (doublet) S-wave phase with energy: $k\cot\delta_{20}$ as a function of k^2 (using 10^{-12} cm as length unit). Small circles represent points calculated by Buckingham and Massey for a neutral (curve labeled O) and a symmetric (curve labeled X) type of interaction. See Eq. (19).

to Eq. (19) with $k\cot\delta_{20}$ plotted as a function of k^2 is shown in Fig. 4. The calculated points fall very close to the straight line so determined in each case. The values of the constants in Eq. (19) for each type of interaction turn out to be:

$$\text{Neutral:}\quad a_2=0.374;\ \beta_2=0.108; \tag{21a}$$

$$\text{Symmetric:}\quad a_2=0.381;\ \beta_2=0.129; \tag{21b}$$

using 10^{-12} cm as a unit of length.

The P-wave phases, as determined from Eq. (20)

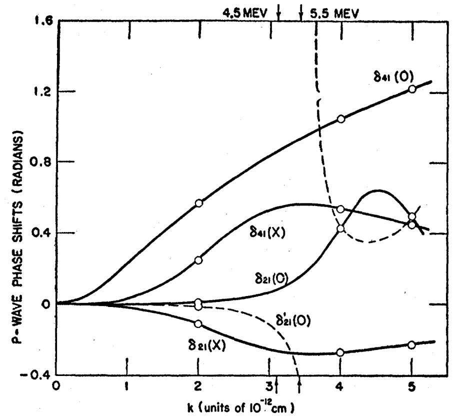

FIG. 5. Variation of P-wave phases with energy: graphs of Eqs. (20a), (20b) as fitted to points (indicated by small circles) calculated by Buckingham and Massey. Reading from top to bottom: $\delta_{41}(O)$, $\delta_{41}(X)$, $\delta_{21}(O)$, $\delta_{21}'(O)$, $\delta_{21}(X)$. The letters O and X respectively stand for the neutral and symmetric types of interaction.

and the calculated values, are graphed as functions of k in Fig. 5. Two curves are shown for δ_{21} in the neutral (O) interaction case. The broken line curve is that which was obtained by taking the Buckingham and Massey calculated values exactly as given. This curve exhibits an anomalous behavior, however—the upper, right-hand branch approaches π as $k\to 0$—and was consequently ruled out here. The other curve, the one adopted for use here, was obtained by replacing their given value of -0.01 radian by $+0.01$ radian at $k=2\times(10^{-12}\text{ cm})^{-1}$. This value was rather arbitrarily chosen as being the nearest adjacent positive value proceeding in steps of 0.01 radian—a simple means of starting the curve off in the right direction.[12] The constants in Eq. (20) determined in this way for the two types of interactions are:

Neutral:

$$A_2=1220;\ B_2=-117;\ C_2=3.09; \tag{22a}$$

$$A_4=1.31;\ B_4=2.98;\ C_4=-0.05; \tag{22b}$$

TABLE II. Values of the S and P phase shifts interpolated from the theoretical calculations of Buckingham and Massey for neutral and symmetric types of interactions, together with the corresponding "experimental" values from Table I.

$E_{\text{lab}}=4.5$ Mev	δ_{20}	δ_{21}	δ_{41}
Neutral	2.07	0.08	0.86
Symmetric	1.99	−0.26	0.55
A	2.25	−0.95	0.12
B	2.30	0.71	−0.41
$E_{\text{lab}}=5.5$ Mev	δ_{20}	δ_{21}	δ_{41}
Neutral	1.96	0.15	0.94
Symmetric	1.88	−0.27	0.57
A	2.12	−0.94	0.15
B	2.15	0.73	−0.38

[10] C. L. Critchfield, Phys. Rev. **73**, 1 (1948).

[11] The derivation of these relationships, which will be omitted here, follows directly from the integro-differential equations used by Buckinham and Massey according to methods which are similar to those devised for obtaining the corresponding relationships for the n-p case by: H. A. Bethe, Phys. Rev. **76**, 38 (1949); G. F. Chew and M. L. Goldberger, Phys. Rev. **75**, 1637 (1949). The relationship (19) has also been derived by: A. Troesch and M. Verde, Helv. Phys. Acta **24**, 39 (1951).

[12] This does not imply that a negative value is necessarily incorrect—it may well be that more terms in the expansion (20a) are necessary to give a detailed description of the variation of δ_{21} with energy in this case.

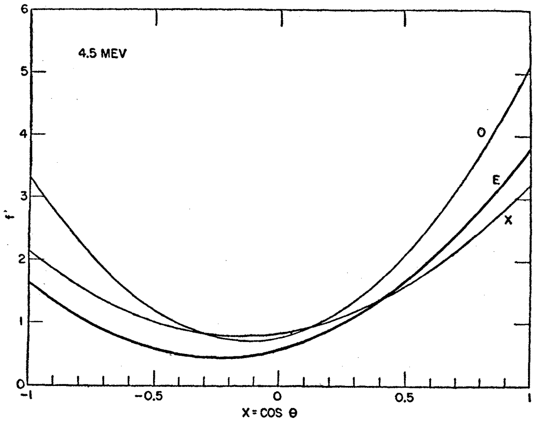

FIG. 6. Angular distribution for *n-d* scattering at 4.5 Mev ($f=k^2\sigma$ *versus* $x=\cos\theta$) obtained from phases in Table II alone: curve E represents experimental data minus D-wave effects; curves O and X represent calculations of Buckingham and Massey for a neutral and symmetric type of interaction, respectively.

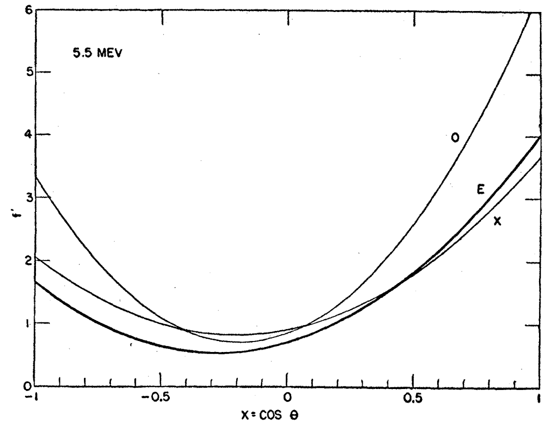

FIG. 7. Same as Fig. 6, for *n-d* scattering at 5.5 Mev.

Symmetric:

$$A_2=-81.6;\quad B_2=6.18;\quad C_2=-0.97; \tag{23a}$$

$$A_4=38.5;\quad B_4=-3.80;\quad C_4=0.50; \tag{23b}$$

with 10^{-12} cm as a unit of length.

The values of the Buckingham and Massey phases interpolated to 4.5 and 5.5 Mev by the use of Eqs. (19) and (20) with the constants of (21), (22), (23) are given in Table II together with the corresponding "experimental" phases from Table I. It is apparent from Table II that no possible agreement exists between either of the sets A and B of "experimental" P-wave phases and the corresponding calculated values for a neutral type of interaction. On the other hand, there is a qualitative agreement between the set A of "experimental" phases and the values calculated for a symmetric type of interaction. The lack of quantitative agreement in the latter case could be attributed to the range, depth, and possibly shape of the inter-nucleon potential chosen by Buckingham and Massey (see below).

It is instructive to compare the angular distributions which are obtained from the sets of S and P phases in Table II alone. Such a procedure, in effect, is a comparison of the theoretical calculations with the experimental angular distributions *minus* the D-wave contributions. The resulting curves are shown in Figs. 6 and 7. The sets A and B of phases in Table II produce essentially the same angular distributions for a given energy since δ_{20} is nearly the same for both—hence, only one curve representing the experimental data is shown for each energy. From these curves it is apparent that the experimental angular distributions minus D-wave effects definitely prefer the theoretical curves obtained from the symmetric type of interaction. Thus, the discrepancies which were found by Wantuch between his experimental angular distributions and those predicted by the Buckingham and Massey calculations for a symmetric type of interaction can indeed be accounted for by the presence of small D-wave effects. On the other hand, it should be noted that the general appearance of the curves in Figs. 6 and 7 tends to belittle the discrepancies in the P-wave phases found above. Moreover, these curves indicate how the qualitative agreement between the "experimental" and symmetric-interaction-calculated phases can produce a quantitative agreement between the resulting total cross-section values (i.e., the areas under the curves). It seems, therefore, that if a decisive comparison between experiment and theory is to be obtained by means of *n-d* scattering at low energies, a phase shift analysis of the experimental data must be carried out in each case.

In conclusion, we should like to point out that Eq. (19) and its counterpart for δ_{40}, taken together with Eq. (12), requires, in particular, that the doublet and quartet scattering lengths of the deuteron (a_2 and a_4) must be approximately equal, independent of the force-type considered. Now the experimental evidence from slow-neutron scattering reveals that:[13] $a_2=0.83\times10^{-12}$ cm and $a_4=0.24\times10^{-12}$ cm so that the validity of Eq. (12) cannot be taken too seriously. Insensitive as it is to the force-type considered, this result strongly suggests that in future calculations the form (i.e., range, depth, and shape) of the inter-nucleon potential used by Buckingham and Massey should be modified.[14] One may then find (upon analyzing the experimental phases anew) that a symmetric interaction can produce still better agreement with the experimental P-wave phases than that found above.

[13] Wollan, Shull, and Koehler, Phys. Rev. **83**, 700 (1951). In addition, compare this value of a_2 with those given in (21).

[14] In support of which, we note that A. Troesch and M. Verde (see reference 11), using a Gauss potential of 45-Mev depth and 1.9×10^{-13} cm range, found (in units of 10^{-12} cm): $(a_2, a_4)=(0.68, 0.28)$ and $(0.70, 0.32)$ for the neutral and symmetric interactions, respectively.