Letters to the Editor

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Third-Order Elastic Coefficients in Trigonal and Hexagonal Crystals

FAUSTO G. FUMI
Institute of Theoretical Physics, University of Milan, Milan, Italy
(Received March 28, 1952)

THE independent third-order elastic coefficients $pqr=prq=qpr$ ($p, q, r=1$ to 6) have been reported previously for twenty-five of the thirty-two crystal classes.[1] Table I completes these results. The numbers of coefficients obtained agree, of course, with those predicted by group theory.[2] Symmetries C_3, S_6, C_{3v}, D_3, and D_{3d} are treated again but with reference to the usual frames having $z\|C_3$.[3]

TABLE I. Third-order elastic coefficients: sixth-order polar tensor $pqr=qpr=prq$.[a, b]

Symmetry group	C_1, S_2	C_3, S_6	D_3, D_{3d}	C_{3v}	C_{3h}, C_6, C_{6h}	D_6, D_{6h}, D_{3h}	C_{6v}
Reference frame	Any	$z\|C_3$	$z\|C_2'$, $z\|C_3$	$z\|C_3$, $yz\|\sigma_v$	$z\|C_3$, C_6	$x\|C_2'$, $z\|C_3$, C_6	$z\|C_6$, $yz\|\sigma_v$
111	111	111	111		111	111	
112	112	112	112		112	112	
113	113	113	113		113	113	
114	114	114	114		0	0	
115	115	0	0		0	0	
116	116	0	0		116	0	
122	111−222+112	111−222+112	111−222+112		111−222+112	111−222+112	
123	123	123	123		123	123	
124	124	124	124		0	0	
125	125	0	0		0	0	
126	−116	0	0		−116	0	
133	133	133	133		133	133	
134	134	134	134		0	0	
135	135	0	0		0	0	
136	0	0	0		0	0	
144	144	144	144		144	144	
145	145	0	145		0	0	
146	$-\frac{1}{2}\cdot 115-\frac{3}{2}\cdot 125$	0	0		0	0	
155	155	155	155		155	155	
156	$\frac{1}{2}\cdot 114+\frac{3}{2}\cdot 124$	$\frac{1}{2}\cdot 114+\frac{3}{2}\cdot 124$	0		0	0	
166	$(\frac{3}{4}\cdot 222-\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 112)$	$(\frac{3}{4}\cdot 222-\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 112)$	$(\frac{3}{4}\cdot 222-\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 112)$		$(\frac{3}{4}\cdot 222-\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 112)$	$(\frac{3}{4}\cdot 222-\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 112)$	
222	222	222	222		222	222	
223	113	113	113		113	113	
224	$-114-2\cdot 124$	$-114-2\cdot 124$	0		0	0	
225	$-115-2\cdot 125$	0	0		0	0	
226	116	0	116		0	0	
233	133	133	133		133	133	
234	−134	−134	0		0	0	
235	−135	0	0		0	0	
236	0	0	0		0	0	
244	155	155	155		155	155	
245	−145	0	−145		0	0	
246	$\frac{1}{2}(125-115)$	0	0		0	0	
255	144	144	144		144	144	
256	$\frac{1}{2}(114-124)$	$\frac{1}{2}(114-124)$	0		0	0	
266	$(\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 222-\frac{1}{4}\cdot 112)$	$(\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 222-\frac{1}{4}\cdot 112)$	$(\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 222-\frac{1}{4}\cdot 112)$		$(\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 222-\frac{1}{4}\cdot 112)$	$(\frac{1}{2}\cdot 111-\frac{1}{4}\cdot 222-\frac{1}{4}\cdot 112)$	
333	333	333	333		333	333	
334	0	0	0		0	0	
335	0	0	0		0	0	
336	0	0	0		0	0	
344	344	344	344		344	344	
345	0	0	0		0	0	
346	−135	0	0		0	0	
355	344	344	344		344	344	
356	134	134	0		0	0	
366	$\frac{1}{2}(113-123)$	$\frac{1}{2}(113-123)$	$\frac{1}{2}(113-123)$		$\frac{1}{2}(113-123)$	$\frac{1}{2}(113-123)$	
444	444	444	0		0	0	
445	445	0	0		0	0	
446	145	0	145		0	0	
455	−444	−444	0		0	0	
456	$\frac{1}{2}(155-144)$	$\frac{1}{2}(155-144)$	$\frac{1}{2}(155-144)$		$\frac{1}{2}(155-144)$	$\frac{1}{2}(155-144)$	
466	124	124	0		0	0	
555	−445	0	0		0	0	
556	−145	0	−145		0	0	
566	125	0	0		0	0	
666	−116	0	−116		0	0	

[a] $p, q, r=1, 2, 3, 4, 5, 6$.
[b] The symmetries listed at the top of each column are equivalent for the third-order elastic coefficients (and generally for polar properties of even order).

The last three columns of Table I have been derived from column C_3 by the "direct inspection" method.[4] Indeed, the Cartesian orthogonal coordinates chosen (which are of the usual type) do not transform into linear combinations of themselves under the generating elements to be added to the group C_3 to obtain the groups D_3 and C_{3h} or to be added to the group C_6 to obtain the group D_6. Column C_3 has been obtained by imposing invariance directly on each tensor component.[5]

The results of Table I have been obtained also by group theory. The method used can be applied to any tensor property of a material system of finite symmetry; it consists essentially in constructing the totally symmetrical linear combinations of tensor components in each group of interest, and in equating to zero the nontotally symmetrical linear combinations of the components contained in them. The equations of invariance that the method yields are very simple, since the vanishing components are eliminated from the start. Important simplifications are provided by the correlation tables of irreducible representations of groups and subgroups and by the replacement of Cartesian orthogonal components by coordinate products; the results for all subgroups can be obtained from the results for a group treated with proper completeness and the results for higher order tensors can be obtained from those for lower order tensors. Details of this method will be given in a paper to be published in *Nuovo Cimento*.

[1] F. G. Fumi, Phys. Rev. **83**, 1274 (1951).
[2] S. Bhagavantam and D. Suryanarayana, Acta Cryst. **2**(1), 21 (1949); H. A. Jahn, Acta Cryst. **2**(1), 30 (1949).
[3] The reference frames which are not distinguishable in relation to the symmetry elements of a given group are equivalent for it.
[4] F. G. Fumi, Acta Cryst. **5**, 44 (1942).
[5] C. Hermann, Z. Kryst. **89**, 32 (1934).

The Decay Energy of Si^{31}

A. H. WAPSTRA
Nobel Institute of Physics, Stockholm, Sweden *
(Received March 27, 1952)

THE difference in binding energies of P^{31} and Si^{28}, computed from Ewald's[1] mass spectrographical measurements, is 26.314±0.036 Mev. This is in disagreement with the Q-values of some nuclear reactions studied at M.I.T.;[2] from $P^{31}(p, \alpha)Si^{28}$ Mev it follows that $P^{31}-Si^{28}=26.374\pm0.011$ Mev; from $P^{31}(d, \alpha)Si^{29}$ and $Si^{28}(d, p)Si^{29}$ it follows that $P^{31}-Si^{28}=26.373\pm0.015$ Mev. A check on these values can be made with the Q-values for the d, p reactions on the three Si isotopes in combination with the disintegration energy of Si^{31}. However, existing values[3-5] for this decay energy were not very precise.

We first tried to compare the Si^{31} and P^{32} absorption curves by the Feather method. We hoped that the similar end points and shapes of the β-spectrum (Si^{31} was expected to have an allowed form) would result in an accurate Feather plot. This hope was not substantiated in agreement with the later results of Wennerblom *et al.*[6] We found a range of 690±20 mg/cm² in Al; with Glendenin's[7] range-energy relation this yields a β-energy of 1.53±0.04 Mev. This part of the work was done in the Instituut voor Kernphysisch Onderzoek, Amsterdam, Netherlands.

We have now measured the β-spectrum of Si^{31} in an intermediate image β-spectrometer,[8] calibrated for this purpose with the internal conversion lines of Cs^{137} [9,10] and Co^{60}.[11,12] The Si^{31} samples were prepared in the way described by Wennerblom *et al.*[6] The best sample used had a mean thickness of 0.5 mg/cm² and a backing of 0.15 mg/cm² Al; with this sample the Fermi plot was linear above 700 kev (lower energies were not measured for lack of suffi-