with the maximum shearing stress. This cross slip occurs because glide on the original slip system gradually uses up the long sources there until eventually, as the applied stress increases, the resolved shearing stress in some other slip system is large enough to induce loop generation there. The amount of glide which occurs on the cross slip lamellas will depend on whether the locking process which has stopped lamellas in the principal slip system is also at least partially effective in stopping glide on the second slip system. The present theory is consistent with the finding that during the transition from single to double slip no discontinuity is observed in the stress-strain curve of a pure metal. The treatment given in this paper implies that the glide produced by cross slip is much less than that on the primary slip system. This assumption is in accord with experiment.[32]

In conclusion, I would like to thank Dr. L. Slifken, Professor F. Seitz, and Mr. J. Marx for numerous stimulating discussions.

[32] G. I. Taylor and C. F. Elam, Proc. Roy. Soc. (London) **A102**, 643 (1923).

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High Energy Electron-Electron Scattering

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Eradicated electron sensitive nuclear emulsions were exposed to 200-Mev electrons at the Berkeley synchrotron. In scanning the electron tracks 427 events were observed in which the scattered electron of lower energy, or knock-on electron, had an energy greater than 30 kev. The observed differential cross section was found to agree in absolute value with Møller's theoretical cross section, although an insufficient number of high energy knock-on electrons were observed to distinguish between the Møller, relativistic Mott, and relativistic Rutherford formulas. Two pairs initiated by primary electrons and two cases in which primary electrons vanished in the emulsion were also observed in 102.6 cm of track. No heavy particle events were seen.

I. INTRODUCTION

HISTORICALLY, the electron is the best known of the fundamental particles. However, a lack of information still exists concerning its actual structure. An electron-electron scattering experiment would seem to be the ideal way to investigate the boundaries of the electron and the possibility of non-Coulomb electron-electron forces. To find deviations from a Coulomb potential, one would roughly estimate that it is necessary to have an impact parameter of the order of the classical electron radius. In order for the impact parameter to be well defined the de Broglie wavelength, $\lambda\!\!\!^{-}$, of the electron in the relativistic center-of-mass system must be of the order of 2.8×10^{-13} cm or less. A simple calculation shows that such a wavelength would require an energy of about 19 Bev in the laboratory system. In the present experiment 200-Mev electron primaries were used which have a de Broglie wavelength of about 10 times the classical electron radius in the relativistic center-of-mass system. Even for this wavelength, the possibility seemed to exist of observing a deviation from the Coulomb potential if the effect were strong.

The generally accepted formula giving the scattering cross section of electrons by electrons has been derived by Møller.[1] This formula in terms of the scattering angle, θ, in the relativistic center-of-mass system is the following:

$$\sigma(\theta)d\theta=\frac{(\gamma+1)\pi r_0^2\sin\theta d\theta}{\gamma^2\beta^4}\left[\csc^4\frac{\theta}{2}+\sec^4\frac{\theta}{2}-\csc^2\frac{\theta}{2}\sec^2\frac{\theta}{2}+\frac{(\gamma-1)^2}{\gamma^2}(1+4\csc^2\theta)\right],\quad(1)$$

where r_0 is the classical electron radius, $\beta=v/c$, $\gamma=1/(1-\beta^2)^{\frac{1}{2}}$, and v is the velocity of the primary electron in the laboratory system. The first two terms in the bracket correspond to the classical, relativistic Rutherford scattering formula. The third term is the quantum-mechanical exchange term. The inclusion of this term with the Rutherford formula gives the relativistic Mott formula. The fourth term represents retardation and spin interaction effects.

Equation (1) is more conveniently expressed in terms of the parameter A, defined as the ratio of kinetic energy given to the secondary or knock-on electron to the kinetic energy of the primary electron. It is not possible to distinguish between the primary and secondary electrons after collision. The knock-on electron is by definition the lower energy electron after collision. The maximum value of A is obviously 0.5. By a simple

[1] C. Møller, Z. Physik **70**, 786 (1931); C. Møller, Ann. physik. **14**, 531 (1932); K. C. Kar and C. Basu, Indian J. Phys. **18**, 223 (1944).

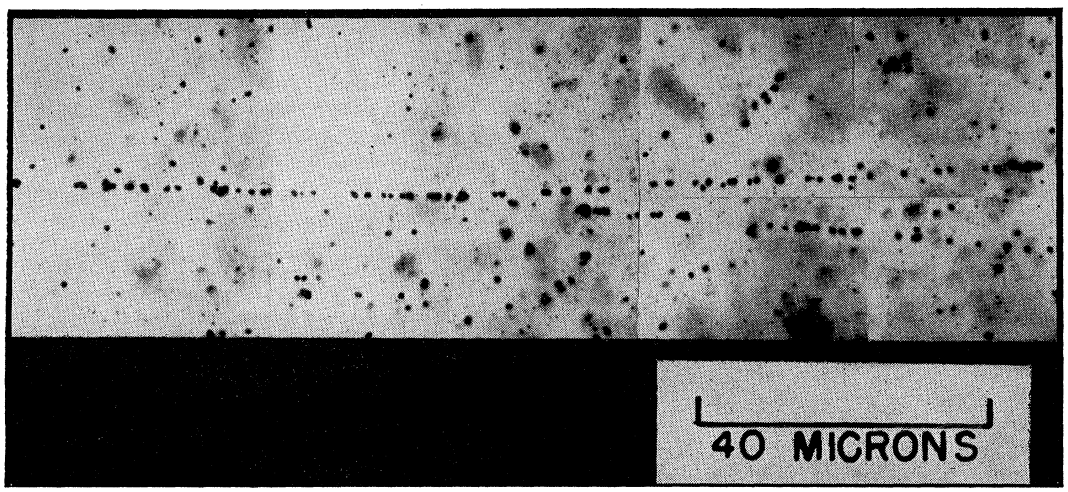


FIG. 1. Microphotograph mosaic of an electron-electron collision of large energy transfer initiated by an ~185-Mev primary electron. The angles of scatter are 9° and 2° corresponding to 32 Mev for the electron of lower energy.

transformation, as shown by Møller,[1] Eq. (1) becomes

$$\sigma(A)dA=\frac{2\pi r_0^2}{\beta^2(\gamma-1)}\left[\frac{1}{A^2(A-1)^2}-\frac{3}{A(1-A)}+\frac{(\gamma-1)^2}{\gamma^2}\left(1+\frac{1}{A(1-A)}\right)\right]dA. \quad (2)$$

The corresponding relativistic Rutherford cross section is

$$\sigma(A)dA=\frac{2\pi r_0^2}{\beta^2(\gamma-1)}\left[\frac{1}{A^2(A-1)^2}-\frac{2}{A(1-A)}\right]dA. \quad (3)$$

The relativistic Mott cross section is

$$\sigma(A)dA=\frac{2\pi r_0^2}{\beta^2(\gamma-1)}\left[\frac{1}{A^2(A-1)^2}-\frac{3}{A(1-A)}\right]dA. \quad (4)$$

Several previous experiments[2–6] have been performed to verify Møller's theory. The primary energies used in these experiments have ranged from 0.05 to only 2.64 Mev. Except for the experiment of Williams and Terroux[2] all results are in good agreement with Møller's theory. Champion,[3] Groetzinger *et al.*[6] attempted to find discrepancies between the Rutherford, Mott, and Møller formulas, Eqs. (3), (4), and (2). Champion found good agreement with the Møller equation but not with the other two equations. Groetzinger *et al.* were not able to discriminate between any of the three. However, combining their data with Champion's, they ruled out the Rutherford equation; but within statistical error they could not discriminate between the Mott and Møller equations.*

For a 200-Mev electron Eq. (2) can be approximated by the following:

$$\sigma(A)dA=\frac{2\pi r_0^2}{\beta^2(\gamma-1)}\left[\frac{1}{A^2(1-A)^2}-\frac{2}{A(1-A)}+1\right]dA. \quad (5)$$

Comparison of Eqs. (3), (4), and (5) shows that in the region of A less than 0.01 the three equations are indistinguishable. For A in the region between 0.1 to 0.5, the percentage deviation of Rutherford's cross section from Møller's cross section varies from 1 percent to 11 percent, while that of Mott's to Møller's varies from 12 percent to 56 percent. The expected number of knock-ons in photographic emulsions in the entire region from $A=0.1$ to $A=0.5$ (Fig. 1) is about one per

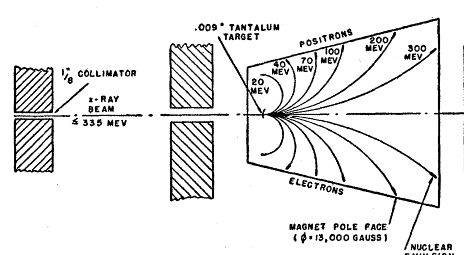


FIG. 2. Arrangement of the photographic emulsions in the magnetic field of the synchrotron pair spectrometer.

[2] E. J. Williams and F. R. Terroux, Proc. Roy. Soc. (London) **A126**, 289 (1929/1930). [Williams and Terroux were attempting to verify the theory of Thompson (J. J. Thompson, Phil. Mag. **29**, 449 (1912)). According to Hornbeck and Howell, the results of the Williams and Terroux experiment lead to cross sections which are more than twice as great as predicted from Møller's theory.]

[3] F. C. Champion, Proc. Roy. Soc. (London) **A137**, 688 (1932).

[4] G. Hornbeck and I. Howell, Proc. Am. Phil. Soc. **84**, 33 (1941).

[5] P. E. Shearing and T. E. Pardue, Proc. Am. Phil. Soc. **85**, 243 (1942).

[6] Groetzinger, Leder, Ribe, and Berger, Phys. Rev. **79**, 454 (1950).

* *Note added in proof:*—Since this paper was submitted for publication, an important new electron-electron scattering experiment employing a beam of 15.7-Mev electrons has been reported [Scott, Hanson, and Lyman, Phys. Rev. **84**, 638 (1951)]. Only a two percent deviation from Møller scattering was found. This they attribute to experimental error.

100 cm of track. Therefore one cannot hope to resolve these three equations without scanning enormous quantities of track. The scope of this experiment therefore has been limited to verifying Møller's formula for the absolute scattering cross section of 200-Mev electrons, realizing that the Rutherford and Mott formulas are equivalent to Møller's in the region where most of the data can be obtained.

II. EXPERIMENTAL PROCEDURE

The existence of electron-sensitive emulsions and a technique for eradicating accumulated background has made possible this study of high energy electron processes taking place within the nuclear emulsion. 200-micron Ilford G-5 plates were exposed to 200-Mev electrons obtained by magnetic separation in the pair spectrometer at the Berkeley synchrotron (Fig. 2). The plates were exposed so that electrons from the target entered the emulsion at a slight angle to the surface and perpendicular to the leading edge of the plate. In order to insure that only electrons which came directly from the converter were accepted, only tracks whose initial directions lay within $2\frac{1}{2}°$ of the perpendicular were scanned. This criterion included over 90 percent of all the high energy electrons entering the plate. On plates exposed with no converter in the beam the number of acceptable tracks found was less than one percent of that found on plates exposed with the converter in place.

Because of the high background of low energy electrons found in all but freshly prepared electron sensitive emulsions, it was necessary to eradicate[7] the latent image of old tracks immediately before exposure. The eradication was accomplished by storing the plates in a warm, water-saturated atmosphere for several days before use. The temperature was controlled at about 97°F by immersing a watertight box containing the plates in a thermostatically controlled water bath. The relative humidity was maintained at 10 percent by placing a wet sponge in the box with the plates. Immediately after exposure the plates were developed by a temperature cycle process[8] in order to obtain a uniform and highly sensitive development.

In order to reconstruct stereoscopically the ranges and angles of the knock-on electrons, it was necessary to measure the shrinkage factor of the emulsion. This was accomplished by passing 380-Mev alpha-particles through the undeveloped emulsion at an angle of 45° to the emulsion surface[9] and then measuring the ratio of the horizontal projection to the vertical projection of the alpha-track after development. This ratio gave the shrinkage factor directly as 2.5 ± 0.1.

[7] H. Yagoda and N. Kaplan, Phys. Rev. **73**, 634 (1948).
[8] Dilworth, Occhialini, and Vermaesen, Bull. Cen. Phys. Nuc. Brussels (1950).
[9] M. Weissbluth, University of California Radiation Laboratory Report No. 568 (1950).

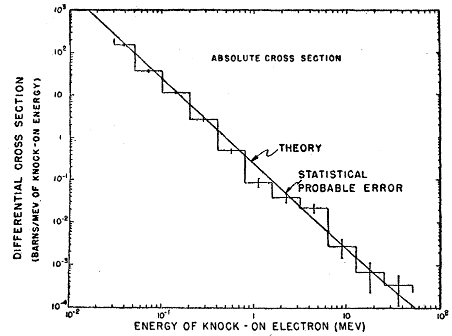


FIG. 3. Histogram of the experimental results shown with statistical probable errors. The effect of the energy resolution upon the magnitude of the absolute cross section is negligible in comparison with the statistical error.

III. METHOD OF ANALYSIS OF PLATES

The plates were scanned under $\sim 500\times$ magnification, and all events of interest were measured under $\sim 2500\times$ magnification. In these plates the grain density of a 200-Mev electron is 41.9 ± 1.0 grains per 100 μ of track. The length of primary track scanned was measured by means of the microscope stage coordinates. In order to reduce the percentage of knock-on electrons missed, each track used was scanned independently by two observers and all questionable events were examined by a third observer before a decision was reached. No track was scanned for more than 0.8 cm or beyond a detectable single scatter or a high energy electron-electron scatter. Tracks were not scanned and no event was recorded within 10μ of either surface of the emulsion. The average track length in emulsion was 0.40 cm giving an average loss due to both ionization and radiation of 30 Mev. Thus the average primary electron has a mean energy of 185 Mev. The energies of some primary electrons were measured by their multiple scattering and found to be consistent with the above calculated values.

In order to insure that no events were being missed (especially those in which the knock-on electron track was nearly vertical in the emulsion), a plot was made of the distribution of the azimuthal angles of the knock-ons about the direction of the incident electron. This distribution was found not to be significantly different from a symmetric distribution.

To determine the energy of the knock-on electron both its range and the angle between its direction and the direction of the incident electron were measured wherever possible. For very low energy knock-on electrons the angle became difficult to measure because of nuclear scattering. Therefore, the range was the principal means of determining the energy up to about 0.6 Mev.[10] Above this energy few knock-ons stayed in the emulsion, but the angle became a practical means of determining the energy. The angle, θ, is related to the knock-on kinetic energy, Q, and the incident electron

[10] B. Zajac and M. Ross, Nature **164**, 311 (1949).

kinetic energy, E, by $Q=E\cos^2\theta/[1+(E/2mc^2)\sin^2\theta]$, where mc^2 is the rest energy of the electron. For $E\sin^2\theta/2mc^2\gg1$, the knock-on energy as determined by the angle θ is nearly independent of the primary energy. For a 200-Mev primary electron this condition is met by all observed events, so we have disregarded the variation in primary energy caused by losses in the emulsion in calculating the knock-on energy. In the region where the angle and range methods of determining energy overlap, good agreement was found for the knock-on energy considering the large electron range straggle.

Knock-on electrons of energy less than 30 kev were not included in this study because of their small range ($<7\ \mu$) and because of the effect of electron binding.

IV. EXPERIMENTAL RESULTS

In Fig. 3 is shown a histogram of the results compared with the cross section as predicted by Møller. In the energy range from 30 kev to 0.1 Mev there were 182 events found in 33.4 cm of electron track. The rest of the histogram represents 245 events found in 102.6 cm of electron track. The number of electrons per cubic centimeter of emulsion was calculated to be 1.07×10^{24} from the emulsion composition given by Ilford Ltd. The effect of water absorbed in the emulsion from the atmosphere on the electron density has been measured and is negligible in this experiment.

The fact that the experimental data provides such a good fit to Møller's curve indicates that with these conditions there is no measurable deviation from a Coulomb potential for 185 ± 15 Mev electron primaries.

A similar study is being carried out using primary positrons of 200 Mev. Preliminary results[11] indicate that positron-electron scattering is similar to the electron-electron scattering in the range of knock-on energies studied here.

V. OTHER HIGH ENERGY ELECTRON PROCESSES

In the course of scanning for electron-electron collisions the following events were also noted:

In 102.6 cm of electron track two events were found in which the primary electron track divided into three tracks (Fig. 4), suggesting pair production in the field of the nucleus. By an approximate calculation[12] one would expect 1.1 pairs for this length of track.

An event was found on each of two separate plates (total path length of 102.6 cm) in which the electron track terminated in the center of the emulsion. Figure 5 is a photograph of one of the disappearances. The lengths of track before disappearance were 0.7 and 1.5 mm. The experimental arrangement and selection criteria rule out the possibility that these tracks were positrons. It is improbable that the tracks traversed an insensitive volume of the emulsion since the single grain background remains uniform and other primary tracks have no apparent change in grain density in the region of the disappearance. A short distance back on one of the disappearing electrons there is a knock-on coming off in the forward direction, confirming the assumed direction of this primary; this rules out the possibility of a Compton electron in the backward direction for this case. The fact the endings are near the center of the emulsion reduces the probability of

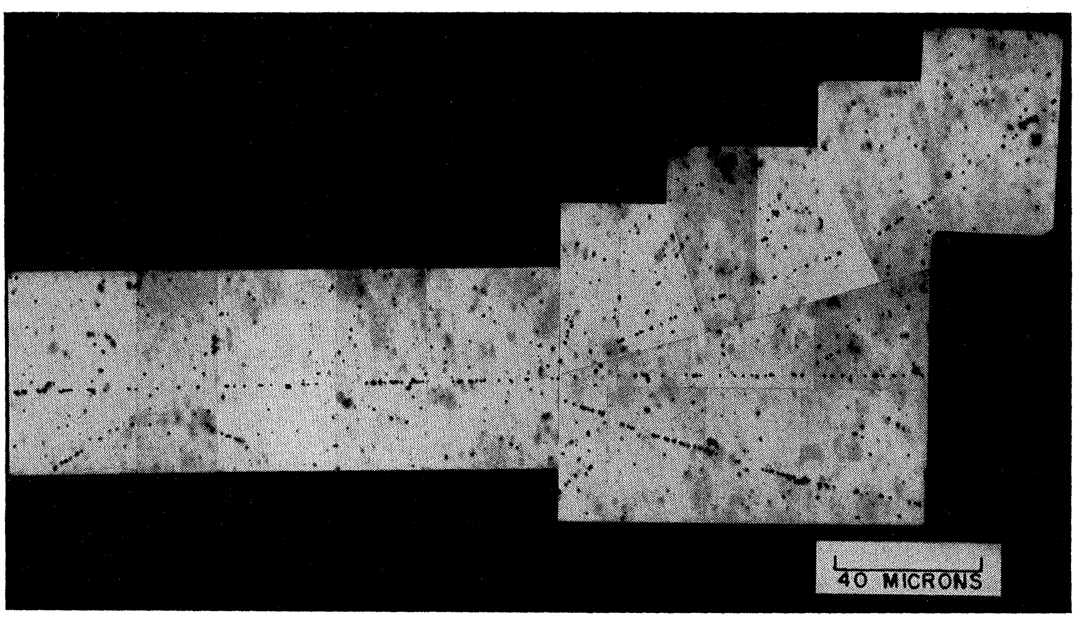


FIG. 4. Microphotograph mosaic of an electron-positron pair apparently produced in the field of a nucleus by an $\sim$185-Mev electron.

[11] Gilbert, Violet, and Barkas, Phys. Rev. **81**, 656 (1951).
[12] Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York), second edition.

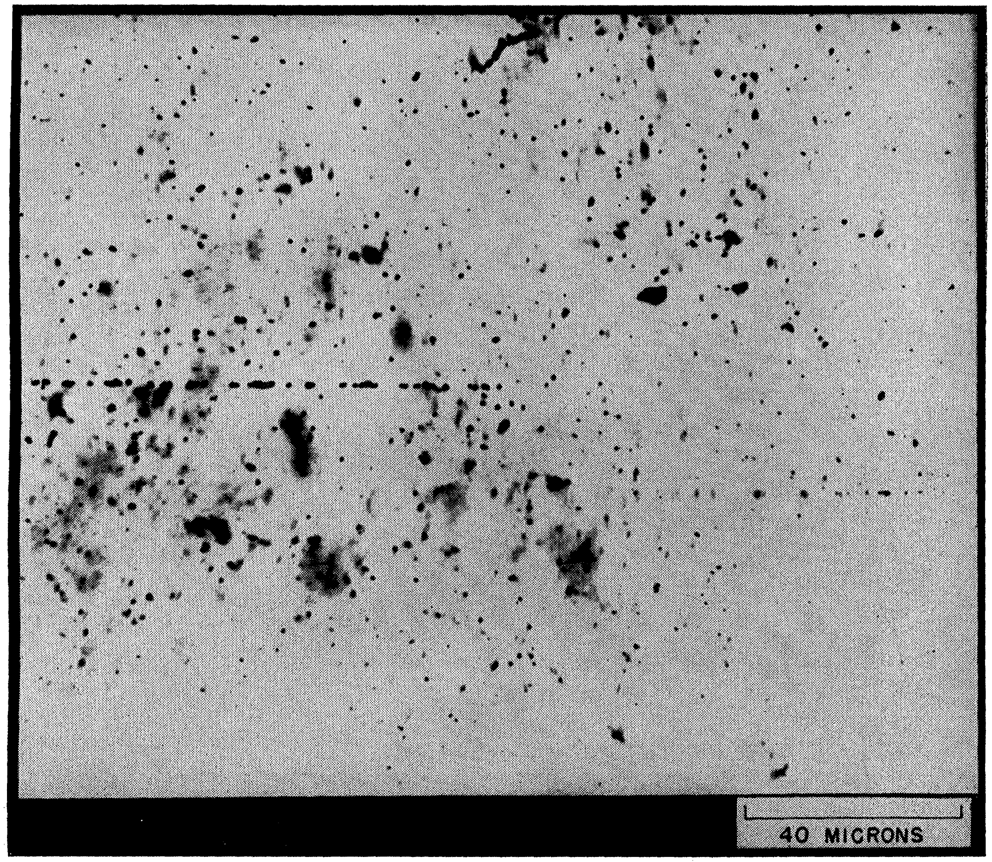


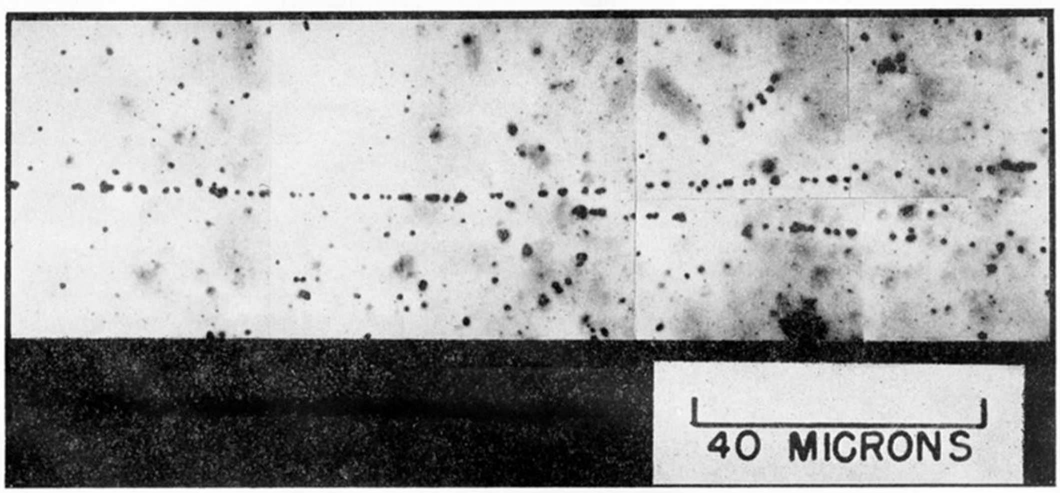
FIG. 5. Microphotograph of the disappearance of an ~185-Mev electron near the center of the emulsion.

not observing a large angle scatter out of the emulsion. The mechanism by which a high energy electron could disappear in emulsion has not been satisfactorily explained.

In scanning about 230 cm of electron track, no events were found in which protons or mesons were ejected from nuclei. Large angle nuclear scattering has been observed, but the study of such events has not been completed.

VI. ACKNOWLEDGMENTS

The emulsion method of studying electron-electron scattering was first found to be practical using fast electrons from a linear accelerator at Stanford University. Dr. G. E. Becker kindly assisted one of us (W.H.B.) in this experiment. Much of the microscope work of the preliminary experiments was done by Professor Lawrence Germain and Mrs. Edith Goodwin, and we are indebted to Mr. A. Oliver for his microphotographs and helpful advice on emulsion technique. The work was assisted greatly by the continued interest and encouragement given the film program by Professor E. O. Lawrence. We have enjoyed the invaluable cooperation of the synchrotron operating crew under the direction of Mr. George McFarland and Professor A. C. Helmholz.



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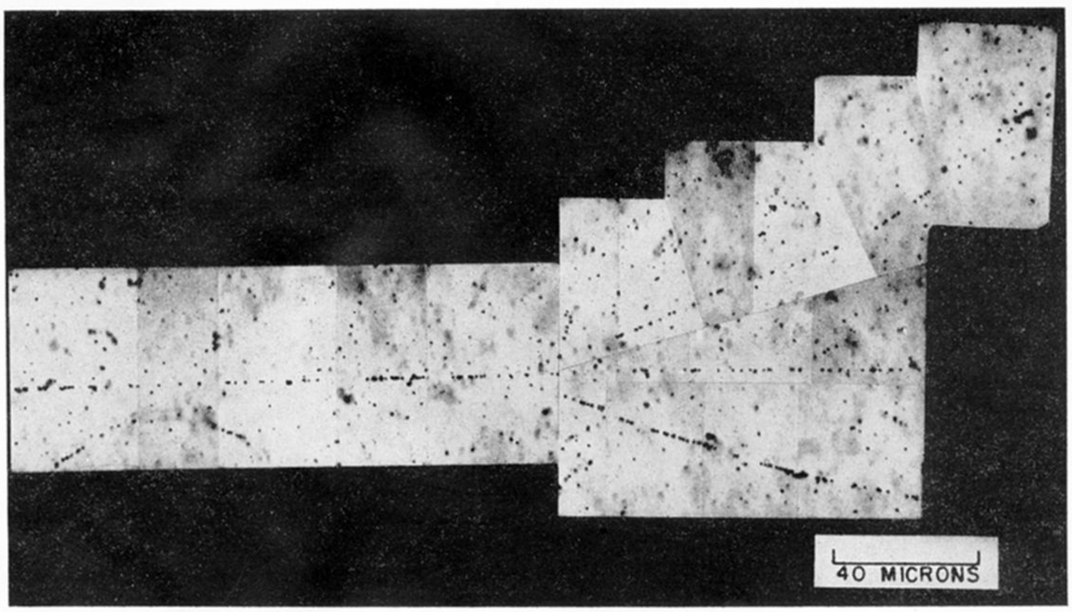

FIG. 4. Microphotograph mosaic of an electron-positron pair apparently produced in the field of a nucleus by an $\sim$185-Mev electron.

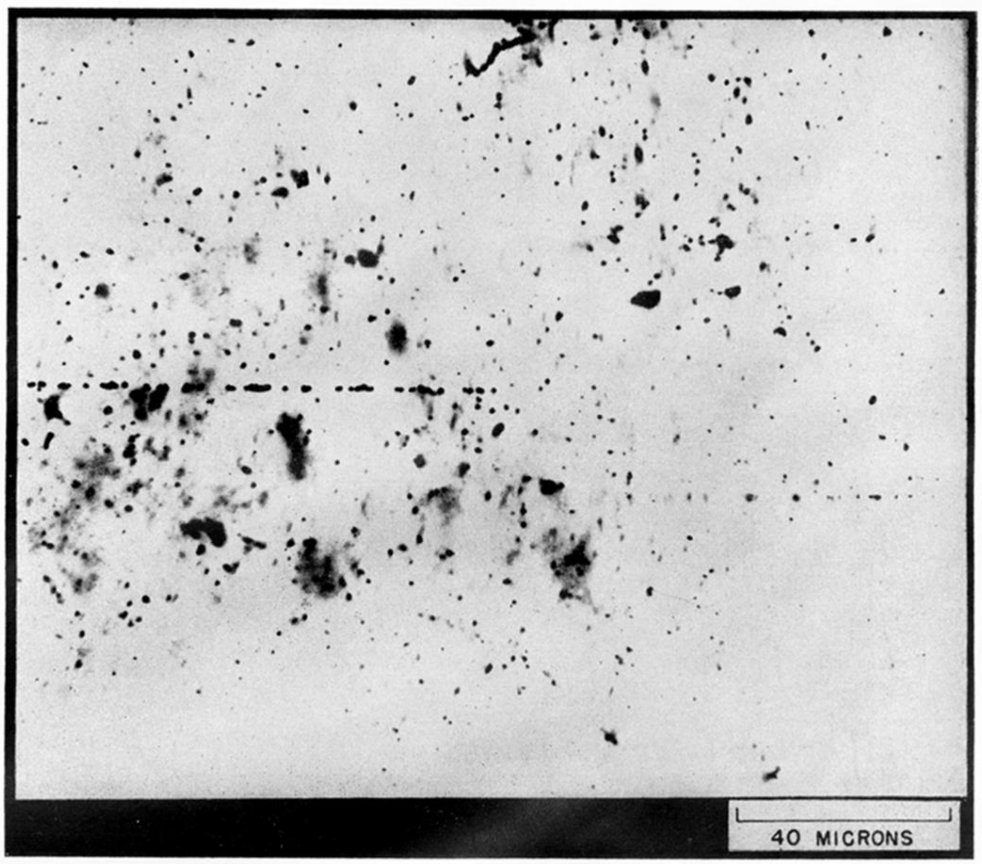

FIG. 5. Microphotograph of the disappearance of an $\sim$185-Mev electron near the center of the emulsion.