and $\lambda = 69°$ (where the vertical cut-off energies are 0.57 Bev per nucleon and 0.025 Bev per nucleon, respectively) are absorbed as a result of energy loss by ionization in the 10 g/cm² residual atmosphere above the apparatus.

V. ACKNOWLEDGMENTS

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The Emission of L X-Rays of Lead in Po^{210} Decay*

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By use of a proportional counter x-ray spectrometer, it is established that Po^{210} sources emit characteristic L photons of Pb, in yield $2.93\times10^{-4}(\pm15$ percent) per alpha-particle. The excitation of these photons is ascribed to the ejection of L electrons from the electronic cortege of the nucleus in the act of alpha-decay, in a manner treated by Migdal's theory. The probability of L electron ejection is about 8.8×10^{-4} per alpha-particle, as computed from the observed photon yield and Kinsey's fluorescence yields. The theory in its present state accounts for only 13 percent of the photons observed.

I. INTRODUCTION

THE emission of low energy photons of low intensity from Po^{210} sources was observed and studied experimentally by Curie and Joliot[1] and has been noticed by others.[2,3] Curie and Joliot detected the photons by means of an ionization chamber connected to an electroscope, and identified photon energies by absorption coefficients. They found, within the limitations of this technique, that the photon energies were those expected for the L and M x-radiations of Po. Further, they thought that the intensities of the radiations increased with the thickness of the Po^{210} source (they used sources of roughly constant area and of strengths up to about 50 mC), and hence, were inclined to believe that the radiations were Po x-rays excited by alpha-particle bombardment of the undecayed Po atoms in the source.

We study these radiations by means of a proportional counter x-ray spectrometer, and establish that the L x-rays, at least, are the characteristic L radiations of Pb. We ascribe their production to the ejection of L electrons from the electronic cortege of the nucleus in the act of alpha-decay, a process which has been the subject of a theoretical study by Migdal.[4]

II. THE APPARATUS

The proportional counter x-ray spectrometer is an improved version of the one which has been described elsewhere.[5] The counter tube is a cylinder of brass, or brass with an inner cylindrical liner of aluminum (to eliminate fluorescence radiations from the brass wall), about four inches in diameter and twelve inches long, with an axial center-wire of stainless steel 0.004 inch in diameter. The window is a hole, one inch in diameter, drilled through a flat surface built up at the center of the cylindrical counter wall, and covered with a beryllium disk 0.005 inch thick. The counter gas is a 9:1 mixture of noble gas (A or Kr) and methane or ethane, at a total pressure of one atmosphere. With the argon-methane filling, pulses of convenient height are obtained with the center wire at about +2600 volts. The counter wall is grounded.

The stability requirements on the high voltage supply are quite stringent; if one demands that the pulse height be stable to better than 0.5 percent during the course of a run, the high voltage supply must be stable to

* Research carried out under the auspices of the AEC.
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[1] I. Curie and F. Joliot, J. phys. et radium **VII**, 20 (1931).
[2] S. DeBenedetti and E. H. Kerner, Phys. Rev. **71**, 122 (1947).
[3] Zajac, Broda, and Feather, Proc. Phys. Soc. (London) **60**, 501 (1948).
[4] A. Migdal, J. Phys. (U.S.S.R.) **IV**, 449 (1941). This paper also contains the theory of the corresponding process for beta-decay. The theory of ejection of K electrons in beta-decay has been worked out very elaborately by E. L. Feinberg, J. Phys. (U.S.S.R.) **IV**, 424 (1941).
[5] Bernstein, Brewer, and Rubinson, Nucleonics **6**, 39 (1950).

better than 0.01 percent. It must also be completely free of noise or ripple. Our present supply, designed by Higinbotham,[6] meets these requirements admirably.

The preamplifier and the non-overloading linear amplifier are described in reference 5. The essential feature of the amplifier is that it registers pulses of a given height in the presence of pulses several thousand times higher without distortion of the pulses of interest, displacement of the base line, or the introduction of spurious pulses. This permits the study of the K, L, and M radiations from Po^{210} sources without interference from each other or from the 0.77-Mev gamma emitted in a small fraction of the Po^{210} decays. The maximum output of the amplifier is 60 volts.

The amplifier output goes to a single sliding-channel pulse-height analyzer of Oak Ridge design.[7] The channel width, which we ordinarily set at 1.7 volts, shows excellent stability and independence of channel position. The analyzer output is fed to a commercial scale of 64.

Provision is also made for automatic operation, which is very convenient for exploratory experiments. A motor moves the channel at constant speed across the pulse-height range of interest, and the analyzer output is fed to a counting-rate meter,[8] whose output, in turn, is fed to a 0–25 mv Brown recorder. The chart drive of the Brown recorder is synchronized with the channel drive so that the chart scale reads directly in volts.

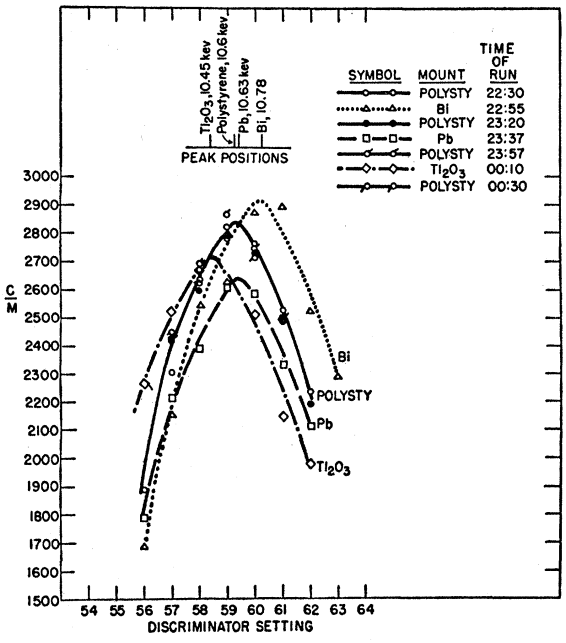

FIG. 1. Comparison of the 10.6-kev radiations from Po^{210} mounted on polystyrene, Bi, Pb, and Tl_2O_3.

[6] W. A. Higinbotham, Rev. Sci. Instr. **22**, 429 (1951).
[7] Francis, Bell, and Gundlach, Rev. Sci. Instr. **22**, 133 (1951).
[8] W. C. Elmore and M. Sands, *Electronics, Experimental Techniques* (McGraw-Hill Book Company, Inc., New York, 1949), National Nuclear Energy Series, Plutonium Project Record, Vol. 1, Div. IV, p. 254.

The line voltage to the amplifier and the analyzer is stabilized by means of a Sorenson voltage regulator.

As an example of what the instrument can do at its best, we have obtained a curve of the 10.6-kev arsenic K line (from Se^{75} K capture) with a width at half-height of 12.5 percent, and drifts have been negligible over a period of six hours.

III. THE Po^{210} SOURCES

The Po^{210} sources were prepared from a stock solution that consisted of Oak Ridge Po^{210} made up to 10 ml with 0.5 N HNO_3. The stock solution was assayed with an accuracy of ± 3 percent by the method of dilution and alpha-counting. Its specific activity was about 10 mC/ml.

To prepare a source, an aliquot of the stock solution was placed in a depression 1 mm deep and 15 mm in diameter in a polystyrene disk, evaporated under a heat lamp, and covered with Cellophane 2.7 mg/cm^2 thick. Polystyrene was chosen as a source backing because of the softness of the radiations excited in it by the alpha-particles. The strengths of the different sources were between 0.5 and 4 mC, as computed from the assay of stock solution. The sources prepared from small aliquots showed practically no visible deposit. With large aliquots, there was some chemical reaction between the polystyrene and the nitric acid of the solvent.

The stock solution was not tested for radiochemical purity beyond insuring that the half-life was correct. That the x-radiations we study are associated in some way with Po^{210} decay is adequately established by the fact that their intensity decreases in time with the Po^{210} half-life.

IV. PROOF THAT THE OBSERVED PHOTONS ARE CHARACTERISTIC L X-RAYS OF Pb

A critical absorption experiment (experiment 4, below) establishes that the radiations we study are characteristic L x-rays of Pb. We describe a few other experiments for their value as confirmatory evidence, and as illustrations of technique that may be useful to others.

Experiment 1

The observed pulse height (and, therefore, energy) of the most prominent x-ray line from a Po^{210} source is experimentally identical with that of the 10.56-kev arsenic K line from a Se^{75} (K capture) source. The listed L_α line of Pb has energy 10.57 kev.[9] This establishes that the radiations from Po^{210} have approximately the correct energy to be L x-rays of Pb.

Experiment 2

The intensity of the radiations from a Po^{210} source containing 1 mg of added Bi was only slightly greater

[9] A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand Company, Inc., New York, 1935), p. 787. We use the factor 12.393 to convert from angstroms to kev.

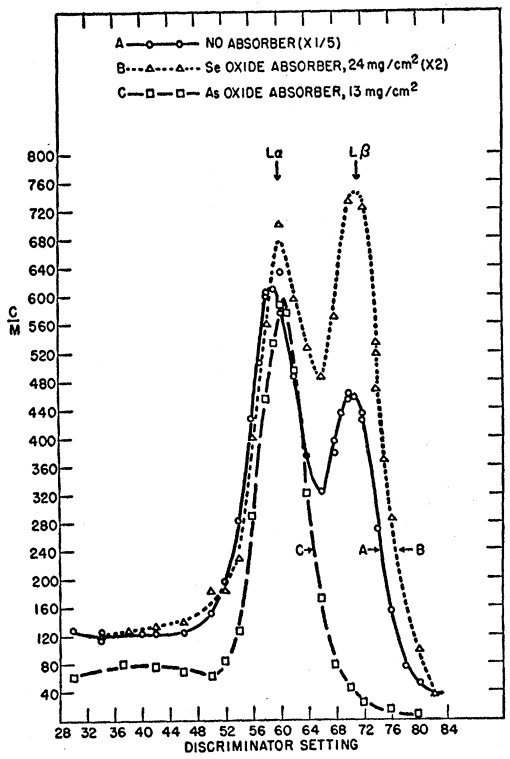

FIG. 2. L-radiations from Po^{210}; absorption in Se and As.

than that from a source of identical strength containing no added Bi, although the former source contained 40,000 times as much Bi as Po. Therefore, the observed radiations cannot be Po L lines excited by alpha-particle bombardment of Po atoms in the source.

Experiment 3

Four identical aliquots, ~1 mC each, of the Po^{210} stock solution were evaporated on thick plates of polystyrene, Pb, Bi, and Tl_2O_3, respectively. Alpha-particle bombardment of the plate material excites its characteristic x-rays, which we observe superimposed on the radiations from Po^{210}. (This is not inconsistent with experiment 2; in the present case, a much greater number of heavy element atoms is available for excitation by alpha-particles.) The observed intensities from the four sources were adjusted to approximate equality by means of beryllium absorbers, and then the different ~10.6-kev peaks were run in rapid succession, each run with a source on Pb, Bi, or Tl_2O_3 being sandwiched between two runs with the source on polystyrene. The results are shown in Fig. 1. The four runs with the source on polystyrene are essentially identical, showing that the instrument did not drift during the two hours required to take the data. It is clearly seen in Fig. 1 that the source on polystyrene gives a peak position experimentally identical with that of the source on Pb, and different from the peak positions given by the sources on Tl_2O_3 and Bi. Therefore, the ~10.6-kev radiation from Po^{210} is the characteristic $L\alpha$ x-radiation of Pb.

Experiment 4

There is no critical absorber that distinguishes between the $L\alpha$ lines of Pb and Bi, but Se, which has its K edge at 12.675 kev, can distinguish between the $L\beta$ lines: Pb $L\beta_1=12.635$ kev, $L\beta_2=12.635$ kev; Bi $L\beta_1=13.045$ kev, $L\beta_2=13.001$ kev. Curves A and B in Fig. 2 show the radiations from a Po^{210} source before and after passage through 24 mg/cm² of selenium oxide absorber. The $L\beta$ energy is obviously less than the energy of the Se K edge. (The clean effect that would have been obtained had Se absorbed $L\beta$ critically can be seen in curve C, Fig. 2, where the absorber was 13 mg/cm² of As_2O_3. The arsenic K edge is at 11.886 kev.) Therefore the $L\beta$ line cannot be that of an element higher in atomic number than Pb. The source certainly contained no quantities of elements below Pb that could give rise to the observed intensity of radiation. Therefore the radiations are Pb radiations.

V. THE MEASUREMENT OF THE L PHOTON YIELD

The L photon yield, i.e., the probability of L photon emission per alpha-particle emitted, was determined as the ratio of the rates of photon and alpha emission. The measurement of the rate of alpha emission from the source is discussed in Sec. III above. The rate of photon emission was obtained from the observed photon

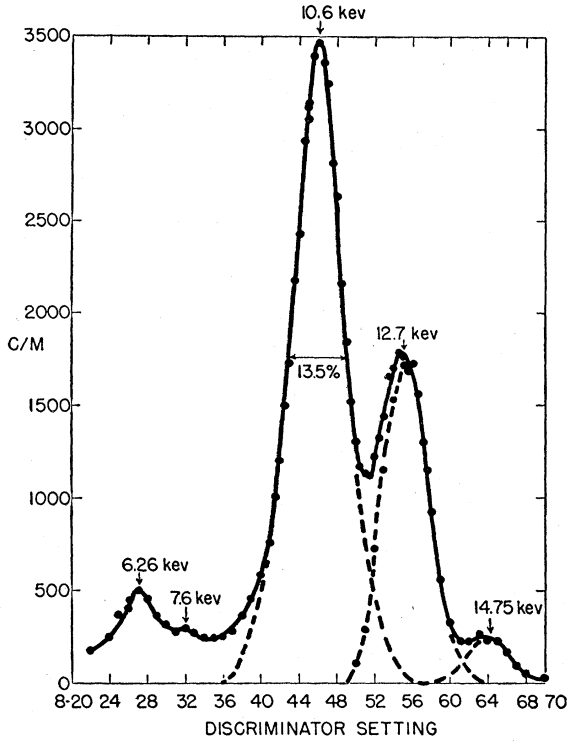

FIG. 3. Pb L-lines from Po^{210}. Counter gas: 650 mm A+70 mm CH_4.

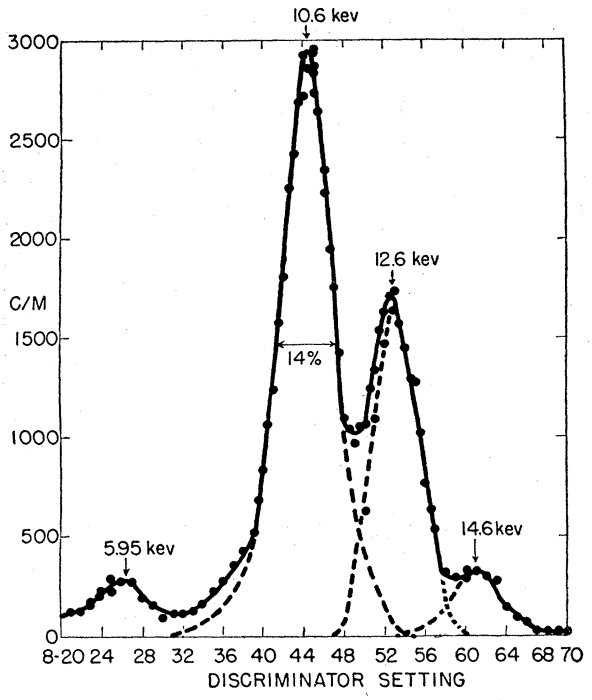

FIG. 4. Pb L-lines from Po^{210}. Counter gas: 670 mm Kr+70 mm C_2H_6.

counting rate and a computed counting efficiency. The photon counting rate was obtained by graphical integration of the experimental curves, two examples of which are shown as Figs. 3 and 4.

We digress at this point to discuss these curves in some detail, in order to display the reliability and limitations of the instrument. Figures 3 and 4 show the L peaks from a Po^{210} source of $\sim$3 mC strength. Both curves were taken with the same counter, but in the case of Fig. 3 the counter gas was A+CH_4 and the source was 1.69 inches from the counter window, while in the case of Fig. 4 the counter gas was Kr+C_2H_6 and the source was 2.19 inches from the counter window. One-minute counts were taken at each discriminator setting. Solid lines are drawn through the data points; the dotted lines represent our analyses of the curves.

The most prominent peak can be identified with the Pb $L\alpha$ line, as we have established above. It consists predominantly of the 10.569-kev $L\alpha_1$ and 10.466-kev $L\alpha_2$ lines, the latter having about one-tenth the intensity of the former. The two lines cannot be resolved by the instrument. The peak at about 12.6 kev consists predominantly of $L\beta_1$ and $L\beta_2$, the latter being half as intense as the former, and the two having identical energies, 12.635 kev. The small peak at 14.6 kev must be $L\gamma_1$, 14.789 kev.

TABLE I. Absorption coefficients used to compute counting efficiencies for Pb L photons.

Substance	Absorption coefficients 10.57 kev	12.64 kev	14.79 kev
Argon	52.0 cm²/g	31.5 cm²/g	19.8 cm²/g
Krypton	47.0	29.2	138.0
Beryllium	0.83	0.56	0.37
Air	4.2	2.5	1.5
Cellophane	0.0134(=μt)	0.008(=μt)	0.006(=μt)

The peak energies shown in Figs. 3 and 4 are computed from an assumed value of 10.6 kev for the main peak, and the linearity of the discriminator scale. We consider such agreement between the observed and the known Pb line energies as indication of very satisfactory behavior of the instrument.

The peak at about 6 kev in both figures is 6.4-kev Fe fluorescence radiation from a small amount of iron in the beryllium window (about 0.5 percent, according to a spectrographic analysis by M. Slavin of this laboratory). The intensity of this peak relative to the 10.6-kev peaks is greater in the argon counter (Fig. 3) than in the krypton counter (Fig. 4). This is ascribable to the facts that the Fe radiation is absorbed almost completely in both counter gases, and the 10.6-kev radiation is absorbed less efficiently in argon than in krypton.

The argon counter shows a peak at 7.6 kev which does not appear in the krypton counter. This is ascribable to the loss of argon K radiation from the counter subsequent to K ionization of argon atoms by the 10.6-kev x-rays from the source. Since the K ionization edge of argon is at about 3.2 kev, a peak is expected to appear at about 7.4 kev, and to be of low intensity because of the low K fluorescence yield in argon.

In analyzing the experimental curves into their components, the peak shapes were modeled after the shape of the clean arsenic K peak from a Se^{75} (K capture) source. There is, however, considerable arbitrariness in the drawing of the wings, which may lead to an error of several percent in the intensity.

With the original curves drawn to a fairly large scale, the areas under each of the peaks were measured (in units of "counts/min×discriminator unit") by planimeter or by counting squares, and divided by the channel width (calibrated in discriminator units) to give the "observed intensities" (counts/min) listed in Table II.

To calculate the counting efficiency ϵ, suppose that the source is a point source at the origin of a coordinate system. Let the counter axis be parallel to the y axis, with the window centered on, and perpendicular to, the z axis. Let θ be the angle between a photon path and the z axis, and let ϕ be the angle between the x axis and the plane of the z axis and the photon path. Then

$$\epsilon=\frac{1}{4\pi}\int_0^{2\pi}d\phi\int_0^{\theta\max}\sin\theta e^{-\sec\theta\Sigma\mu t}(1-e^{-\mu\rho\sec\theta(z-h)})d\theta,$$

with

$$z=\{p+[p^2-(p^2-a^2)(\tan^2\theta\cos^2\phi+1)]^{\frac{1}{2}}\}/(\tan^2\theta\cos^2\phi+1),$$

where a is the counter radius, h is the distance from the

source to the window, p is the distance from the source to the counter axis, μ_g is the absorption coefficient of the counter gas, μt denotes absorption coefficient times thickness, and $\Sigma\mu t$ is the sum of the μt's of the beryllium window, the air between source and window, and the cellophane cover of the source.

The various absorption coefficients were obtained by graphical interpolation of tabulated data,[10,11] and are listed in Table I.

The integral was evaluated by numerical integration. For a point source 2.19 inches from the counter window, the counting efficiencies computed in this fairly elaborate fashion differ by only a few percent from those computed by simply taking an average photon path length equal to the geometric mean of the minimum and maximum path lengths in the counter.

The assumption of a point source (our sources were $\sim$15 mm in diameter) introduces a negligible error in the geometry,[12] but it may underestimate the counting efficiency by a few percent because it underestimates the average path length in the counter.

Table II is a synopsis of our data on six curves obtained with the use of two different counters, two different counting gases, three different distances between source and window, and three different Po^{210} sources of independently determined source strength.

Table III lists the photon yields computed from the data in Table II. Note that the results obtained with different sources under different counting conditions agree almost as well as do the two results, curves 5 and 6, obtained with the same source under identical counting conditions.

The total L photon yield,[13] 2.93×10^{-4}, is probably reliable to better than ±15 percent. Most of the sources of systematic error that can be imagined would tend to make this yield an underestimate. The L photon yield found by Curie and Joliot was 4×10^{-4}, about 35 percent greater than our result.

It is interesting to compare the relative intensities of the Pb L lines from Po^{210} with those obtained from targets in x-ray tubes. We find for Po^{210}

$$L\alpha:L\beta:L\gamma=1:0.635:0.12.$$

The corresponding ratios for heavy element targets in x-ray tubes are

$$L\alpha:L\beta:L\gamma=1:0.786:0.09,$$

as computed from Jönsson's[14] values for Pt by assuming that the sum of the l, α_1, and α_2 intensities correspond to our 10.6-kev peak, the sum of the η, β_1, β_2, β_3, β_4, and β_6 intensities to our 12.6-kev peak, and the γ_1 intensity to our 14.6-kev peak.

[10] A. H. Compton and S. K. Allison, reference 9, Chapter VII and Appendix IX.

[11] C. S. Barret, *Structure of Metals* (McGraw-Hill Book Company, Inc., New York, 1943), Appendix III.

[12] See Blachman's equation for the geometry of an extended source in the Appendix to a paper by B. P. Burtt, Nucleonics **5**, 28 (1949).

[13] In Phys. Rev. **82**, 334 (1951) and Brookhaven National Laboratory Quarterly Progress Report, Oct. 1–Dec. 31, 1950, p. 98, we reported a total L photon yield of 7.2×10^{-4}, which is incorrect because of an arithmetic error. We reported the correct result at the September 1951 New York Meeting of the XIIth International Congress of Pure and Applied Chemistry.

[14] See reference 9, p. 645.

TABLE II. Data for computing L photon yields in Po^{210} decay.

Experiment	Line	Observed intensity (counts/min)	Counting efficiency	Po^{210} source strength (disinteg/min)
1. June 12, 1950	$L\alpha$	2970	0.0140	1.24×10^9
A+CH_4 counter	$L\beta$	1437	0.00978	
1.41 in. from window	$L\gamma$	140	0.00670	
2. April 13, 1951	$L\alpha$	13,912	0.00972	7.95×10^9
A+CH_4 counter	$L\beta$	5957	0.00688	
1.69 in. from window	$L\gamma$	810	0.00463	
3. April 20, 1951	$L\alpha$	11,586	0.0088	7.68×10^9
Kr+C_2H_6 counter	$L\beta$	5312	0.0068	
2.19 in. from window	$L\gamma$	1215	0.0120	
4. May 4, 1951	$L\alpha$	7283	0.0059	7.56×10^9
A+CH_4 counter	$L\beta$	3106	0.00414	
2.19 in. from window	$L\gamma$	461	0.00284	
5. May 8, 1951	$L\alpha$	5110	0.00458	7.48×10^9
A+CH_4 counter	$L\beta$	2742	0.00348	
0.3071 g/cm² Be	$L\gamma$	443	0.00253	
2.19 in. from window				
6. May 10, 1951	$L\alpha$	5615	0.00458	7.40×10^9
A+CH_4 counter	$L\beta$	2664	0.00348	
0.3071 g/cm² Be	$L\gamma$	421	0.00253	
2.19 in. from window				

VI. THE ORIGIN OF THE Pb L PHOTONS

The Pb L photons are in some way associated with Po^{210} decay because their intensity decreases in time with the Po^{210} half-life. They cannot be attributed to alpha-bombardment of Pb impurity in the source, as shown by Experiment 2, Sec. III, and by the fact that sources prepared from both fresh and aged stock solution gave the same photon yield although the aged source contained forty times the impurities of the fresh source.

The Pb L photons cannot be attributed to L conversion of the well-known 0.77-Mev gamma-ray of Po^{210}, whose intensity has recently been measured[15] to be $1.8\pm0.14\times10^{-5}$ per alpha, because even complete L

TABLE III. L photon yields in Po^{210} decay (computed from the data in Table II).

Experiment	Photon yield $L\alpha$	$L\beta$	$L\gamma$	Total
1	1.71×10^{-4}	1.18×10^{-4}	0.17×10^{-4}	3.06×10^{-4}
2	1.80	1.09	0.22	3.11
3	1.71	1.02	0.13	2.86
4	1.63	0.99	0.21	2.83
5	1.49	1.05	0.23	2.77
6	1.66	1.04	0.23	2.92
Mean	1.67×10^{-4}	1.06×10^{-4}	0.20×10^{-4}	$2.93\times10^{-}$
% spread	18.5	17.0	50	11.4
% standard deviation	6.9	6.3	20	4.6

[15] Grace, Allen, West, and Halban, Proc. Phys. Soc. (London), **64**, 493 (1951).

conversion of this gamma could account for only a few percent of the L photon intensity we observe.[16]

It is highly unlikely that some hitherto unobserved gamma is being completely converted in the L shell, and this mode of excitation of Pb L lines becomes even less likely in view of the fact that we have also observed what are presumably Pb K photons with considerably smaller intensity, and Pb M photons with considerably greater intensity,[17] than the L photon intensity, in qualitative agreement with Migdal's theory.

We conclude that the Pb L radiations are excited by ejection of L electrons from the electronic cortege of the nucleus in the act of alpha-decay in a manner described by Migdal's theory.[4]

VII. THE THEORETICAL L PHOTON YIELD

Migdal[4] deduces the probability of ejection of an extra-nuclear electron in the alpha-decay process by a perturbation calculation in which the potential change due to the change of nuclear charge, and the potential due to the moving alpha-particle, are treated as small perturbations on the nuclear potential. The probability of transition of an electron from a given initial state to an allowed continuum state is derived, and integration is then made over the continuum states. The approximations of the theory are:

(1) $\Delta Z^*/Z^* \ll 1$, where Z^* is the effective nuclear charge of the daughter nucleus. In the present case, $\Delta Z^*/Z^* = 0.025$.

(2) $v_\alpha/v_e \ll 1$, where v_α is the speed of the alpha-particle, and v_e is the speed of the electron in the bound state. For Po^{210} alphas and Pb L electrons, this ratio is ~ 0.2.

(3) The use of nonrelativistic hydrogenic wave functions. The hydrogenic wave function approximation is reasonably good for Pb L electrons. As concerns the nonrelativistic approximation, the speed of the electron in the bound state is about $0.3c$, and is about the same in the continuum state, according to Migdal's theory. At these speeds, relativity effects should not be prominent.[18]

With these approximations, the probability W_{nl} for ejection of an nl electron is

$$W_{nl} = (v_\alpha/v_e)^2 (C_{nl}/Z^{*2}), \qquad (1)$$

where, for the L shell, the sum of the two C_{20}'s is equal to 7.2, and the sum of the six C_{21}'s is equal to 13.3.

In applying this equation to the ejection of L electrons in Po^{210} decay, we have $v_\alpha = 1.6 \times 10^9$ cm/sec (the speed of a 5.3-Mev alpha-particle), and v_e, the speed of an electron in the second hydrogenic orbit about a nucleus of effective charge Z^*, is $\frac{1}{2}Z^*$ times the average electron speed in the first hydrogen orbit, or $v_e = 1.09 \times 10^8 Z^*$ cm/sec. With these numbers, and changing the notation so that W_i (i=I, II, III) denotes the probability of electron ejection from the L_i shell, Eq. (1) becomes

$$W_i = 215 C_i/Z^{*4}, \quad (C_{\rm I} = 7.2;\ C_{\rm II,III} = 13.3). \qquad (2)$$

The choice of Z^* in this equation offers some trouble. Imagine the L shells to lie entirely between the K and the M, N, $\cdots$ shells. Then the L shell eigenfunctions are relatively unaffected by the presence of the outer shells, so that the correct Z^* is the nuclear Z decreased by only the inner screening constant. But the presence of electrons in the outer shells has some effect on the amplitude of the continuum state eigenfunctions at the position of the L shell, and, therefore, on the magnitude of the transition probability. And, in fact, the magnitude of the transition probability may be quite sensitive to the existence of external screening, because the overlap of the bound state and continuum state eigenfunctions is small (as shown by our measured photon yields), so that a small absolute change in the amplitude of the continuum state eigenfunctions may result in a large relative change in the overlap. There is further discussion of this point below.

However, a literal interpretation of Migdal's theory seems to require that only the inner screening be used. We can expect this to result in an electron ejection probability that is too low.

Using the semi-empirical inner screening constants deduced by Sommerfeld and Wentzel,[19] we get $Z^* = 80$ for the Pb $L_{\rm I}$ shell, and $Z^* = 78.5$ for the $L_{\rm II,III}$ shell. With these Z^*'s, Eq. (2) gives the following theoretical electron ejection probabilities:

$$\begin{aligned} W_{\rm I}(L_{\rm I}\ \text{shell}):&\ 0.38\times10^{-4} \\ W_{\rm II,III}(L_{\rm II,III}\ \text{shell}):&\ 0.75\times10^{-4} \\ \text{Total}:&\ 1.13\times10^{-4}. \end{aligned} \qquad (3)$$

These numbers must be multiplied by the appropriate fluorescence yields in order to get the theoretical photon yields for comparison with our experiments. With the L fluorescence yields deduced by Kinsey,[20] who has

[16] D. E. Alburger and G. Friedlander, Phys. Rev. **81**, 523 (1951), report that the 0.77-Mev Po^{210} gamma is about 10 percent K converted and about 2 percent L converted.

[17] Curie and Joliot, reference 1, report an M photon yield of 15×10^{-4}. They were not able to detect the K photons.

[18] In a theoretical study of the Auger effect in Au, where an L electron is ejected with a speed of $\sim 0.5c$, H. S. W. Massey and E. H. S. Burhop, Proc. Roy. Soc. (London) **A153**, 661 (1936), find that a relativistic calculation gives a result twice as large as that of the nonrelativistic calculation, and in good agreement with experiment. In a similar calculation for Ag, where the electron is ejected with speed $\sim 0.27c$, the difference between the relativistic and nonrelativistic calculations is much less.

[19] See H. E. White, *Introduction to Atomic Spectra* (McGraw-Hill Book Company, Inc., New York, 1934), p. 318.

[20] B. B. Kinsey, Can. J. Research **26A**, 404 (1948). In his analysis, Kinsey takes account of the "Coster-Kronig transitions," which are radiationless transitions whereby a hole in the $L_{\rm I}$ shell is filled by an electron from $L_{\rm II}$ or $L_{\rm III}$ with simultaneous emission of an electron from an M or higher shell. This effect, which accounts for the very low intensity of $L_{\rm I}$ lines and the high intensity of $L_{\rm II}$ and $L_{\rm III}$ satellite lines in Pb and its neighboring elements, was first recognized by D. Coster and R. Kronig, Physica **2**, 13 (1935). See the review article by F. R. Hirsch, Jr., Revs. Modern Phys. **14**, 45 (1942).

made a careful analysis of the available empirical and theoretical information on the fluorescence yields of heavy elements, the theoretical photon yields, q_i, are

$$\begin{aligned} q_{\mathrm{I}} &= 0.095 W_{\mathrm{I}} = 0.036 \times 10^{-4} \\ q_{\mathrm{II}} &= 0.495 W_{\mathrm{II}} = 0.124 \times 10^{-4} \\ q_{\mathrm{III}} &= 0.285 (W_{\mathrm{III}} + 0.685 W_{\mathrm{I}}) = 0.217 \times 10^{-4} \\ \text{Total} &= 0.377 \times 10^{-4}. \end{aligned} \tag{4}$$

In this computation, we have assigned one-third of $W_{\mathrm{II,III}}$ to the L_{II} shell and the other two-thirds to the L_{III} shell.

This theoretical total photon yield is about 13 percent of the experimental value, 2.93×10^{-4}, which we believe to be reliable to 15 percent.

Two sources of this large discrepancy suggest themselves—the use of nonrelativistic eigenfunctions, and the neglect of the influence of the outer screening on the shape of the continuum eigenfunctions at the position of the L shell.

The effect of relativistic eigenfunctions has been discussed above, where we concluded tentatively that these would not make any large change in the theoretical result.

The major portion of the discrepancy is probably ascribable to neglect of the outer screening. In a theoretical study of the K ionization of heavy atoms by collision with relatively slow electrons, Soden[21] found that the continuum eigenfunctions deep within a Fermi-Thomas potential were considerably contracted toward the nucleus, as compared with the corresponding eigenfunctions in the absence of external screening. Such an inward contraction of the continuum eigenfunctions would increase the theoretical transition probabilities, but whether it would do so sufficiently to give satisfactory agreement with our measurements cannot be seen without an accurate calculation.

VIII. THE PROBABILITY OF L ELECTRON EJECTION IN Po^{210} DECAY

From our data on the L photon yield, we can compute the probability of L electron ejection if we can assume that Migdal's theory gives the correct ratios for the ejection of electrons from the different L shells. For, from Eqs. (3) and (4), the average L fluorescence yield is $\frac{1}{3}$. Then, since the observed total L photon yield is 2.93×10^{-4}, the total L electron yield is 8.8×10^{-4}, and the electron yields from the different L shells are

$$L_{\mathrm{I}}: 3.0 \times 10^{-4}; \quad L_{\mathrm{II}}: 1.9 \times 10^{-4}; \quad L_{\mathrm{III}}: 3.9 \times 10^{-4}.$$

Thus, the ejection probability per electron in L_{I} is 1.5 the corresponding probability of ejection from L_{II} or L_{III}.

IX. CONCLUDING REMARKS

In the course of these experiments, we have observed that Po^{210} sources also emit what are presumably K and M radiations of Pb, but we have not as yet established their identities or measured their intensities. Using a scintillation counter, we were not able to find a radiation reported to be at 105 kev with an intensity about 25 percent that of the K radiation,[3] although we would have been able to detect one-fifth the reported intensity.

In their investigations on the radiations from Po^{210}, Grace *et al.*[15] established by critical absorption experiments that at least part of the ~80-kev radiations from the source are characteristic K lines of Pb, with a yield of $\sim 1.5 \times 10^{-6}$ K photons per alpha-particle. They ascribe these to internal conversion of the 0.77-Mev Po^{210} gamma-ray. This implies a K conversion coefficient of about 7 percent, from which Grace *et al.* drew certain conclusions concerning the multipole nature and parity change of the 0.77-Mev gamma-ray. However, Migdal's theory predicts that a K photon intensity of the order of 10^{-6} per alpha is excited by ejection of K electrons by alpha-particles, so that considerable doubt attaches to their conclusions. The fact that they also observed the internal conversion electrons (which can be easily distinguished from electrons ejected by alphas) in numbers equal to their observed number of K photons cannot be reconciled with our own results on L photons, unless the theory overestimates electron ejections from the K shell as badly as it underestimates those from the L shell.

We wish to thank Dr. R. Sternheimer of this Laboratory for illumination on some points of the theory.

[21] D. G. Soden, Ann. Physik. **19**, 409 (1934).