The Nature of Work-Hardening*

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Two kinds of work-hardening are discussed, interaction-hardening and source-hardening. Interaction-hardening arises because a large stress must be applied to move a dislocation through a material which already contains a large number of dislocations. This paper is mainly concerned with source-hardening which occurs because the sources of easy glide become inactive. Additional glide occurs by the use of sources which act at a higher stress level. Source-hardening is the most important kind of hardening at small strains since then the dislocation density is small.

The experimental stress-strain curves obtained by Rosi and Mathewson on pure aluminum crystals can be calculated theoretically as follows: At very low temperatures one assumes that one has a distribution of "free lengths" of dislocations. The Frank-Read mechanism for the production of dislocations is used. It is also assumed in accordance with electron microscope data that generation ceases after about 500 dislocation loops have been produced. Source-hardening therefore occurs as the sources of greatest "free length" are gradually used. At finite temperatures the following modifications are introduced: The total stress is the sum of the applied stress and the stress resulting from thermal fluctuations. In addition, Brown's electron microscope data indicates that a given source produces much more glide at high temperature than at low temperature. The present theory does not predict an equation of state and does check Rosi and Mathewson's data on this topic.

Suggestions are made to explain the reason for the increased glide per source with increased temperature and to understand the way in which a source becomes inactive as a result of vacancy production.

Numerous experiments are suggested.

The major accomplishment of the paper is that it is able to show that the lamella structure found in the electron microscope pictures can be related to the mechanical properties of the metal single crystals.

I. INTRODUCTION

WE would like to describe a theory which makes it possible to understand certain aspects of work-hardening. Let us first summarize some of the available experimental data.

Rosi and Mathewson[1] have recently published resolved stress-strain curves for 99.996 percent aluminum crystals at four temperatures the lowest being 77°K. Their yield-stress data is as follows:

77°K	$\sigma_0=421.3$ g/mm^2
205°K	$\sigma_0=124.1$ g/mm^2
293°K	$\sigma_0=104.5$ g/mm^2
353°K	$\sigma_0=91.8$ g/mm^2.

Their stress-strain data extends only to about one percent strain. The curves appropriate for various temperatures are given in Fig. 1.

Brown[2] obtained electron microscope pictures on pure polycrystalline aluminum. After 15 percent extension, Brown found: at -180°C slip zones separated $\frac{1}{2}$ to 1μ each zone contains 1 to 2 lamellas; at 20°C slip zones separated 2μ each zone contains 3 to 4 lamellas; at 250°C slip zones separated 4μ each zone contains 5 to 6 lamellas. In most cases the average glide per lamella was about 2000A. The separation of the lamellas was from 200 to 800A.

Let us consider the temperature dependence of the yield stress and of the initial slope of the stress-strain curve. According to Becker[3] the probability W of a thermal stress fluctuation over a volume V which produces a total stress in excess of σ_1 is

$$W=\exp[-V(\sigma_1-\sigma)^2/2GkT], \quad (1)$$

where σ is the applied stress and G is the modulus of rigidity. If we suppose[4] that W must reach a value W_0 before glide occurs then the stress σ required for glide is

$$\sigma=\sigma_1-[2GkT/V\log(1/W_0)]^{\frac{1}{2}}=\sigma_1-BT^{\frac{1}{2}}. \quad (2)$$

Kuhlmann[4] has shown that Eq. (2) is in agreement with the yield stress data obtained on hexagonal metals. This relation does not fit the yield-stress data of Rosi and Mathewson given above. They fit their yield-stress data to the following relation:

$$\sigma_0=A\exp[Q/RT] \quad (3)$$

obtaining $A=61$ g/mm^2 and $Q=300$ cal/g atom.

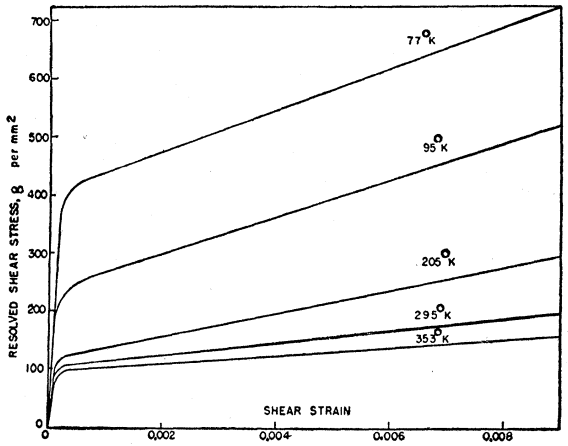

FIG. 1. The temperature dependence of the resolved stress-strain curves of pure aluminum (after Rosi and Mathewson).

* This work has been partially supported by the ONR.

[1] F. D. Rosi and C. H. Mathewson, J. Metals **188**, 1159 (1950).

[2] A. F. Brown, *Metallurgical Applications of the Electron Microscope* (The Institute of Metals, London, 1950), p. 110.

[3] R. Becker, Physik. Z. Sowjetunion **26**, 919 (1925).

[4] D. Kuhlmann, Proc. Phys. Soc. (London) **A64**, 145 (1951).

The yield-stress data of Rosi[5] on pure silver single crystals does fit relation (2). The data of Schmid and his co-workers on zinc and cadmium also fits Eq. (2). There is a further difficulty with the data of Rosi and Mathewson at very low strain. These investigators measured the temperature dependence of the Young's modulus; they found that Young's modulus increased by about a factor of two in going from 350°K to 77°K. Goens[6] measured the ratio of Young's modulus of this essentially isotropic material at 78°K to the value at 291°K; he found $E_{78}/E_{291}=1.098$. Since the low temperature data of Rosi and Mathewson appears questionable, we have fit the yield stress data at the three highest temperatures to relation (2) and used the low temperature values thus obtained for comparison with theory.

The initial slope of the stress-strain curves of aluminum and silver depends upon temperature in the following fashion:[7]

$$d\sigma/d\epsilon=(d\sigma/d\epsilon)_0\{1-CT^{\frac{1}{2}}\}. \quad (4)$$

This result probably indicates that thermal fluctuations are also of importance in determining the rate of work hardening in face-centered cubic metals at high temperatures.

Figure 2 shows the variation of the initial slope of the stress-strain curve of cadmium with temperature.[8] The dependence is clearly not in agreement with that predicted by Eq. (4). The temperature dependence of the initial slope of the stress-strain curves of zinc[9] is also not given by Eq. (4). In the case of magnesium[10] the data can be fit to Eq. (4) without undue error (i.e., deviations are less than 5 percent). Since magnesium has essentially a close-packed lattice whereas zinc and cadmium do not, the data indicate that the initial slopes of the stress-strain curves of all close-packed metals vary with temperature in accordance with relation (4). If the material is not close packed, then the temperature dependence is apparently more complex.

II. THEORY

A. In General

In devising theoretical explanations for the observed data it is necessary to make various assumptions. In the present paper an attempt will be made to begin with those assumptions which seem most plausible. The experimental consequences of the assumptions will then be discussed. Later, further less certain suggestions will be made to obtain a description of additional experimental results.

In the entire paper it is assumed that dislocations are generated by the Frank-Read[11] mechanism; i.e., a dislocation loop has sufficient length in the slip plane so that it bows out under an applied stress and produces dislocation rings in the slip plane as shown in Fig. 3. Since in principle the Frank-Read dislocation source is the same after the generation of a loop as it was before, any number of loops can be generated from a single source. The Frank-Read process will operate if one starts with either an edge or a screw-type dislocation; but since screw-type dislocations are not as restricted in their motion, an annealed metal probably contains more edge than screw dislocations.

It will be assumed that there are two types of work-hardening. At low strains, when the dislocation density is low, it will be assumed that a metal work-hardens because a given Frank-Read source can only produce a finite number of dislocation loops. This assumption was suggested by the fact that Brown[12] found only a finite amount of glide per slip lamella in aluminum.

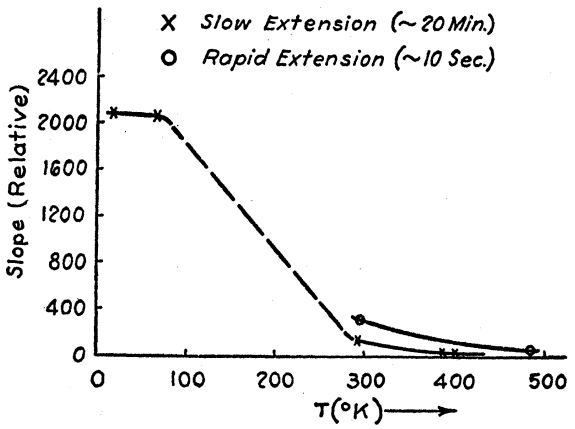

FIG. 2. The variation of the initial slope of the stress-strain curves curves of cadmium with temperature (after Schmid and Boas).

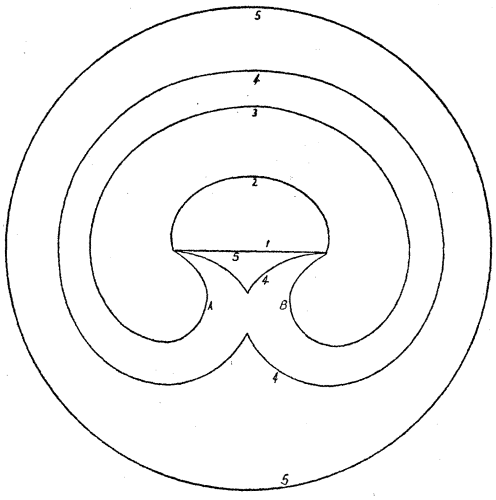

FIG. 3. Successive stages in the Frank-Read process for the generation of dislocation loops. The plane of the figure is the slip plane. The portion of the dislocation shown is anchored at its ends.

[5] F. D. Rosi, unpublished data.
[6] E. Goens, Ann. Physik Series 5, 4, 733 (1930).
[7] See reference 1, p. 1166; see also reference 5.
[8] W. Boas and E. Schmid, Z. Physik **61**, 777 (1930).
[9] See reference 8, p. 779.
[10] E. Schmid and G. Siebel, Z. Elektrochem. **37**, 447 (1931).
[11] F. C. Frank and W. T. Read, Phys. Rev. **79**, 722 (1950).
[12] See reference 2, p. 110.

Since a given source can only produce a finite amount of glide, the sources which generate at low stress are used up and further glide can only occur by using sources which operate at higher stress levels.

When the dislocation density is large, especially in cases where glide on several intersecting slip planes has occurred, the interactions of the dislocations give rise to work-hardening. This kind of work-hardening is probably important at strains where asterism becomes noticeable (in the face-centered cubic metals above strains of a few percent). This kind of work-hardening was first considered by Taylor[13] and has recently been discussed by Mott.[14] In the present paper an attempt will be made to consider only those experiments in which interaction-hardening is not important.

B. The Low Temperature Stress-Strain Curve at Small Strains

Frank and Read estimate that a dislocation of free length l will generate if the total resolved shearing stress on it is

$$\sigma = Ga/l, \tag{5}$$

where G is the modulus of rigidity and a is the smallest distance between atoms of the lattice. If a constant driving stress is maintained, then the following consideration indicates that the expansion of a dislocation loop once started is difficult to stop. Since $E_0 = Ga^2$ is the energy per unit length of dislocation line,[15] the energy associated with a circular loop of radius r is

$$E = 2\pi r G a^2 + E'.$$

The increment of work dW done by the applied stress σ is expanding the loop from r to $r+dr$ is

$$dW = \sigma a 2\pi r dr.$$

Thus the total work done in expanding the loop to radius r is

$$W = \sigma a \pi r^2 + W'.$$

Hence the external work available goes up more rapidly than the required potential energy of the loop. The additional energy available will either be converted into kinetic energy or it will be dissipated.

Consider next the form of the low temperature stress-strain curve at small strains. The picture presented here claims that glide occurs using the sources of greatest length first. As deformation proceeds, shorter and shorter dislocations operate as sources and hence according to (5) the required stress becomes larger. Thus the form of the stress-strain curve is determined by Eq. (5), by the amount of glide which occurs on each lamella before locking, and by the distribution of the free lengths of the dislocations in the solid.

At present very little is known about the free length distribution. A distribution which seems to give reasonable results can be obtained as follows:[16] Suppose that there are N atomic lengths of dislocation line in one cubic centimeter and that along the total length of dislocation line there are n points arranged at random at which the dislocation is pinned down. The probability of finding two locking points separated by s atoms is approximately

$$P(r) = (n/N)(1-n/N)^s = c(1-c)^{l/a} \cong c \exp[-cl/a],$$

where $c = n/N$ is the concentration of locked points along the dislocation line. The number of dislocations having lengths between l and $l+dl$ is therefore

$$N(l)dl = (Nc^2/a) \exp[-cl/a], \tag{6}$$

where the factor in front has been picked to give the proper total length of dislocation line (i.e., Na).

The low temperature stress-strain curve at small strains can now be obtained as follows: The increment $d\epsilon$ of plastic strain accomplished by using the dislocation sources having lengths between l and $l+dl$ is

$$d\epsilon = \epsilon_1 N(l)dl,$$

where ϵ_1 is the shearing strain which occurs upon using a single source. The total shearing strain accomplished by using all sources of length greater than l is

$$\epsilon = \epsilon_1 \int_l^{l_m} N(l)dl = \epsilon_1 Nc\{e^{-cl/a} - e^{-cl_m/a}\}.$$

If $\epsilon_0 = \epsilon_1 Nc \exp(-cl_m/a)$ then

$$\epsilon + \epsilon_0 = \epsilon_1 Nc e^{-cl/a}. \tag{7}$$

Solving this for l and inserting the result into Eq. (5) the resulting expression for the low temperature stress-strain curve is

$$\sigma = Gc/[\log(\epsilon_1 Nc) - \log(\epsilon + \epsilon_0)]. \tag{8}$$

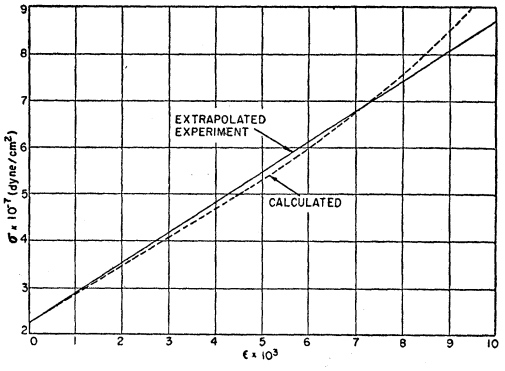

FIG. 4. Comparison of the experiment and theory at 0°K. Full line is extrapolated using stress-strain data of Rosi and Mathewson on aluminum single crystals. Dashed curve is calculated.

[13] G. I. Taylor, Proc. Roy. Soc. (London) **145**, 362 (1934).
[14] N. F. Mott, 35th Guthrie Lecture, Proc. Phys. Soc. (London) **B64**, 729 (1951).
[15] N. F. Mott and F. R. N. Nabarro, *Report of Conference on the Strength of Solids*, Phys. Soc., London (1948), p. 8.
[16] J. S. Koehler, *Imperfections in Nearly Perfect Crystals* (John Wiley and Sons, Inc., New York, to be published).

This expression can now be fit to the low temperature aluminum data.

Brown[17] found that the average amount of glide on a single lamella varied slightly with the temperature. He found

Temperature	Average slip distance
−180°C (93°K)	1600A
20°C (293°K)	2000A
250°C (523°K)	2200A

Extrapolating to 0°K, the average slip distance there is 1330A or 465 atomic distances. Hence ϵ_1, the strain resulting from the use of a single source, is 1.33×10^{-5}.

The yield stress at absolute zero is 2.3×10^{7} dyne/cm^2. This figure used in Eq. (5) yields the value of 3.80×10^{-4} cm for l_m, the largest free length of dislocation line, where the low temperature value of G has been used (i.e., $G_0=3.06\times10^{11}$ dyne/cm^2).

The theoretical expression (8) has then been fitted to the experimental 0°K curve by choosing N and c. One finds that $N=0.73\times10^{7}$ and $c=2.30\times10^{-4}$. The resulting theoretical curve is shown in Fig. 4. The value of c indicates that the pinned down points are separated on the average by about 4300 atomic distances. Na represents the total length of dislocation line per cc in the original annealed specimen. One finds that $Na=0.21$ cm per cc. This is much smaller than the usually accepted value[18] (i.e., about 10^8 cm per cc). Fitting to the 77°K data leads to very similar results.

The cadmium stress-strain curve obtained at −253°C has been fitted in Eq. (8). The parameters which give a good fit are $l_m=4.87\times10^{-4}$ cm, $c=1.05\times10^{-4}$, $N=1.32\times10^{9}$. The total length of dislocation line is therefore 3.93 cm. Since data was not available giving the amount of glide per lamella, it was assumed in these calculations that the glide in cadmium per lamella at low temperature was the same number of atomic distances as that found in aluminum at low temperature (i.e., 465 atomic distances). The situation is therefore qualitatively similar for the hexagonal and the face-centered cubic metals at low temperatures. Data on the lamella structure of the hexagonal metals would be a valuable aid for further calculations. It would be especially significant if magnesium displayed such a structure and zinc and cadmium did not.

Recently in his Guthrie lecture Mott[19] has considered the way in which screw dislocations on different slip systems impede one another's motion. The net result of the crossing of a pair of such dislocations is the production of a line of vacancies joining the two. This introduces a restriction which favors the motion of edge dislocations over that of screw dislocations. Seitz[20] in his paper points out that as a result of this, glide over the entire cross section of a macroscopic specimen may require the simultaneous action of a large number of sources. Seitz estimates that for a specimen which is one square centimeter in cross section about 10^4 sources each of about one micron in length may be required to produce the slip lamella. Such an effect would increase the estimate of the length of dislocation line to a value much nearer the usually accepted figure.

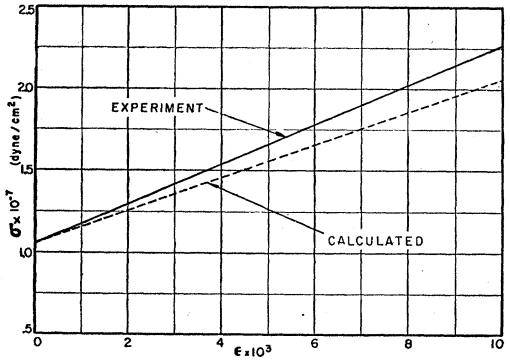

FIG. 5. Comparison of experiment and theory at room temperature. Full line is Rosi and Mathewson's data on aluminum single crystals. Theoretical curve is dashed.

C. The High Temperature Stress-Strain Curve at Small Strains

The room temperature stress-strain curve will be calculated next. This calculation is based on the low temperature theory and on Brown's information about the zone structure in aluminum at various temperatures.

Brown found that on the average each slip zone contains 1.5 lamellas at −180°C and 3.5 lamellas at room temperature. It will be assumed that each slip zone is produced by glide at a single generator. If the arguments of Mott and Seitz about the number of sources required per lamella are correct, then some revision of the above assumption will be required. The basic idea, i.e., that at high temperatures more glide is obtained for each long source used, is still admissible. The room temperature curve can therefore be calculated by modifying the low temperature calculation as follows:

1. The glide per source should be increased. In fact, the room temperature value should be $\epsilon_3=2\times10^{-5}\times3.5=7\times10^{-5}$.
2. The total stress acting to produce slip should be the sum of the applied stress and the stress arising from thermal fluctuations. The thermal stress can probably be measured by the decrease in the yield stress as the temperature is raised from 0°K to room temperature.
3. The room temperature value of G should be used.

The resulting stress-strain curve at any temperature can therefore be written as

$$\sigma=Gc/[\log(\epsilon_3 Nc)-\log(\epsilon+\epsilon_4)]-FT^{\frac{1}{2}}, \qquad (9)$$

where $\epsilon_4=\epsilon_3 Nc\exp[-cl_m/a]$. The expression which is subtracted takes the thermal stresses into account. Ac-

[17] See reference 2, p. 110.
[18] F. Seitz, *The Physics of Metals* (McGraw-Hill Book Company, Inc., New York, 1943), p. 93. This figure is obtained by assuming that the mosaic blocks have dislocations at their edges. See also F. Seitz and T. A. Read, J. Appl. Phys. **12**, 476 (1941).
[19] See reference 14, p. 733.
[20] F. Seitz, Quarterly Supplement of Phil. Mag. (January, 1952, to be published).

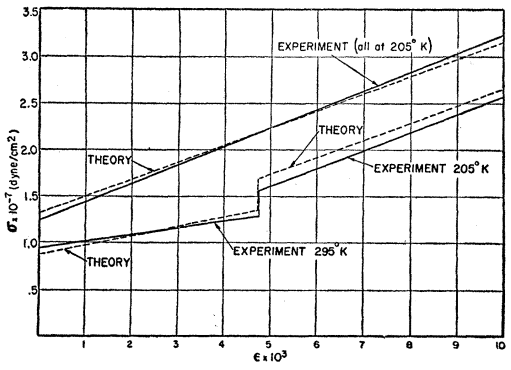

FIG. 6. Comparison of Rosi and Mathewson's equation of state data on aluminum single crystals with the present theory. Full lines are experiment. Dashed lines are calculated.

cording to our estimate, the yield stress of aluminum drops from 2.3×10^7 to 1.04×10^7 dynes/cm^2 in going from 0°K to room temperature. Inserting the other numerical constants into Eq. (9), we obtain the dashed curve shown in Fig. 5. The room temperature value of G has been used and F has been adjusted to give the required yield stress. The result is $F(293)^{\frac{1}{2}}=0.96\times10^7$ dyne/cm^2. The theoretical expression fits the experimental curve quite well. The calculated stress-strain curve at 205°K also fits the experimental curve well as we shall see later.

D. The Equation of State

If an equation of state existed for plastic flow, then the stress necessary to produce further plastic strain should be independent of the past history of the specimen; the stress required should depend only on the prior strain, the strain rate, and the temperature. Actually the experiments of Dorn, Goldberg, and Tietz[21] on polycrystals and those of Rosi and Mathewson on single crystals[22] show that the past history is important and that the plastic flow of metals cannot be described by an equation of state.

According to the views presented here, the past history is important. For example, if a specimen is strained in single slip through a strain ϵ at low temperature, the resulting state of the specimen as given by the remaining dislocation free length distribution is quite different from the result obtained by straining through ϵ at a high temperature. Qualitatively the low temperature strain uses more of the dislocation length distribution, so that if such a specimen is then warmed to the high temperature, a larger stress is required for further strain than if the prior strain had occurred at the high temperature. This prediction is in qualitative agreement with the experiments.

Rosi and Mathewson give the stress-strain curves of a single crystal pulled in tension through a strain of about one percent at 205°K and of a second crystal pulled in two stages, first at room temperature and then through the remaining strain at 205°K. The resolved stress-strain curves obtained using their data[23] are given by the full lines in Fig. 6.

The stress-strain curve which was all run at 205°K was fitted to the theory just used for the room temperature data. The glide required per source was $\epsilon_3'=1.86\times10^{-5}\times2.10=3.91\times10^{-5}$. The resulting stress-strain curve is shown dashed in Fig. 6. This figure 2.10 for the average number of glide lamellas per slip zone agrees well with Brown's electron microscope data (see Fig. 8) when it is extrapolated to 205°K.

The calculation of the stress-strain curve of a crystal which has been extended in two stages can be done as follows: Using the room temperature theory, the longest unused dislocation length l_5 at the end of the room temperature strain ϵ_5 can be obtained from the equation:

$$\epsilon_5+\epsilon_4=N\epsilon_3 c\exp[-cl_5/a], \tag{10}$$

where ϵ_4 and ϵ_3 are the room temperature values of these parameters. This together with the values of G and $F(205)^{\frac{1}{2}}$ will give the stress at which the low temperature plastic strain begins [using Eq. (5)]. The remainder of the stress-strain curve is described by Eq. (9) where ϵ in the present case represents only that portion of the strain which occurs after the drop in temperature and the value of ϵ_4' to be used is

$$\epsilon_4'=\epsilon_3'Nc\exp[-cl_5/a], \tag{11}$$

where ϵ_3' is the value of ϵ_3 appropriate at 205°K. The resulting theoretical curve is shown dashed in Fig. 6. It should be mentioned that inasmuch as this particular crystal showed an abnormally low yield stress at room temperature, the theory was adjusted accordingly by using a somewhat larger l_m and a somewhat smaller c. N and the product l_mc were kept the same as before. These changes are reasaonable for a specimen of somewhat higher purity than most of those used by Rosi and Mathewson. The values used were $l_m=3.98\times10^{-4}$ cm and $c=2.296\times10^{-4}$. The theoretical results for the two stage case are shown in Fig. 6 by the lower dashed curve. The calculation should predict two things correctly, the magnitude of the increment in stress when the temperature is changed and the value of the slope at low temperature. Both quantities are reasonably well given by the theory.

If the basic ideas used in the calculations made thus far are correct, then the distribution of "free lengths" of dislocation which exists in a crystal can be obtained experimentally as follows: The number dN of glide steps produced per unit distance measured normal to the glide plane during the stress increment $d\sigma$ is measured at low temperature. This number represents the number of dislocations having length between l and $l+dl$ where the stress used and the dislocation length are associated

[21] Dorn, Goldberg, and Tietz, Trans. Am. Inst. Mining Met. Engrs. **180**, 205 (1949).
[22] See reference 1, p. 1162.
[23] There exists a typographical error in Rosi and Mathewson's Table I. Crystal A should have $\sin x\cos\lambda$ equal to 0.4685.

by Eq. (5). The assumptions made above about the effect of temperature upon the strain resulting from the use of a single source can also be checked experimentally by making measurements on the number of slip zones produced per stress increment $d\sigma$ in a two stage stress-strain curve. This information together with electron microscope data on the number of glide lamellas produced per slip zone at various stresses would furnish the necessary data for the test. The data should be obtained on single crystals.

E. Interaction Hardening and Source Hardening

The present theory implies that glide on planes other than the one having the highest yield stress should occur when the resolved shearing stress on these other planes exceeds their yield stress. In the face-centered cubic and the body-centered cubic crystals which have several equivalent glide planes, this implies considerable cross slip, particularly at large strains. This glide on several intersecting slip systems leads to the entrapment of large numbers of dislocations in the specimen as a result of the interaction of the crossed dislocations with one another.[24] The entrapment makes it harder to force additional dislocations through the specimen and hence results in work-hardening. This second kind of work-hardening should probably be called interaction-hardening. The kind of hardening discussed earlier in the paper could properly be called source-hardening. Source-hardening is clearly the only kind of hardening at very small strains, whereas both are important at large strains. Rohm and Kochendorfer[25] have attempted to perform an interesting experiment in which interaction-hardening would be reduced or eliminated by constraining an aluminum crystal in such a way that glide can only occur on one slip system. They then apparently obtain no asterism and the stress-strain curve is linear. This and other evidence[4,14] indicates that interaction hardening is probably associated with asterism, deformation bands, etc.

III. FURTHER SUGGESTIONS

Certain tentative proposals will now be made to explain experimental points not thus far considered. In general, not enough data is available to decide for or against the explanations given. It is hoped that the present discussion will stimulate further decisive experimental work in this field.

A. The Increase in the Glide per Source with Temperature

The experiments of Brown and the stress-strain data both indicate that the glide per source increases with increasing temperature. Earlier in the paper it was noted that the decrease in the initial slope of the stress-strain curves of close-packed crystals is proportional to $T^{\frac{1}{2}}$. The temperature dependence of the initial slope is more complex in the case of zinc and cadmium. This behavior can be understood as follows:

After a given source has produced a large number of loops, the process of generation fails to carry through. Thermal fluctuations then produce stresses which, when acting on the portion of the dislocation that is screw type, carry a sufficient length of the dislocation into a new glide plane which is far enough from the original lamella so that a large amount of slip can occur in the new lamella (see Fig. 7 for the geometry). The separation of the lamellas in a zone is determined by the interaction force between dislocations on neighboring lamellas. The total stress must be large enough to drive the dislocations past one another. Thus[26]

$$\sigma = Ga/[8\pi(1-s)d], \tag{9}$$

where s is Poisson's ratio and d is the separation. Brown found lamella spacings from 200 to 800A after 15 percent extension. Data on polycrystalline aluminum at room temperature[27] shows that the resolved shearing stress required for 15 percent extension is 2.9×10^8 dyne/cm^2. The stress given by Eq. (9), assuming a separation of 200A, is 2.5×10^8 dyne/cm^2. At 78°K the experimental shearing stress required is about 7.1×10^8 dyne/cm^2 whereas the theoretical value, again for a spacing of 200A, is 2.9×10^8 dyne/cm^2. The present theory indicates that the spacing should be almost inversely proportional to the applied shearing stress no matter what the temperature. Further measurements on single crystals would be desirable.

The number of lamellas in a single slip zone will depend on the stress, the temperature, and the speed of deformation. Consider the temperature dependence for samples strained at the same rate. The probability W of a thermal fluctuation which produces a stress σ_1 in the cross slip system if σ is the component of the shearing stress in the cross slip system is given by Eq. (1). Since $n-1$ of the lamellas in a zone are produced by fluctuations, this number should be proportional to W. The number of lamellas per zone should, according

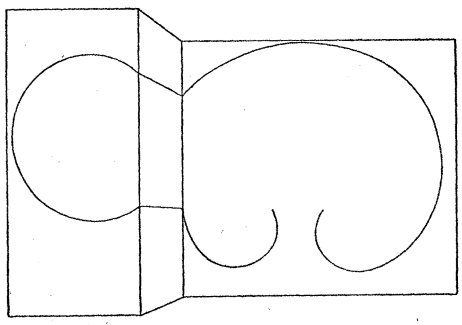

FIG. 7. Illustrating the glide produced by thermal fluctuations on slip planes near the plane of the original source.

[24] See reference 14, Sec. 8 and 10.
[25] F. Rohm and A. Kochendorfer, Z. Metallkunde **41**, 265 (1950).
[26] A. H. Cottrell, *Progress in Metal Physics* (Butterworths Scientific Publications, London, 1949), Vol. I, p. 103.
[27] See reference 21, p. 219.

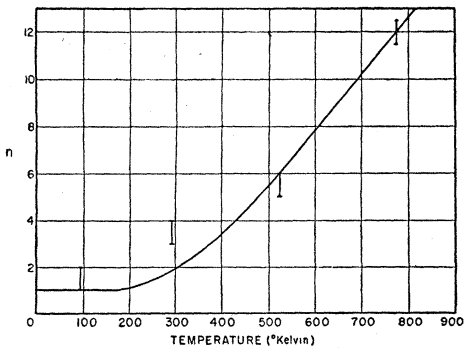

FIG. 8. The number n of glide lamellas per zone as a function of temperature. Brown's experimental values are given by the vertical lines. The solid curve is calculated.

to this theory, be compared at the same resolved shearing stress on the cross slip system at different temperatures. Brown's data is not immediately applicable because the number of lamellas per zone are compared after the same extension at the different temperatures. Nevertheless we have fitted Brown's data to an exponential. The results are shown in Fig. 8. If the above theory is correct, then a single crystal of a face-centered cubic material which has its axis nearly parallel to the [111] direction should have an abnormally large number of lamellas per zone at room temperature because the shearing stresses on the original and on the particular cross slip system having the same slip direction as the original system are nearly equal. The thermal stresses should therefore find it easy to produce the necessary cross slip. The initial slope of the stress-strain curve should also be unusually small for such a specimen.

B. The Locking of a Frank-Read Source

Brown's data[28] indicates that at all temperatures the glide per lamella ranges from 1500A to 2500A. He found that the average glide per lamella increases with temperature as follows:

−180°C	average glide = 1600A
20°C	average glide = 2000A
250°C	average glide = 2200A

The present data,[28] although not decisive, indicates that once a lamella has been formed, little, if any, further glide occurs there.

In several recent publications Seitz[29] has discussed data which indicates that moving dislocations generate vacancies. These vacancies produced in the vicinity of the glide plane may be associated with the locking process. Seitz's viewpoint is summed up by him as follows: "The generator will encounter resistance to motion as a result of vacant sites or interstitial atoms produced in the generating area only if (a) the imperfections are formed as a result of dynamical instability during motion in regions where it attains velocities comparable with the velocity of sound or (b) if vacancies and interstitials are produced in pairs in the heated annihilation zone and diffuse apart. It seems entirely feasible, however, that the limited life of a generator is determined by the fact that debris accumulates in its region of operation, particularly in the annihilation region where it attains maximum velocity." Seitz[30] explains the fact that once a generator locks, the application of larger stresses will not reactivate it, as follows: "Once a dislocation has come to rest in a region where there is a high density of vacancies, it will probably become locked into position because the vacancies in the vicinity will migrate to it and form clusters in order to relieve the local stress field in the manner suggested by Koehler in another connection."

It may be that Seitz's proposals should be extended as follows: It seems quite reasonable to suppose as Seitz did that vacancies are produced by dynamical instability at a jog in a moving dislocation. In addition, it seems plausible that the production of a vacancy or an interstitial at such a jog will be more likely to occur during the time the jog is passing an impurity, a vacancy, or an interstitial atom. The local differences in the inertial force and the binding force should make it difficult for the entire configuration to move smoothly.

It is also possible that locking occurs not so much because of debris left on the glide plane, but because after a sufficient number of jogs have been produced, the spiralling dislocation has become so ragged that the two portions in Fig. 3 which should annihilate one another no longer fit together. As a result, further generation does not occur. Such a locked annihilation center should be quite stable and should require considerable well-correlated diffusion to unlock the center.

Seitz's suggestions and the above proposals both give a qualitative explanation of the Molenaar and Aarts experiments.[31] In these experiments a polycrystalline metal wire is pulled in tension at liquid air temperature. Subsequent annealing for about ten minutes at room temperature results in an appreciable drop in the electrical resistance at liquid air temperature but no change occurs in the low temperature stress required for further extension.

If the first additional proposal made above is correct, then a small amount of lead dissolved in an aluminum crystal should produce a decrease in the amount of glide per lamella.

It must be admitted that a present the locking mechanism is not well understood.

C. Cross Slip

The theory presented in this paper implies that slip also occurs on systems which are not those associated

[28] See reference 2, p. 109.
[29] See reference 20, Part 6, Sec. I.
[30] See reference 20, Part 6, Sec. H.
[31] J. Molenaar and W. H. Aarts, Nature **166**, 690 (1950).

with the maximum shearing stress. This cross slip occurs because glide on the original slip system gradually uses up the long sources there until eventually, as the applied stress increases, the resolved shearing stress in some other slip system is large enough to induce loop generation there. The amount of glide which occurs on the cross slip lamellas will depend on whether the locking process which has stopped lamellas in the principal slip system is also at least partially effective in stopping glide on the second slip system. The present theory is consistent with the finding that during the transition from single to double slip no discontinuity is observed in the stress-strain curve of a pure metal. The treatment given in this paper implies that the glide produced by cross slip is much less than that on the primary slip system. This assumption is in accord with experiment.[32]

In conclusion, I would like to thank Dr. L. Slifken, Professor F. Seitz, and Mr. J. Marx for numerous stimulating discussions.

[32] G. I. Taylor and C. F. Elam, Proc. Roy. Soc. (London) **A102**, 643 (1923).

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High Energy Electron-Electron Scattering

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Eradicated electron sensitive nuclear emulsions were exposed to 200-Mev electrons at the Berkeley synchrotron. In scanning the electron tracks 427 events were observed in which the scattered electron of lower energy, or knock-on electron, had an energy greater than 30 kev. The observed differential cross section was found to agree in absolute value with Møller's theoretical cross section, although an insufficient number of high energy knock-on electrons were observed to distinguish between the Møller, relativistic Mott, and relativistic Rutherford formulas. Two pairs initiated by primary electrons and two cases in which primary electrons vanished in the emulsion were also observed in 102.6 cm of track. No heavy particle events were seen.

I. INTRODUCTION

HISTORICALLY, the electron is the best known of the fundamental particles. However, a lack of information still exists concerning its actual structure. An electron-electron scattering experiment would seem to be the ideal way to investigate the boundaries of the electron and the possibility of non-Coulomb electron-electron forces. To find deviations from a Coulomb potential, one would roughly estimate that it is necessary to have an impact parameter of the order of the classical electron radius. In order for the impact parameter to be well defined the de Broglie wavelength, λ, of the electron in the relativistic center-of-mass system must be of the order of 2.8×10^{-13} cm or less. A simple calculation shows that such a wavelength would require an energy of about 19 Bev in the laboratory system. In the present experiment 200-Mev electron primaries were used which have a de Broglie wavelength of about 10 times the classical electron radius in the relativistic center-of-mass system. Even for this wavelength, the possibility seemed to exist of observing a deviation from the Coulomb potential if the effect were strong.

The generally accepted formula giving the scattering cross section of electrons by electrons has been derived by Møller.[1] This formula in terms of the scattering angle, θ, in the relativistic center-of-mass system is the following:

$$\sigma(\theta)d\theta=\frac{(\gamma+1)\pi r_0^2\sin\theta d\theta}{\gamma^2\beta^4}\left[\csc^4\frac{\theta}{2}+\sec^4\frac{\theta}{2}-\csc^2\frac{\theta}{2}\sec^2\frac{\theta}{2}+\frac{(\gamma-1)^2}{\gamma^2}(1+4\csc^2\theta)\right], \quad (1)$$

where r_0 is the classical electron radius, $\beta=v/c$, $\gamma=1/(1-\beta^2)^{\frac{1}{2}}$, and v is the velocity of the primary electron in the laboratory system. The first two terms in the bracket correspond to the classical, relativistic Rutherford scattering formula. The third term is the quantum-mechanical exchange term. The inclusion of this term with the Rutherford formula gives the relativistic Mott formula. The fourth term represents retardation and spin interaction effects.

Equation (1) is more conveniently expressed in terms of the parameter A, defined as the ratio of kinetic energy given to the secondary or knock-on electron to the kinetic energy of the primary electron. It is not possible to distinguish between the primary and secondary electrons after collision. The knock-on electron is by definition the lower energy electron after collision. The maximum value of A is obviously 0.5. By a simple

[1] C. Møller, Z. Physik **70**, 786 (1931); C. Møller, Ann. physik. **14**, 531 (1932); K. C. Kar and C. Basu, Indian J. Phys. **18**, 223 (1944).