

## The High Frequency Resistance of Metals in the Normal and Superconducting State

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The high frequency resistance of lead, indium, and tin has been measured at frequencies in the vicinity of 9000 megacycles per second using resonant cavity techniques. Preliminary measurements on tin at 24,000 megacycles per second also have been made. The experimental techniques and measurement procedure are described. The effect of surface finishes is discussed for both the normal and superconducting regions. Data for the normal state are in general agreement with the Reuter-Sondheimer theory of the anomalous skin effect.

The high frequency resistance in the superconducting region was found to vary with frequency as the three-halves power rather than the predicted variation of the second power.

### INTRODUCTION

THE results reported here are part of a general program of investigation of microwave conductivity of metals at low temperatures. The original objective of the investigation was to determine whether the twilight zone of superconductivity existed in the microwave region. After preliminary experiments it became evident that the bulk conductivity could easily be masked by the surface treatment given the metal. Consequently, a study of the effect of surface treatment on conductivity was undertaken.

The London<sup>1</sup> model of superconductivity postulates the simultaneous existence of normal and superconducting electrons. The superconducting electrons behave according to the phenomenological equations of F. London and H. London, whereas the normal conduction electrons exhibit the customary resistive behavior. If a high frequency alternating electromagnetic field is applied to a superconductor, both normal and superconducting electrons should flow in the surface of the superconductor.<sup>2</sup> As the temperature is reduced below the transition temperature, the fraction of normal conduction electrons remaining should decrease until at zero degrees Kelvin all normal conduction electrons should be converted into superconduction electrons. The first very high frequency resistance experiment on a superconductor was carried out by H. London<sup>3</sup> in 1940. A number of investigators have extended London's work both theoretically and experimentally.<sup>4-7</sup> Some of their results will be compared to ours.

### APPARATUS AND TECHNIQUES

The present work has been carried out at two frequencies for tin and one frequency for indium and lead. The effects of surface finishes on the resistance was studied; also, the behavior of films of superconducting

metals, electroplated on nonsuperconductors, were investigated. A cylindrical resonant transmission-type cavity is constructed of the metal to be investigated. The logarithmic decrement is measured by exciting the cavity in the proper mode and then observing the rate of decay of the amplitude of oscillation after the excitation is removed.

The cavities were fabricated a number of ways. Some were cast in carbon molds and then the inner surfaces turned to as good a finish as is obtainable in a lathe. Others were given various degrees of polishing with a ball mill, or polished with alumina, gamal cloth, and distilled water. Several cavities were made of a brass base on which several thousandths of an inch of the metal under study was electrodeposited. One specimen was cold-pressed from a block of tin. The die in this case was given a good surface finish and carefully cleaned before pressing. The cavities operate in the lowest mode, the TE<sub>111</sub> mode, and are made in two pieces. The plane at which the two parts are joined is a section where no transverse currents flow provided symmetry is maintained. Coupling to the cavity is made through two small irises accurately located in the center of each end plate. Stainless steel wave guides are soldered to the cavity and are sealed off with mica windows. The cavity is continuously evacuated after it is assembled. A sectional view of the cavity and dewars is shown in Fig. 1.

Most of the experiments were conducted in external helium Dewar flasks surrounded by liquid nitrogen. However, the experiments with lead were conducted in the chamber of the Collins helium liquefier. The temperature measurements in the liquid helium region were determined from vapor pressure measurements using the 1937 Leiden scale. Other temperature measurements were made by a constant volume helium gas thermometer.

The rapid and accurate measurement of very high  $Q$  factors in the microwave region presents a difficult problem. Our measurement procedure uses a decay method in which the resonator is excited with a short pulse of rf energy, and the decay of energy in the cavity

<sup>1</sup> F. London and H. London, *Physica* **2**, 341 (1935).

<sup>2</sup> H. London, *Nature* **2**, 497 (1934).

<sup>3</sup> H. London, *Proc. Roy. Soc. (London)* **A176**, 522 (1940).

<sup>4</sup> A. B. Pippard, *Nature* **162**, 68 (1948); *Physica* **15**, 40 (1949); *Proc. Roy. Soc. (London)* **A203**, 98 (1950).

<sup>5</sup> W. M. Fairbank, *Phys. Rev.* **76**, 1106 (1949).

<sup>6</sup> G. E. H. Reuter and E. H. Sondheimer, *Proc. Roy. Soc. (London)* **A195**, 336 (1948).

<sup>7</sup> Maxwell, Marcus, and Slater, *Phys. Rev.* **76**, 1332 (1949).

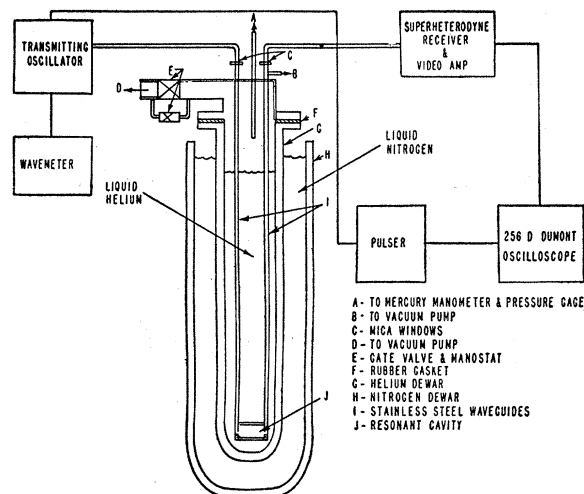


FIG. 1. Schematic diagram of apparatus.

is observed after the oscillator is shut off. The requirements of this system are:

1. The signal source should have a reasonable short time stability.

2. The response of the receiver be known.

3. An accurate system of time measurement be used.

The signal source used is a reflex klystron oscillator. The receiver is a conventional superheterodyne, and the oscilloscope is a Dumont type 256D.

The receiver may be calibrated in terms of input voltage required to give various amplitudes on the oscilloscope, or in terms of the attenuation necessary to change the amplitude at the oscilloscope from an arbitrary point *A* to another arbitrary point *B*. Care must be taken so that the receiver is linear in the usable portion of the decay trace. In most of our measurements we record about six or eight points in terms of amplitude *versus* time. Then for a decay trace, the output voltage is plotted on semi-log paper as a function of time. The best straight line is drawn through the points and *Q* calculated from the expression

$$Q = \frac{n\pi}{\ln(A_1/A_2)},$$

where  $n = \theta f$  = number of cycles in time interval  $\theta$ ,  $f$  = frequency,  $A_1$  = initial amplitude at  $\theta_1$ ,  $A_2$  = amplitude at  $\theta_2$ , and  $\theta = \theta_2 - \theta_1$ .

An alternate method may be used which involves only two points; with this system both amplitude and time must be measured more accurately. This entails measuring the time required for the signal to decay from amplitude  $A_1$  to amplitude  $A_2$ , and the attenuation required to produce a change in amplitude from  $A_1$  to  $A_2$ .  $Q$  is given by the expression

$$Q = 27.3\theta f / \Delta \text{ db},$$

where  $\theta$  = time interval,  $f$  = frequency, and  $\Delta \text{ db} = \text{db}$

attenuation required to produce a change in the amplitude from  $A_1$  to  $A_2$ . Time intervals may be measured with a precision delay circuit, or by timing markers generated by a crystal-controlled oscillator. The signal source should be reasonably stable for short periods of time. This measurement procedure is rapid, and if desired the decay trace with the appropriate timing markers can be photographed.

To measure the very high  $Q$  factors encountered here by the resonance curve method would require a degree of frequency stability difficult to obtain.

Several photographs of oscilloscope patterns of the decay of oscillations in the cavity are shown in Fig. 2. Timing markers are also shown on the photographs. The decay trace labeled *A* was taken at 78°K for a tin cavity; trace *B* is for the same cavity at a temperature just above transition. This curve corresponds to a  $Q$  of approximately 40,000. The trace labeled *C* is for the same cavity at a temperature somewhat below the transition temperature, the  $Q$  in this case being about  $2.3 \times 10^6$ . The accuracy of our measurements of  $Q$  in the 10,000 megacycles per second region is about 5 percent, dependent on the values of  $Q$ , being greater for the higher values of  $Q$ .

With the preceding techniques it is possible to couple very loosely to the cavity, so corrections for energy radiated back into the wave guides can be kept small.

The radiation  $Q$  is given by the expression

$$Q_r = 2\pi \frac{\text{energy stored in the cavity}}{\text{energy radiated through the irises per cycle}}$$

Table I gives the radiation  $Q$  as a function of hole diameter. A number of hole sizes were used in the 9000 Mc/sec cavities. The highest calculated radiation  $Q$  was  $62 \times 10^6$  and the lowest radiation  $Q$  was  $8.2 \times 10^6$ .

The  $Q$  factor of a superconducting cavity was measured at a given temperature for a number of hole

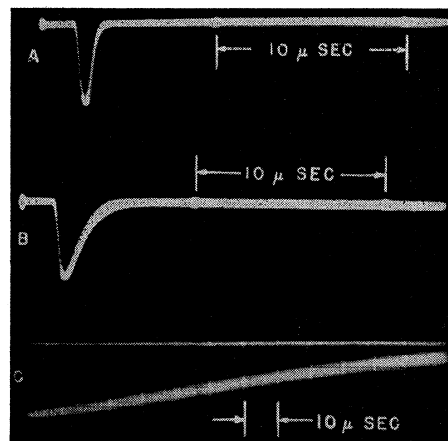


FIG. 2. Oscilloscope patterns of decay traces for a tin cavity at various temperatures: *A*, 78°K, sweep length 20 microseconds; *B*, 3.8°K, sweep length 20 microseconds; *C*, below transition temperature, sweep length 130 microseconds.

sizes. By this means it was possible to deduce values of  $Q$  for zero coupling, and, thus, determine experimental values of radiation  $Q$  for a given hole size. The experimental results yielded somewhat lower values of radiation  $Q$  than the calculated values. We used the calculated value of radiation  $Q$  to convert our measured loaded  $Q$  to unloaded  $Q$ . The correction amounted to but a few percent except for the extremely high  $Q$  factors. The unloaded  $Q$  or  $Q_0$  is obtained from the measured  $Q$  corrected for the radiation  $Q$ .

$$Q_0 = 2\pi \frac{\text{energy stored in the cavity}}{\text{energy dissipated in the walls of the cavity per cycle}}$$

The measured  $Q$  is given by

$$Q = 2\pi \frac{\text{energy stored in cavity}}{\text{energy dissipated in walls and energy radiated per cycle}}$$

The relationship of  $Q_0$  to  $Q$  and  $Q_r$  is given by the expression

$$1/Q_0 = 1/Q - 1/Q_r.$$

The surface resistance is obtained from the measured  $Q$  corrected for radiation loss and a knowledge of the geometry of the cavity.<sup>8</sup>

$$R_s = 2\pi K \mu_0 f / Q_0,$$

where  $R_s$  = surface resistance in ohms per unit square,  $f$  = resonant frequency,  $\mu_0 = 4\pi \times 10^{-7}$  henry/meter, and  $K$  is a constant dependent on the geometry of the cavity. For a particular geometry this may be reduced to

$$R_s = C/Q_0,$$

where  $C$  is a constant for that geometry and mode. In the region where classical theory still prevails

$$R_s = (2\pi\omega 10^{-7}/\sigma)^{\frac{1}{2}},$$

where  $\sigma$  is the dc conductivity.

#### NORMAL STATE

The results of measurements on tin, lead, and indium indicate a pronounced anomaly in the normal region just above the transition temperature, in that the conductivity is lower than that calculated by classical theory. This is in agreement with the experiments of London, Pippard, Maxwell *et al.*, Fairbank, and others.

TABLE I. Radiation  $Q$  as a function of hole size.

Frequency Mc/sec	Hole diameter inches	Calculated radiation $Q$
9150	0.078	$62 \times 10^6$
9150	0.088	$22 \times 10^6$
9150	0.099	$8.2 \times 10^6$
24000	0.060	$3.6 \times 10^6$

<sup>8</sup> J. C. Slater, *Revs. Modern Phys.* **18**, 475 (1946).

TABLE II. Surface conductivity of metals in the normal state just above the transition temperature.

Fre- quency Mc/sec	Temper- ature °Kelvin	Metal	Surface conductivity ohm <sup>-1</sup>	Remarks	State
9105	4.2	tin	117	cold pressed	normal
9155	4.2	tin	130	electropolated	normal
9160	4.2	tin	133	cast and machined	normal
9145	4.2	tin	150	polished electroplated	normal
9160	4.2	tin	160	polished cast	normal
9400		tin	148-220	Pippard-single crystal	normal
9050		tin	143	Simon—cast	normal
9400		tin	115	Fairbank cold pressed	normal
24000	4.2	tin	57	Maxwell <i>et al.</i> — cold pressed	normal
24300	4.2	tin	80	preliminary NRL	normal
24000		tin	83	NRL at 9160 Mc/sec extrapolated to 24000 Mc/sec	normal
9165	8	lead	176	machined cast	normal
9100		lead	136	Simon—cast	normal
9137	3.5	indium	40	electrodeposited	normal

Reuter and Sondheimer<sup>6</sup> have derived an expression which replaces Ohms' law in the low temperature region and have satisfactorily explained the anomaly occurring in the normal state.

Table II gives the various surface conductivities, in the normal state, as a function of surface finish and frequency. Also included in this table are the results of Pippard, Fairbank, Simon, and Maxwell *et al.* The specimen of tin that yielded the lowest conductivity was the one formed by cold-pressing the metal into a suitable shape. The measured conductivity is in good agreement with the results obtained by Fairbank and the extrapolated results of Maxwell, Marcus, and Slater. In these cases, the cavities were formed by cold-pressing and yielded low values of conductivities. The electrodeposited surface yielded about the same conductivity as the machined cast specimen. Electrodeposition of the tin was carried out in an acid tin bath at a rather slow rate so as to obtain a dense deposit. By mechanically polishing the inner surfaces of the cast and the electroplated specimens, the conductivity was improved. Pippard's results on single crystals show that the conductivity varies with crystal orientation. The surface conductivity varies from 148-220 ohms<sup>-1</sup>.

Preliminary data on tin at 24,000 Mc/sec have been obtained to date. The conductivities obtained by us were higher than those obtained by Maxwell *et al.* The surface conductivity in the normal state should vary<sup>6</sup> as  $\omega^{-\frac{1}{2}}$ . If our results at 9160 Mc/sec, for a polished surface, are extrapolated to 24,300 Mc/sec, we obtain good agreement with the measured values for a polished surface at the higher frequency.

Whether poor normal conductivity is due to surface roughness or to surface strains and dislocations, is difficult to say. For the pressed specimen, in our case, the surface appeared smooth. With any milling, machining, hobbing, or polishing process, some strains and dislocations will be set up in the surface of the material. By annealing the metal in vacuum for a sufficient length of time some of the strains in the metal should be relieved. From our measurements it would appear that

TABLE III. Experimental results in the superconducting state.

Frequency extrapolated to Mc/sec	$R/R_n$ absolute zero	$A(\omega)$	Metal	Remarks
9145	0	0.127	tin	polished electroplated
9160	0.004	0.117	tin	cast and polished
9160	0.0075	0.118	tin	cast and machined
9105	0.028	0.110	tin	cold pressed
9200	0(?)	0.0865	tin	Pippard—cast polycrystals
9400	less than 0.001	0.105–0.20	tin	Pippard—single crystals
9400	0.019	...	tin	Fairbank—cold pressed
1200	0.007	0.0195	tin	Pippard polycrystals
24,000	0.09	...	tin	Maxwell <i>et al.</i> —cold pressed
24,300	0.006	0.265	tin	cast polished
9137	0.005	0.081	indium	electroplated
9165	0.010	0.046	lead	cast machined

cold-pressing sets up too many dislocations, thus giving rise to abnormally high resistance. It would also appear that surface roughness plays a part, since by polishing the surface the conductivity is increased. In the case of electroplated films surface strains are probably minimized while the surface tends to be less smooth. If Fairbank's results are extrapolated to 24,000 megacycles per second, a value of surface conductivity of approximately  $61 \text{ ohm}^{-1}$  is obtained; this compares with the value of  $57 \text{ ohm}^{-1}$  measured by Maxwell, Marcus, and Slater. Fairbanks and Maxwell used the same method of forming cavities. Thus, it appears that for both types of surface, the variation of conductivity with the frequency follows the law  $\sigma = a\omega^{-\frac{2}{3}}$ , where  $a$  is a constant.

Similar measurements were carried out with machined cast lead and electroplated indium surfaces in the 9000 megacycles per second region. The normal surface conductivity of lead just above the transition temperature was about  $176 \text{ ohm}^{-1}$ . Simon<sup>9</sup> obtained a value of  $136 \text{ ohm}^{-1}$  for lead at 9200 megacycles per second.

The value of normal conductivity for indium was considerably lower than that of tin. Two electroplated specimens yielded consistently low values of conductivity. Whether this is caused by a thin film, the plating process, or by some other mechanism remains to be resolved. Although the normal conductivity of indium

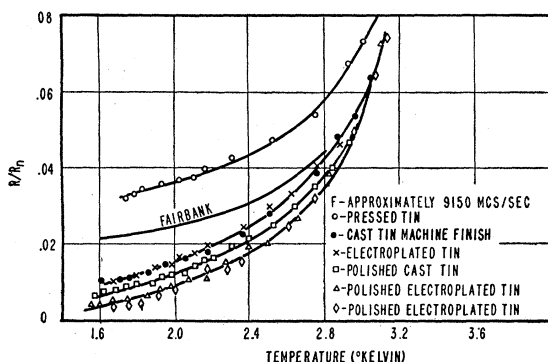


FIG. 3. Surface resistance of tin in the superconducting state for different types of surface treatment.

<sup>9</sup> I. Simon, Phys. Rev. 77, 384 (1950).

was poor, the conductivity in the superconducting state was very high.

### SUPERCONDUCTING STATE

The experimental results obtained for the superconducting state are collected in Table III. The table includes values of normalized surface resistance extrapolated to absolute zero and values of  $A(\omega)$ . The normalized surface resistance is defined as equal to  $R/R_n$ , where  $R$  is the superconducting surface resistance and  $R_n$  is the surface resistance in the normal state just above the transition temperature.  $A(\omega)$  is the rate of variation of  $R/R_n$  with the function of temperature  $t^4(1-t^2)/(1-t^4)^2$ , where  $t$  is the reduced temperature  $T/T_c$ ,  $T$ =temperature, and  $T_c$ =transition temperature. The results of Pippard and others are also included in the table. The data for the individual specimens are shown in Figs. 3 to 10.

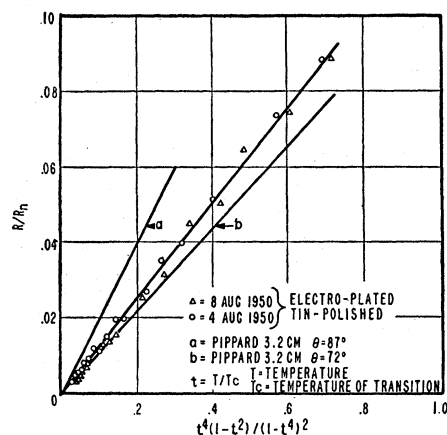


FIG. 4. Linear plot of low temperature resistance at 9145 Mc in superconducting state (polished electroplated tin specimen).

In the superconducting state the effect of surface finish is clearly evident in Fig. 3. Here the surface resistance ratio,  $r=R/R_n$ , of tin is plotted against temperature. Only the lower portion of the curves are shown since they are in essential agreement near the transition and start to diverge near  $3.2^\circ\text{K}$ . The pressed tin surface yielded the highest resistance ratio, somewhat higher than Fairbank's value; this difference may be due to differences in measurement techniques or to differences in the actual surfaces. The machined surface yielded a better value than the pressed surface and is about similar to the electrodeposited surface. The effect of mechanically polishing a cast tin specimen whose surface was turned in a lathe is also shown in this figure. The mechanical polishing was done by placing a number of clean, random sized, steel balls in the cavity and rotating it for a number of hours. A considerable improvement in the conductivity has been effected by this process. The lower curve is for an electrodeposited surface after it had been mechanically polished and then

etched with hydrochloric acid and a light tin plate deposited over the polished surface.

In Fig. 4, the surface resistance ratio is extrapolated to absolute zero in terms of a function of temperature first suggested by Pippard. The function of temperature is given by the expression  $t^4(1-t^2)/(1-t^4)^2$ , where  $t$  is the reduced temperature  $T/T_c$ . It is apparent that the polished electrodeposited specimen does extrapolate to zero resistance at zero degrees Kelvin. Pippard's curves for single crystals of tin are also shown in the figure.

Figure 5 shows the same type extrapolation for the machined surface and the same specimen after mechanical polishing. Pippard's work on polycrystals at this frequency is also shown. When the normalized surface resistance is extrapolated to zero degrees Kelvin, it is found that the machined cast tin specimen yields a value of resistance equal to 0.0075 times its resistance just before the transition point. After polishing, this

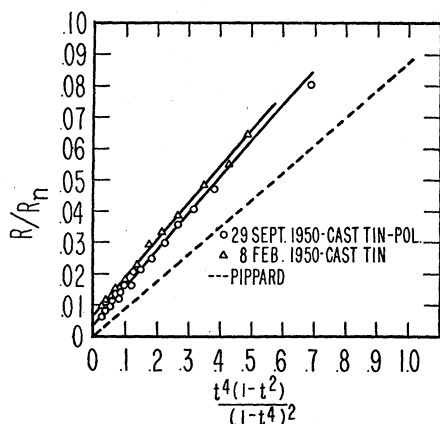


Fig. 5. Linear plot of low temperature resistance at 9160 Mc in the superconducting state (machined cast tin specimen and a polished, machined cast tin specimen).

same specimen yields at absolute zero a value of 0.004 times its initial resistance.

The experimental results on an electrodeposited indium surface are shown in Fig. 6. Indium is quite similar to tin except that the initial conductivity, for two specimens measured, was low. The normalized surface resistance extrapolated to zero degrees Kelvin yields a residual resistance of 0.005.

The results on lead are shown in Fig. 7. The curve shown is a plot of loaded  $Q$  versus temperature. The lead specimen was of high purity, cast in a carbon mold and then the inner surface turned in a lathe. The experiment was carried out in the Collins helium liquefier. The temperature measurements in this case are not precise since no attempt was made to stabilize the temperature. The scatter on the lead data is somewhat worse than that of the tin data. Part of the scatter is due to temperature variations. When  $r$  is extrapolated to zero degrees Kelvin, a residual resistance of 0.01 is obtained. The result is about what one would expect

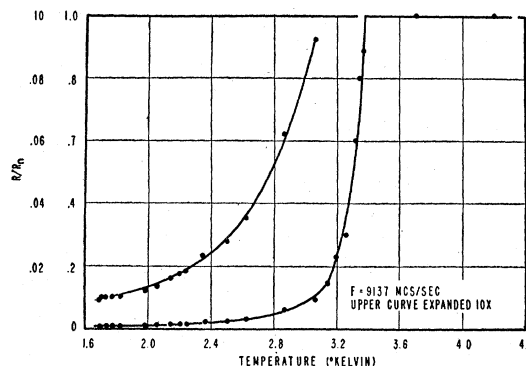


Fig. 6. Surface resistance of electrodeposited indium in the superconducting state.

from this type of surface treatment. It appears then that lead is about as good a conductor as tin in the superconducting region.

Preliminary measurements made on tin in the 24,000 megacycles per second region using a cavity cylindrical in shape and operated in its lowest mode yields results shown in Fig. 8. Also shown are the results of Maxwell *et al.*, and our results at 9160 megacycles per second for a similar surface treatment. Our results show considerably higher conductivity than those of Maxwell. On the basis of our work at 9000 megacycles and Fairbank at the same frequency it appears that the cold-pressing method of fabrication used by Maxwell and Fairbank imparts strains and dislocations, thus giving rise to abnormally high resistance in both the normal and superconducting regions.

In Table III,  $R/R_n$  is extrapolated to absolute zero to determine the residual resistance. Also included are the values of  $A(\omega)$  for the different cases. Our values of  $A(\omega)$  in the 9000-Mc/sec region lie between Pippard's values for single crystals and are somewhat higher than his value for polycrystals. The amount of resistance remaining at absolute zero is dependent on the surface treatment given the specimen. It appears that with appropriate surface treatment the conductivity in the superconducting state can be improved substantially.

The normalized surface resistance of tin at 24,000

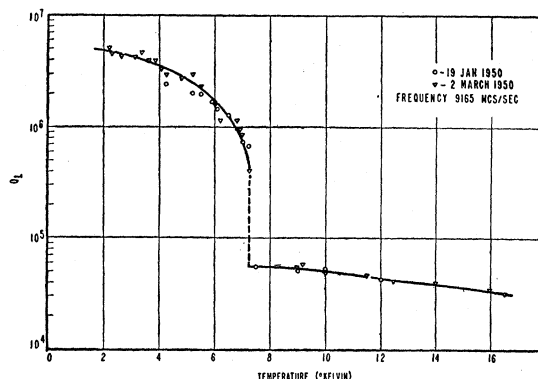


Fig. 7. Plot of loaded  $Q$  against temperature for lead.

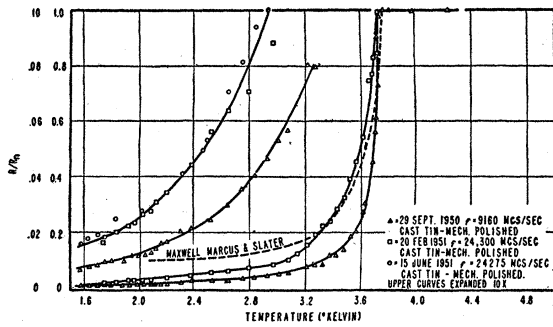


FIG. 8. Surface resistance of tin in the superconducting state.

megacycles per second is extrapolated to zero degrees Kelvin in Fig. 9. Also shown in this figure is a curve for a specimen in 9160 megacycles per second. The two specimens were given very similar surface treatment, being mechanically polished with gamal cloth, alumina, and distilled water. They extrapolate to very nearly the same values of residual resistance. It appears that the twilight region of superconductivity has not been reached at 24,000 megacycles per second. Further investigations at higher frequencies should be carried out.

The normal surface resistance  $R_n$  varies as  $\omega^3$ , which is in essential agreement with the results for these two cavities. If  $r$  is small the normal electrons are considered to have little effect on the distribution of the magnetic field in the metal. The resistive processes may be visualized as the absorption of power by the normal electrons in their collisions with the lattice as they move under the influence of the electric field induced by the varying magnetic field. If, below the transition, the magnetic field distribution is governed only by the superconducting electrons the effect of changing frequency will be to change, at some particular point within the metal, the magnitude of the electric field proportional to the frequency. Consequently,<sup>10</sup> the surface resistance  $R$  should vary as  $\omega^2$ . Therefore,  $r$  would be proportional to  $\omega^{4/3}$ , since  $r = R/R_n$ . It is assumed that

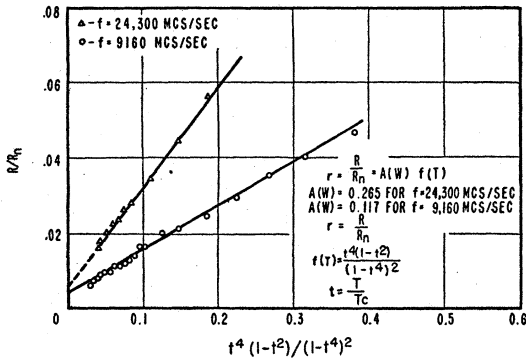


FIG. 9. Linear plot of low temperature resistance of tin in the superconducting state.

<sup>10</sup> F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1950) Vol. 1, p. 90.

relaxation effects may be neglected in obtaining the previous expression.

At temperatures sufficiently far below the transition such that  $r \ll 1$  the curves may be closely represented by the equation<sup>4</sup>  $r = A(\omega)t^4(1-t^2)/(1-t^4)^2$ , where  $t$  is the reduced temperature and  $A$  is a function of frequency only. If  $A(\omega)$  is evaluated from our curves shown in Fig. 9, we obtain a value of  $A(\omega) = 0.117$  for  $f = 9160$  megacycles per second and  $0.265$  for  $f = 24,300$  megacycles per second. Thus the ratio of  $A(\omega)$  for the two cases is 2.26.

The predicted ratio of  $A(\omega)$  in the two cases is 3.67, which corresponds to  $\omega^{4/3}$ . The observed variation of  $A(\omega)$  corresponds to  $\omega^{0.886}$ , or approximately  $\omega^{5/6}$ . Hence,  $R$  in the superconducting region is observed to vary with frequency as  $\omega^{3/2}$ . Pippard,<sup>11</sup> in his experiments at 1200 megacycles per second, obtained a value of  $A(\omega) = 0.0195$ . If we assume  $r$  varies as  $\omega^{5/6}$  and

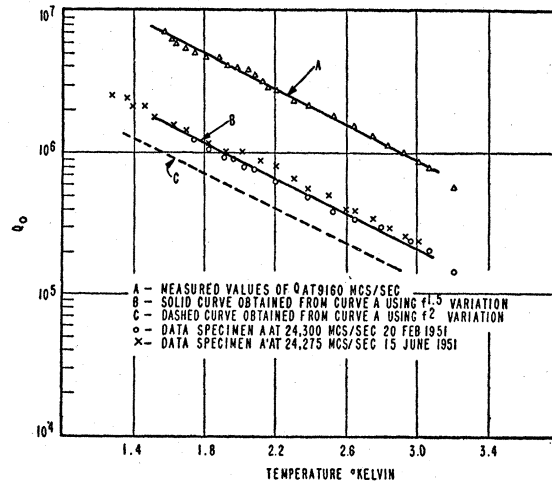


FIG. 10. Plot of the unloaded  $Q$  factor against temperature for two frequencies with curves  $B$  and  $C$  extrapolated on two assumptions from curve  $A$ .

extrapolate his results to 9160 megacycles per second and 24,300 megacycles per second, we obtain for  $A(\omega)$  a value of 0.106 for the former and 0.238 for the latter. This compares with our measured values of 0.117 and 0.265 for the two cases.

A comparison of the unloaded  $Q$  factors obtained from the cavities may be used to determine the experimental variation of  $R$  with frequency. This can be done since the cavities operate in the same mode, have the same ratio of diameter/length, and were given similar surface treatment. Shown in Fig. 10 is the data obtained for a representative specimen operating at 9160 Mc/sec and for two specimens operating in the vicinity of 24,000 Mc/sec. Also shown in this figure are two curves derived from the 9160-Mc/sec data. The curve labeled  $C$  uses the predicted variation of the  $Q$  factor with

<sup>11</sup> A. B. Pippard, Proc. Roy. Soc. (London) A191, 370 (1947).

frequency of  $\omega^{-2}$ . The curve labeled *B* uses a variation of the *Q* factor with frequency of  $\omega^{-3/2}$ . It is apparent that the 24,000 Mc/sec data more nearly fit the curve that uses the  $\omega^{-3/2}$  variation than that predicted by the London theory.

We wish to acknowledge the assistance given by Dr. Rufus G. Fellers, especially for his treatment of the radiation loss problem in the resonant cavities. We also wish to thank Mr. A. Szwed for his assistance in some of the temperature measurements.

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## Low Energy *n-d* Scattering: Comparison of Experiment with Theory

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*D*-wave effects are shown to be significant, though small, in the experimental *n-d* scattering data at 4.5 and 5.5 Mev. Consequently, since the calculations of Buckingham and Massey include only *S* and *P* wave phases, a phase shift analysis of the experimental data is carried out to determine these phases so that a suitable comparison can be effected between experiment and theory. The resulting "experimental" *P*-wave phases agree only qualitatively with those of the symmetric-interaction calculations, but they completely disagree with those of the neutral-interaction calculations. The lack of quantitative agreement in the former case could be attributed to the range, depth, and possibly shape of the inter-nucleon potential chosen by Buckingham and Massey especially since their *S*-wave phases, which they found to be insensitive to force-type, are not in satisfactory agreement with the experimental results of slow neutron scattering.

### INTRODUCTION

A PAPER by Wantuch appeared recently in which experimental results were reported for the absolute values of the differential cross sections for 4.5- and 5.5-Mev neutrons scattered from deuterons.<sup>1</sup> A comparison was made therein of these experimental data and the theoretical calculations of Buckingham and Massey,<sup>2</sup> with the following conclusions. The values of the total cross section, obtained by extrapolation, definitely preferred the symmetric force theory. The actual angular distributions, however, did not agree well with either the symmetric or neutral force theory, but, on the other hand, the ratio  $\sigma(\pi)/\sigma(\theta)$  definitely preferred the neutral force theory. Very much the same sort of contradictory conclusions have been drawn by others from the results of *n-d* scattering experiments at low energies.<sup>3-5</sup>

Rosenfeld has stated that the agreement between the variation of the experimental *n-d* total cross sections with energy and that predicted by the symmetric force theory calculations of Buckingham and Massey pro-

vides the "weightiest argument" in favor of this type of interaction.<sup>6</sup> Since Buckingham and Massey have considered only the *S*- and *P*-wave phase shifts in their calculations, the above mentioned discrepancies might possibly be accounted for by the presence of small *D*-wave phases which would deform the angular distributions to an appreciable extent only near  $\theta = \pi$  or 0 while the total cross-section values would be effectively unchanged. The present paper is the result of an investigation into this possibility, carried out in the hope of clarifying the situation.

A more exhaustive treatment has been given to Wantuch's experimental data in order that an effective comparison could be made with the theoretical calculations of Buckingham and Massey. First, a least-square fit of the data to expansions in  $\cos\theta$  revealed the presence of small *D*-wave phases and provided a reliable extrapolation of these data for calculating the corresponding total cross sections. Second, an attempt has been made to find the actual values of the experimental *S*- and *P*-phase shifts from the coefficients in these expansions in order that a more direct comparison with the theoretical calculations could thereby be effected. Finally, a more reliable method has been employed for interpolating the calculated values given by Buckingham and Massey for these phase shifts to the energies involved in the experiment.

The following notation was adopted.

$\sigma(\theta)$ : differential scattering cross section as a function of  $\theta$ , the neutron scattering angle in the center-of-mass system.

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<sup>1</sup> E. Wantuch, Phys. Rev. 84, 169 (1951). The data used here were obtained from his thesis, New York University, 1950.

<sup>2</sup> R. A. Buckingham and H. S. W. Massey, Proc. Roy. Soc. (London) A179, 123 (1941). Their force-types I and III are referred to here as neutral (or "0") and symmetric (or "X"), respectively (see reference 3).

<sup>3</sup> L. Rosenfeld, *Nuclear Forces* (Interscience Publications, New York, 1948), Sec. 14.12.

<sup>4</sup> Martin, Burhop, Alcock, and Boyd, Proc. Phys. Soc. (London) A63, 884 (1940). See also discussion by R. A. Buckingham, which follows thereafter.

<sup>5</sup> Hamouda, Halter, and Scherrer, Phys. Rev. 79, 539 (1950); and Helv. Phys. Acta 24, 217 (1951); also, I. Hamouda and G. de Montmollin, Phys. Rev. 83, 1277 (1951).

<sup>6</sup> See reference 3, p. 298.

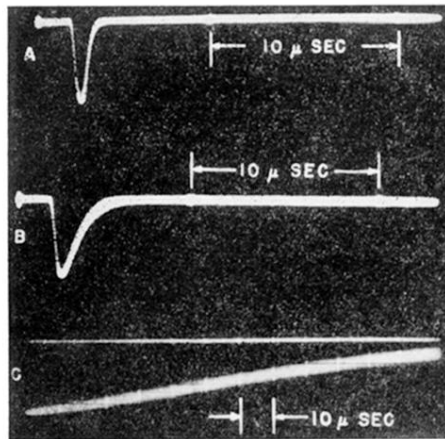


FIG. 2. Oscilloscope patterns of decay traces for a tin cavity at various temperatures: *A*, 78°K, sweep length 20 microseconds; *B*, 3.8°K, sweep length 20 microseconds; *C*, below transition temperature, sweep length 130 microseconds.