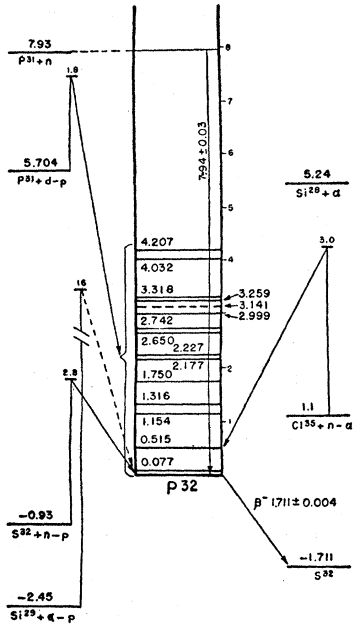

FIG. 3. Energy-level diagram for P^{32}.

The pattern of the established P^{32} levels is quite distinctive. Of particular interest is the discovery of the first excited state at 77.0±1.7 kev. This state has been verified by Kinsey *et al.*,[11] who found two high energy $P^{31}(n,\gamma)P^{32}$ gamma-rays, evidently corresponding to the ground state and an excited state at 80±50 kev. In this region of the periodic table, the only other known case for such a low-lying level is Al^{28}, where the first excited state occurs at 31 kev.[14]

In addition to this ground-state doubtlet, there are three other pairs of closely spaced levels at excitations of 2.2, 2.7, and 3.3 Mev, with spacings of 50±2, 92±7, and 59±3 kev, respectively. At present, none of these doublets has been verified by other workers.

The present results for the $P^{31}(d,p)P^{32}$ reaction can be closely correlated with the results of Kinsey and co-workers for the $P^{31}(n,\gamma)P^{32}$ reaction, as indicated in Table I. The measured energies of twelve of the $P^{31}(n,\gamma)$ gamma-rays agree to 40 kev or less with the established P^{32} levels, when assigned to the transitions indicated in Fig. 2. Except for the case of the ground-state doublet, the relative intensities of the observed gamma-rays have generally the same pattern as the relative intensities of the $P^{31}(d,p)P^{32}$ proton groups.

[14] Enge, Buechner, Sperduto, and Van Patter, Phys. Rev. **83**, 31 (1951).

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The Interpretation of Image Transitions in Beta-Decay Theory*†

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The allowed favored beta-transitions are studied in relation to problems of nuclear structure and beta-decay theory. These problems include (1) the influence of deviations from *L-S* coupling on the beta-decay matrix elements of the light nuclei, (2) the relation between the observed magnetic moments, simple nuclear models and deviations from *L-S* coupling, (3) determination of the ratio of Fermi to Gamow-Teller type coupling constants, and (4) determination of the absolute magnitudes of the coupling constants.

The analysis of all available data yields two mutually supporting conclusions: (a) a substantial Fermi-type component is present in the beta-decay interaction and (b) deviations from *L-S* coupling are an important factor in the interpretation of nuclear magnetic moments.

I. INTRODUCTION

THE theoretical interpretation of beta-decay data is made difficult by two complicating factors: (a) the possibility of linear combinations of the five covariant formulations of the theory and (b) the occurrence of unknown nuclear matrix elements in the derived formulas. Under (b) the difficulties are particularly formidable when two or more nuclear matrix elements are involved in a transition probability.

Because of these general complications the testing of the theory outside of the allowed range has been largely dependent on the occurrence of transitions subject to special selection rules for which only one nuclear matrix element appears in the theoretical transition probability.[1,2] The study of such transitions (especially $\Delta I=\pm 2$, yes) proves the need for a tensor or axial vector component in the general linear combination, but does not exclude the presence of other components (scalar, polar vector, and pseudoscalar).

In allowed transitions the tensor and axial vector components are responsible for the Gamow-Teller

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† Assisted by the joint program of the ONR and AEC.

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[1] C. S. Wu, Revs. Modern Phys. **22**, 386 (1950).

[2] J. P. Davidson, Phys. Rev. **82**, 48 (1951).

selection rules:

$$\Delta I=0, \pm 1 \text{ (no)}, \quad 0\rightarrow 0 \text{ excluded} \tag{1}$$

while the Fermi selection rules

$$\Delta I=0 \text{ (no)} \tag{2}$$

derive from scalar and polar vector components.

An additional important distinction within the allowed group is that between transitions within a supermultiplet (allowed favored or superallowed—$ft \sim 10^3$) and those between states belonging predominantly to different supermultiplets (allowed unfavored or just plain allowed—$ft \sim 10^{5-6}$).[3] The occurrence of these distinct types in widely separated ranges of ft values proves both the approximate nature of the supermultiplet formalism and the general excellence of the approximation—approximate because the allowed unfavored transitions would not occur at all if the supermultiplet formalism were exact and an excellent approximation because the nuclear matrix elements for allowed unfavored transitions are actually quite small.

In this paper the allowed favored group (including the image transitions) is studied in relation to problems of nuclear structure and beta-decay theory, developing further earlier studies by Wigner[3,4] and Grönblom.[5] These problems include:

1. The influence of deviations from L-S coupling on the beta-decay matrix elements of the light nuclei.
2. The relation between the observed magnetic moments, simple nuclear models and deviations from L-S coupling.
3. Determination of the ratio of Fermi to Gamow-Teller type coupling constants.
4. Determination of the absolute magnitudes of the coupling constants.

II. THE METHOD OF CALCULATION

In the general formulation of the theory the half-life t for an allowed transition is given by

$$\frac{1}{t}=\frac{1}{2\pi^3 \ln 2}\frac{mc^2}{\hbar}\left[(G_S^2+G_V^2)\left|\int 1\right|^2 + (G_T^2+G_A^2)\left|\int \boldsymbol{\sigma}\right|^2\right] f(Z, W_0), \tag{3}$$

where

$$f(Z, W_0)=\int_1^{W_0}(W_0-W)^2 pW \frac{2(1+S_0)}{\Gamma^2(2S_0+1)}(2pR)^{2S_0-2} \times e^{\pi\gamma W/p}\cdot|\Gamma(S_0+i\gamma W/p)|^2 dW, \tag{4}$$

$$\gamma=Ze^2/\hbar c, \quad S_0=(1-\gamma^2)^{\frac{1}{2}}$$

and G_S, G_V, G_T, and G_A are the coupling constants for scalar, vector, tensor, and axial vector couplings, respectively. These constants are real numbers in consequence of the postulated invariance of the interaction Hamiltonian under the operation of time reversal[6] or of charge conjugation.[7] Fierz's conditions[8]

$$G_S G_V=G_T G_A=0 \tag{5}$$

insure that the theoretical Kurie plots for allowed transitions are straight lines. They are also necessary conditions for complete symmetry between positron and negatron emission when the effect of the Coulomb field of the product nucleus on the wavefunctions of the emitted particle is neglected.[8] The experimental evidence[1,9] cannot establish Eq. (5), but does require strong inequalities

$$2G_S G_V \ll G_S^2+G_V^2, \quad 2G_T G_A \ll G_T^2+G_A^2. \tag{6}$$

The notation

$$G_S^2+G_V^2=G_F^2, \quad G_T^2+G_A^2=G_{GT}^2, \quad G_F^2=rG_{GT}^2 \tag{7}$$

is useful in indicating, respectively, the Fermi and Gamow-Teller coupling types and permits rewriting Eq. (3) as

$$ft\left(r\left|\int 1\right|^2+\left|\int \boldsymbol{\sigma}\right|^2\right)=\frac{\hbar}{mc}\frac{2\pi^3 \ln 2}{G_{GT}^2}. \tag{8}$$

The right side of Eq. (8) contains only constant factors; so the left side must also be constant for all allowed transitions.

In evaluating r and G_{GT}^2 the matrix elements are first computed; then the left side is a known linear function of r, the coefficients varying with the transition as prescribed by the assumed nuclear model. The left side is made as nearly constant as possible in the sense of minimizing the mean square deviation from the average. This fixes the value of r; setting the right side of Eq. (4) equal to the average value of the left side gives a value for G_{GT}^2 and, hence, through Eq. (7), for G_F^2. The mean square deviation from the average provides a figure of merit for the nuclear model used in the calculation. The resulting theory is applied to a number of transitions not included in the least squares evaluation of r and the coupling constants. Three models are tested and compared in this manner. Finally a compromise solution is developed in an attempt to achieve a satisfactory fit with all the data while maintaining a close correspondence with the more plausible of the three nuclear models.

The transitions used in the least squares calculation are those between ground states of mirror nuclei. In evaluating the nuclear matrix elements, these states are assumed to be pure spin doublets. The calculation is sketched below for $|\int \boldsymbol{\sigma}|^2$; that for $|\int 1|^2$ is similar, but simpler.

[3] E. P. Wigner, Phys. Rev. **56**, 519 (1939).
[4] E. P. Wigner and A. M. Feingold, privately circulated notes (1949).
[5] B. O. Grönblom, Phys. Rev. **56**, 508 (1939).
[6] L. C. Biedenharn and M. E. Rose, Phys. Rev. **82**, 982 (1951).
[7] H. A. Tolhoek and S. R. de Groot, Phys. Rev. **84**, 150 (1951).
[8] H. A. Tolhoek and S. R. de Groot, Physica **16**, 456 (1950).
[9] C. S. Cook and G. E. Owen, Am. J. Physics **18**, 453 (1950).

The explicit form of the matrix element, with the charge operator included, is

$$\left|\int \boldsymbol{\sigma}\right|^2=\sum_{m'}|(T_\xi', I, m'|\mathbf{Y}_\xi \pm i\mathbf{Y}_\eta|T_\xi, I, m)|^2, \quad (9)$$

where

$$\mathbf{Y}_\xi=\tfrac{1}{2}\sum_1^A \tau_{\xi k}\boldsymbol{\sigma}_k,$$
$$\mathbf{Y}_\eta=\tfrac{1}{2}\sum_1^A \tau_{\eta k}\boldsymbol{\sigma}_k, \quad (10)$$

with τ_ξ and τ_η the components of the isotopic spin operator, the subscript k referring to the kth nucleon.[10] T_ξ and T_ξ' are eigenvalues of the operator $\frac{1}{2}\sum\tau_{\xi k}$, equal to the initial and final values of $\frac{1}{2}(A-2Z)$. The plus sign in Eq. (9) refers to positron emission and K-capture, the minus sign to negatron emission.

Under the assumption of pure doublet states[11] the wave function can be expressed in the form

$$\psi=\alpha\psi_{L=I-\frac{1}{2}}+(1-\alpha^2)^{\frac{1}{2}}\psi_{L'=I+\frac{1}{2}}, \quad (11)$$

the individual components being assumed normalized. This represents a linear combination of the two L-S coupling states possible for a given value of I. Since parity is a good quantum number, both components in Eq. (11) have the same parity.

Since $\mathbf{Y}_\xi \pm i\mathbf{Y}_\eta$ does not operate on the space coordinates and hence has no matrix elements between states of different L, Eq. (9) becomes

$$\left|\int \boldsymbol{\sigma}\right|^2=\sum_{m'}|\alpha^2(T_\xi', I, L, S, m'|\mathbf{Y}_\xi \pm i\mathbf{Y}_\eta|T_\xi, I, L, S, m)$$
$$+(1-\alpha^2)(T_\xi', I, L', S, m'|\mathbf{Y}_\xi$$
$$\pm i\mathbf{Y}_\eta|T_\xi, I, L', S, m)|^2. \quad (12)$$

These matrix elements can now be evaluated directly by the use of the theory of angular momentum operators and supermultiplet formalism. The result is (see Appendix)

$$\left|\int \boldsymbol{\sigma}\right|^2=|\alpha^2(I+1)-(1-\alpha^2)I|^2/[I(I+1)]. \quad (13)$$

In a similar manner

$$\left|\int 1\right|^2=1. \quad (14)$$

Equation (8) becomes

$$ft\left[r+\frac{1}{I(I+1)}\{\alpha^2(I+1)-(1-\alpha^2)I\}^2\right]$$
$$=\frac{\hbar}{mc}\frac{2\pi^3\ln 2}{G_{GT}^2}=C. \quad (15)$$

The mixing coefficients α^2 must now be determined from a theoretical interpretation of the nuclear magnetic moments. Plots of moments against spin in conjunction with the theoretical single particle (Schmidt) and uniform model (Margenau-Wigner) limits[12] are helpful in devising a reasonable interpolation procedure for α^2. As a rule the observed moments are close enough to one or the other of the Schmidt limits to permit an unambiguous assignment of parity and a predominant orbital angular momentum L. These assignments are strongly supported by the success of shell model considerations in interpreting a wide range of nuclear properties.

In the single particle model the odd particle is assumed to move in the spherically symmetric field produced by a core containing all the other particles; this core is assumed to have zero angular momentum, so that if the odd particle is a proton $g_L=1$, while if it is a neutron $g_L=0$. A difficulty arises when an attempt is made to explain deviations from the moments predicted in this way since states with $L=I-\frac{1}{2}$ and $L'=I+\frac{1}{2}$ have opposite parities in a single particle model and cannot be put into the same linear combination. Direct interpolation between the Schmidt limits is therefore meaningless and some other procedure must be used.

If it is assumed that one pure L-S coupling component can be described more or less accurately as representing a single particle interacting with a symmetrical core of even parity, the other component necessarily has a many particle character in order to maintain the same parity in both components. A reasonable estimate of the extent to which both components are present can be derived from a simple interpolation procedure on the moment diagrams. One component is referred to the nearest Schmidt limit (μ_S) and the other to the opposite Margenau-Wigner limit (μ_{MW}); then one computes the quantity

$$\Delta_{S\text{-}MW}=(\mu_S-\mu)/(\mu_S-\mu_{MW}). \quad (16)$$

This interpolation coefficient is interpreted as α^2 or $1-\alpha^2$ depending on the location of μ relative to the Schmidt limits.[13]

For comparison purposes another coefficient $\Delta_{MW\text{-}MW}$ is defined by linear interpolation between the MW

[10] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, New York, 1949), Chapter IV and Appendix 1.

[11] A consequence of the assumed validity of the supermultiplet analysis.

[12] E. Feenberg, Phys. Rev. **77**, 771 (1950).

[13] Interesting and perhaps significant regularities appear in the application of Eqs. (11) and (16) to the available data on moments and spins (private communication from J. P. Davidson).

TABLE I. $C=ft[r|\int 1|^2+|\int \boldsymbol{\sigma}|^2]$ for L-S coupling.

Transition	I	α^2	$\|\int \boldsymbol{\sigma}\|^2$	ft	C	$\Delta=C-\bar{C}$	$C(r=0)$
$n^1\rightarrow H^1$	1/2	1	3	1400	$1400r+4200$	$-1777r-48$	4200
$H^3\rightarrow He^3$	1/2	1	3	1100	$1100r+3300$	$-2077r-948$	3300
$Be^7\rightarrow Li^7$	3/2	1	5/3	2300	$2300r+3830$	$-877r-418$	3830
$C^{11}\rightarrow B^{11}$	3/2	1	5/3	3900	$3900r+6500$	$723r+2252$	6500
$N^{13}\rightarrow C^{13}$	1/2	0	1/3	4800	$4800r+1600$	$1623r-2648$	1600
$O^{15}\rightarrow N^{15}$	1/2	0	1/3	3700	$3700r+1230$	$523r-3018$	1230
$F^{17}\rightarrow O^{17}$	5/2	1	7/5	2250	$2250r+3150$	$-927r-1098$	3150
$Ne^{19}\rightarrow F^{19}$	1/2	1	3	2000	$2000r+6000$	$-1177r+1752$	6000
$Mg^{23}\rightarrow Na^{23}$	3/2	1	5/3	4600	$4600r+7670$	$1423r+3422$	7670
$Si^{27}\rightarrow Al^{27}$	5/2	1	7/5	3700	$3700r+5180$	$523r+932$	5180
$S^{31}\rightarrow P^{31}$	1/2	1	3	4300	$4300r+12900$	$1123r+8652$	12900
$Cl^{33}\rightarrow S^{33}$	3/2	0	3/5	3500	$3500r+2100$	$323r-2148$	2100
$A^{35}\rightarrow Cl^{35}$	3/2	0	3/5	3400	$3400r+2040$	$223r-2208$	2040
$K^{37}\rightarrow Cl^{37}$	3/2	0	3/5	2700	$2700r+1620$	$-477r-2628$	1620
$Ca^{39}\rightarrow K^{39}$	3/2	0	3/5	4000	$4000r+2400$	$823r-1848$	2400
$Na^{21}\rightarrow Ne^{21}$	(3/2)	1	5/3	3600	$3600r+6000$		6000
$Na^{21}\rightarrow Ne^{21}$	(5/2)	1	7/3	3600	$3600r+8400$	not used in	8400
$Al^{25}\rightarrow Mg^{25}$	5/2	1	7/5	(3000)[a]	$3000r+4200$	calculating	4200
$P^{29}\rightarrow Si^{29}$	(1/2)	1	3	3800	$3800r+11400$	r and $\bar{C}$	11400
$Se^{41}\rightarrow Ca^{41}$	(7/2)	1	9/7	2550	$2550r+3280$		3280
$Be^7\rightarrow Li^{7*}$	3/2→1/2		4/3	3600	${}^2P_{\frac{3}{2}}\rightarrow{}^2P_{\frac{1}{2}};\ \|\int 1\|^2=0$		4800
$He^6\rightarrow Li^6$	0→1		6	550	${}^1S_0\rightarrow{}^3S_1;\ \|\int 1\|^2=0$		3300
$C^{10}\rightarrow B^{10*}$	(0→1)		6	2000	${}^1S_0\rightarrow{}^3S_1;\ \|\int 1\|^2=0$		12000
$C^{10}\rightarrow B^{10**}$	(0→0)		0	—	${}^1S_0\rightarrow{}^1S_0;\ \|\int 1\|^2=2$		0
$O^{14}\rightarrow N^{14*}$	(0→0)		0	3300	${}^1S_0\rightarrow{}^1S_0;\ \|\int 1\|^2=2$		0
$F^{18}\rightarrow O^{18}$	(1→0)		2	3700	${}^3S_1\rightarrow{}^1S_0;\ \|\int 1\|^2=0$		7400

( ) assumed value; ( )[a] estimated, $W_0=7.08mc^2$.
$\bar{C}=3177r+4248$; $\langle\Delta^2\rangle_{Av}=(122r^2+143r+892)\times10^4$.
r (for minimum $\langle\Delta^2\rangle_{Av}$) $=0$; minimum $\langle\Delta^2\rangle_{Av}=8.92\times10^6$.
C (for minimum $\langle\Delta^2\rangle_{Av}$) $=4250$; probable error $=2000$.
All ft values were taken from A. M. Feingold, Revs. Modern Phys. **23**, 10 (1951), except for the neutron in which case ft was computed from data given in reference 14, and the mass numbers $A=23$, 27, 31, 33, 37 and 39 for which the ft values were taken from F. I. Boley and D. J. Zaffarano, Phys. Rev. **84**, 1059 (1951).
All spins and moments were taken from J. E. Mack, Revs. Modern Phys. **22**, 64 (1950) except O^{17}—F. Alder and F. C. Yu, Phys. Rev. **81**, 1067 (1951); and S^{33}—moment from Eshbach, Hillger, and Jen, Phys. Rev. **80**, 1106 (1950).

limits and interpreted as α^2 or $1-\alpha^2$ depending on the location of μ. In the uniform model it is assumed that a large number of nucleons contribute in a more or less random fashion to the total orbital angular momentum which is still a constant of motion. As in the single particle model, all spins but one are paired off resulting in a doublet state with a definite value of L (L-S coupling). The orbital gyromagnetic ratio is just the fraction of the orbital angular momentum carried by the protons, or $g_L\sim Z/A$, which has very nearly the value 0.5 for most mirror nuclei. Since this is a many-particle model, states with $L=I+\frac{1}{2}$ and $L=I-\frac{1}{2}$ can have the same parity and hence can enter into a linear combination.

In a few cases (including Li^7 and Na^{23}) $\Delta_{MW\text{-}MW}$ is probably a better measure of the deviations from pure L-S coupling than $\Delta_{S\text{-}MW}$. For Li^7, g_L between 1/3 and 17/42 appears reasonable on theoretical grounds. For Na^{23} the shell model interpretation of the ground state is predominantly ${}^2P_{\frac{3}{2}}$ of even parity compounded from configurations containing 3, 5, and 7 nucleons in $1d$ orbits. The moment diagram indicates a substantial amount of ${}^2D_{\frac{3}{2}}$ component.

III. RESULTS

The numerical analysis and results are shown in Tables II and III for the S-MW and MW-MW interpolation procedures. Table I gives the results of the same computation made on the basis of pure L-S coupling, for comparison purposes. In the three tables, $\sigma=(\langle\Delta^2\rangle_{Av})^{\frac{1}{2}}$ is the standard (root-mean-square) deviation, while $e=0.6745\sigma$ is the probable error. Table IV summarizes the calculations. No experimental uncertainties are listed for the ft products, but it is likely that few of the given values are in error by as much as 20 percent.

Let us first dispose of L-S coupling (Table I). Here the addition of a Fermi-type term makes matters worse in 9 transitions out of 15 and the best value of r is clearly 0. The square of the standard deviation is 8.9×10^6 and $\bar{C}$ is 4250 with a probable error of 2000. About $\frac{2}{3}$ of $\langle\Delta^2\rangle_{Av}$ is contributed by the $S^{31}\rightarrow P^{31}$ transition. Actually $\langle\Delta^2\rangle_{Av}$ depends only slightly on r and $r=0.5$ is only slightly inferior to $r=0$. The good agreement between the H^3 and He^6 transitions for $r=0$ is balanced by the difficulty with the 0→0 interpretation of the $O^{14}\rightarrow N^{14*}$ transition.

In Table II (S-MW interpolation procedure) the need for a Fermi-type term is clearly indicated by 12 out of 15 transitions. However, even with $r=0$ the square of the standard deviation is only 2.0×10^6 (compare with 8.9×10^6 for L-S coupling). Much of this reduction comes from the greatly reduced value of

TABLE II. $C=ft[r|\int 1|^2+|\int \boldsymbol{\sigma}|^2]$ from *S-MW* interpolation procedure.

Transition	I	α^2	$\|\int \boldsymbol{\sigma}\|^2$	ft	C	$\Delta=C-\bar{C}$	C
$n^1\rightarrow H^1$	1/2	1	3	1400	$1400r+4200$	$-1777r+2152$	5360
$He^3\rightarrow H^3$	1/2	1	3	1100	$1100r+3300$	$-2077r+1252$	4210
$Be^7\rightarrow Li^7$	3/2	0.88	1.09	2300	$2300r+2510$	$-877r+462$	4420
$C^{11}\rightarrow B^{11}$	3/2	0.76	0.61	3900	$3900r+2380$	$723r+332$	5620
$N^{13}\rightarrow C^{13}$	1/2	0	1/3	4800	$4800r+1600$	$1623r-448$	5580
$O^{15}\rightarrow N^{15}$	1/2	0	1/3	3700	$3700r+1230$	$523r-818$	4300
$F^{17}\rightarrow O^{17}$	5/2	0.99	1.37	2250	$2250r+3080$	$-927r+1032$	4950
$Ne^{19}\rightarrow F^{19}$	1/2	0.95	2.65	2000	$2000r+5300$	$-1177r+3252$	6960
$Mg^{23}\rightarrow Na^{23}$	3/2	0.65	0.33	4600	$4600r+1520$	$1423r-528$	5340
$Si^{27}\rightarrow Al^{27}$	5/2	0.78	0.57	3700	$3700r+2110$	$523r+62$	5180
$S^{31}\rightarrow P^{31}$	1/2	0.45	0.22	4300	$4300r+950$	$1123r-1098$	4520
$Cl^{33}\rightarrow S^{33}$	3/2	0.20	0.13	3500	$3500r+450$	$323r-1598$	3350
$A^{35}\rightarrow Cl^{35}$	3/2	0.22	0.10	3400	$3400r+340$	$223r-1708$	3160
$K^{37}\rightarrow Cl^{37}$	3/2	0.18	0.16	2700	$2700r+430$	$-477r-1618$	2670
$Ca^{39}\rightarrow K^{39}$	3/2	0.08	0.33	4000	$4000r+1320$	$823r-728$	4640
$Al^{25}\rightarrow Mg^{25}$	5/2	0.80	0.59	(3000)	$3000r+1770$	not used in calculating $\bar{C}$ and r	4260
$Be^7\rightarrow Li^{7*}$	3/2→1/2		$4/3\times 0.88$	3600	$0.88(^2P_{\frac{3}{2}}\rightarrow{}^2P_{\frac{3}{2}})$		4220
$C^{10}\rightarrow B^{10**}$	(0→0)[a]		0	—	$^1S_0\rightarrow{}^1S_0;\ \|\int 1\|^2=2$		—
$O^{14}\rightarrow N^{14*}$	(0→0)		0	3300	$^1S_0\rightarrow{}^1S_0;\ \|\int 1\|^2=2$		5470

( ) assumed value from Table I; He^6, $C^{10}(0\rightarrow 1)$, F^{18} as in Table I.
$\bar{C}=3177r+2048$; $\langle\Delta^2\rangle_{Av}=(122r^2-203+194)\times 10^4$.
r (for minimum $\langle\Delta^2\rangle_{Av}=0.83$; minimum $\langle\Delta^2\rangle_{Av}=1.1\times 10^6$.
$\bar{C}$ (for minimum $\langle\Delta^2\rangle_{Av}=4700$; probable error $=710$.
[a] A weak branch with energy and intensity about right for the 0→0 interpretation has been observed recently (private communication from Professor R. Sherr).

$|\int \boldsymbol{\sigma}|^2$ for the $Si^{31}\rightarrow P^{31}$ transition. The optimum value of r (0.83) cuts $\langle\Delta^2\rangle_{Av}$ nearly in half and gives $\bar{C}=4700$ with a probable error of 700. A more impressive measure of the improvement resulting from the *S-MW* interpolation procedure is shown in the bottom line of Table IV; the square of the relative standard deviation $\langle\Delta^2\rangle_{Av}/\bar{C}^2$ is reduced by a large factor when the optimum amount of Fermi-type coupling is put into the calculation.

Table III shows that the *MW-MW* interpolation procedure results in a substantially better fit to beta-decay theory than pure *L-S* coupling, but is definitely less satisfactory than the generally more plausible *S-MW* procedure. It is perhaps significant that both interpolation procedures yield substantially the same value of $\bar{C}$, slightly smaller than 5000 and nearly the same value for r.

The still sizeable dispersion found in the two mixed

TABLE III. $C=ft[r|\int 1|^2+|\int \boldsymbol{\sigma}|^2]$ from *MW-MW* interpolation procedure.

Transition	I	α^2	$\|\int \boldsymbol{\sigma}\|^2$	ft	C	$\Delta=C-\bar{C}$	C
$n^1\rightarrow H^1$	1/2	1	3	1400	$1400r+4200$	$-1777r+1980$	5350
$H^3\rightarrow He^3$	1/2	1	3	1100	$1100r+3300$	$-2077r+1080$	4200
$Be^7\rightarrow Li^7$	3/2	0.99	1.61	2300	$2300r+3700$	$-877r+1480$	5580
$C^{11}\rightarrow B^{11}$	3/2	0.85	0.96	3900	$3900r+3740$	$723r+1520$	6940
$N^{13}\rightarrow C^{13}$	1/2	0.10	0.13	4800	$4800r+620$	$1623r-1600$	4550
$O^{15}\rightarrow N^{15}$	1/2	0.09	0.13	3700	$3700r+480$	$523r-1740$	3510
$F^{17}\rightarrow O^{17}$	5/2	1	7/5	2250	$2250r+3150$	$-927r+930$	4990
$Ne^{19}\rightarrow F^{19}$	1/2	0.95	2.65	2000	$2000r+5300$	$-1177r+3080$	6940
$Mg^{23}\rightarrow Na^{23}$	3/2	0.74	0.55	4600	$4600r+2530$	$1423r+310$	6300
$Si^{27}\rightarrow Al^{27}$	5/2	0.96	1.24	3700	$3700r+4590$	$523r+2370$	7620
$S^{31}\rightarrow P^{31}$	1/2	0.51	0.36	4300	$4300r+1550$	$1123r-670$	5070
$Cl^{33}\rightarrow S^{33}$	3/2	0.41	0.01	3500	$3500r+50$	$323r-2170$	2920
$A^{35}\rightarrow Cl^{35}$	3/2	0.39	0.00	3400	$3400r+0$	$223r-2220$	2780
$K^{37}\rightarrow Cl^{37}$	3/2	0.40	0.00	2700	$2700r+0$	$-477r-2220$	2210
$Ca^{39}\rightarrow K^{39}$	3/2	0.29	0.03	4000	$4000r+120$	$823r-2100$	3400
$Al^{25}\rightarrow Mg^{25}$	5/2	1	7/5	(3000)	$3000r+4200$	not used in calculating $\bar{C}$ and r	6660
$Be^7\rightarrow Li^{7*}$	3/2→1/2		$0.99\times 4/3$	3600	$0.99(^2P_{\frac{3}{2}}-{}^2P_{\frac{1}{2}})$		4750

( ) assumed value from Table I; He^6, $C^{10}(0\rightarrow 1)$, F^{18} as in Table I; $C^{10}(0\rightarrow 0)$, O^{14} as in Table II.
$\bar{C}=3177r+2220$; $\langle\Delta^2\rangle_{Av}=(122r^2-199r+338)\times 10^4$.
r (for minimum $\langle\Delta^2\rangle_{Av}=0.82$; minimum $\langle\Delta^2\rangle_{Av}=2.6\times 10^6$.
$\bar{C}$ (for minimum $\langle\Delta^2\rangle_{Av}=4800$; probable error $=1100$.

TABLE IV. Summary of results from Tables I–III.

	L–S coupling		S–MW for α^2		MW–MW for α^2	
r	0	1	0	1	0	1
$\langle\Delta^2\rangle_{Av}$	8.9×10^6	11.6×10^6	1.9×10^6	1.1×10^6	3.4×10^6	2.6×10^6
$\bar{C}$	4250	7420	2050	5220	2220	5400
$\langle\Delta^2\rangle_{Av}/\bar{C}^2$	0.49	0.21	0.45	0.041	0.70	0.089

state models can be assigned to three causes in addition to experimental errors involved in the ft values. One is the fact that in making the calculations, the same mixing of states was tacitly assumed in parent and daughter nuclei. This is the expected relation from the symmetry principle on which the supermultiplet analysis is founded; but it may not be exactly correct. The second, and probably more important, factor is that the wave functions are in any case not exact. The true wave functions probably contain quartet and even higher spin multiplet terms, which may make fairly strong contributions to $|\int\boldsymbol{\sigma}|^2$. Such additional terms may possibly not produce serious alteration in the values of either $\bar{C}$ or r in virtue of the averaging process by which these values are computed.

Actually there is evidence,[14] based on calculations with tensor forces, that the ground states of H^3 and He^3 contain an admixture of a $^4D_{\frac{1}{2}}$ component with a statistical weight of 4 percent. The beta-decay matrix element for the $^4D_{\frac{1}{2}}$ component is identical with that of $^2P_{\frac{1}{2}}$, so that Eq. (13) can be used without change if $1-\alpha^2$ is identified with the statistical weight of the $^4D_{\frac{1}{2}}$ component.[15]

A third and perhaps most important source of error is the probable inaccuracy of the interpolation procedures used to determine the amount of mixing. Recognition of this possibility is helpful in restoring some freedom of action. To discover in which direction to go consider $ft|M|^2$ for $He^6\rightarrow Li^6$ from Table I. This value,[16] 3300, may be compared with $\bar{C}$ computed from Table II:

$$\bar{C}=2050,\quad r=0$$
$$=5220,\quad r=1.$$

To bring $\bar{C}$ into agreement with the He^6 transition requires $r^2\sim0.5$ (or $\bar{C}\sim3600$). Similarly $ft|M|^2$ for $O^{14}\rightarrow N^{14*}$ is reduced to 3300 for $r=0.5$ in line with $\bar{C}$ and the He^6 transition. These considerations suggest seeking a compromise solution with $r=0.5$ and α^2 adjusted slightly from the S-MW values to improve the over-all constancy of $ft|M|^2$. Actually the adjustments, shown in Table V, are generally rather small.

One may conclude that a variety of errors, both theoretical and experimental, tend to an overestimate of r in the analysis of Tables II and III. A compromise solution with $r=0.5$ fits all the data fairly well except for $n^1\rightarrow H^1$, $C^{10}(0)\rightarrow B^{10*}(1)$, $F^{18}(1)\rightarrow O^{18}(0)$ and $Ne^{19}\rightarrow F^{19}$. The first exception may be improved by a substantial reduction in the measured half-life of the neutron just outside the present reported limits of error.[17] The second can be interpreted in terms of a linear combination $^3S_1+^3D_1+^1P_1$ for the excited state of B^{10} with $^3S_1$ having a statistical weight of 30 percent. This makes the lowest $I=1$ state of B^{10} intermediate in mixing properties between the ground states of Li^6 (almost pure $^3S_1$) and N^{14} (relatively little $^3S_1$). A similar explanation fits the F^{18} transition. The fourth exception may perhaps indicate a small error in the measurement of the transition energy.

The end results are values for G_F^2 and G_{GT}^2. Computed from Table II on the basis of the S-MW interpolation procedure they are

$$G_{GT}^2=(1.16\pm0.15)\pm10^{-23},$$
$$G_F^2=(0.96\pm0.13)\pm10^{-23}. \tag{17}$$

The error is simply the statistical error in the evaluation of $\bar{C}$. The compromise solution yields what are thought to be more reliable values

$$G_{GT}^2=1.65\times10^{-23},\quad G_F^2=0.82\times10^{-23}. \tag{18}$$

The compromise ratio, $r=0.5$, is just outside the upper limit derived by Moszkowski from an analysis

TABLE V. Compromise solution: $r=0.5$ and α^2 adjusted for best fit.

Transition	I	α^2 (S-MW)	α^2 (adjusted)	$\lvert\int\boldsymbol{\sigma}\rvert^2$	$\lvert M\rvert^2$	$ft\lvert M\rvert^2$
$n^1\rightarrow H^1$	1/2	1	1	3	3.5	4900
$H^3\rightarrow He^3$	1/2	1	0.95	2.61	3.11	3400
$Be^7\rightarrow Li^7$	3/2	0.85	0.85	0.96	1.46	3350
$C^{11}\rightarrow B^{11}$	3/2	0.76	0.68	0.40	0.90	3500
$N^{13}\rightarrow C^{13}$	1/2	0	0.05	0.21	0.71	3400
$O^{15}\rightarrow N^{15}$	1/2	0	0	0.33	0.83	3100
$F^{17}\rightarrow O^{17}$	5/2	0.99	0.95	1.17	1.67	3800
$Ne^{19}\rightarrow F^{19}$	1/2	0.95	0.90[a]	2.35	2.85	5700
$Na^{21}\rightarrow Ne^{21}$	(3/2)	—	0.70	0.45	0.95	3400
$Na^{21}\rightarrow Ne^{21}$	(5/2)	—	0.75	0.46	0.96	3400
$Mg^{23}\rightarrow Na^{23}$	3/2	0.65	0.60	0.22	0.72	3300
$Al^{25}\rightarrow Mg^{25}$	5/2	0.80	0.80	0.59	1.09	3300
$Si^{27}\rightarrow Al^{27}$	5/2	0.78	0.73	0.40	0.90	3300
$P^{29}\rightarrow Si^{29}$	(1/2)	—	0.52	0.39	0.89	3400
$P^{29}\rightarrow Si^{29}$	(3/2)	—	0.07	0.40	0.90	3400
$Si^{31}\rightarrow P^{31}$	1/2	0.45	0.48	0.28	0.78	3400
$Cl^{33}\rightarrow S^{33}$	3/2	0.20	0.10	0.32	0.82	2900
$A^{35}\rightarrow Cl^{35}$	3/2	0.22	0.12[b]	0.28	0.78	2700
$K^{37}\rightarrow A^{37}$	(3/2)	—	0.00	0.60	1.10	3000
$Ca^{39}\rightarrow K^{39}$	3/2	0.08	0.08	0.33	0.83	3300
$Sc^{41}\rightarrow Ca^{41}$	(7/2)	—	0.88	0.79	1.29	3300
$Be^7\rightarrow Li^{7*}$	3/2→1/2	$x(^2P_{\frac{3}{2}}\rightarrow{}^2P_{\frac{1}{2}})$		$4/3x$	$4/3x$	$4800x$[c]
$He^6\rightarrow Li^6$	0→1	$^1S_0\rightarrow{}^3S_1$		6	6	3300
$C^{10}\rightarrow B^{10*}$	(0→1)	$y(^1S_0\rightarrow{}^3S_1)$		$6y$	$6y$	$12000y$[d]
$C^{10}\rightarrow B^{10**}$	(0→0)	$^1S_0\rightarrow{}^1S_0$		0	1	—
$O^{14}\rightarrow N^{14*}$	(0→0)	$^1S_0\rightarrow{}^1S_0$		0	1	3300
$F^{18}\rightarrow O^{18}$	(1→0)	$z(^3S_1\rightarrow{}^1S_0)$		$2z$	$2z$	$7400z$[e]

( ) assumed value.
[a,b]—a good fit would require excessively large adjustment.
[c]—$x\sim0.7$ consistent with 85 percent $^2P_{3/2}$ and 85 percent $^2P_{1/2}$.
[d]—$y\sim0.3$ consistent with 30 percent $^3S_1$ in the linear combination $^3S_1+^3D_1+^1P_1$.
[e]—$z\sim0.45$ consistent with 45 percent $^3S_1$ in the linear combination $^3S_1+^3D_1+^1P_1$.

[14] H. Feshbach and R. L. Pease, private communication.
[15] Private communication from Dr. E. Feenberg.
[16] Based on data reported by V. Perez-Mendez and H. Brown, Phys. Rev. **77**, 404 (1950).
[17] J. M. Robson, Phys. Rev. **83**, 349 (1951).

using only a small part of the available data.[18] Equation (17) is roughly consistent with the Wigner-Critchfield interaction containing equal weights of the scalar, axial vector, and pseudoscalar combinations. However, recent analyses[19,20] seem to favor the tensor over the axial vector. In any event tentative conclusions that the correct interaction is a pure Gamow-Teller type should be carefully re-examined.

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APPENDIX. DERIVATION OF EQUATION (13)

If a vector operator $\mathbf{Q}$ satisfies the commutation relations

$$[I_x, Q_x]=0, \quad [I_x, Q_y]=iQ_z, \quad [I_x, Q_z]=iQ_y,$$

and others obtained from these by cyclic permutation of subscripts, then the general theory of angular momentum gives[21]

$$(\gamma' Im'|\mathbf{Q}|\gamma Im) = (Im'|\mathbf{I}|Im)(\gamma' Im|\mathbf{I}\cdot\mathbf{Q}|\gamma Im)/[I(I+1)], \quad \text{(A1)}$$

where the first factor on the right is independent of γ and the last independent of m; γ and γ' represent all the additional quantum numbers necessary to specify the state completely. Since the operators $\mathbf{Y}_\xi \pm i\mathbf{Y}_\eta$ are of this type, the rule (A1) can be applied to the matrix elements in Eq. (6), giving

$$\left|\int\sigma\right|^2=\frac{1}{I^2(I+1)^2}\sum_{m'}|(Im'|\mathbf{I}|Im)|^2|\alpha^2(T_\xi' LSI|\mathbf{I}\cdot(\mathbf{Y}_\xi\pm i\mathbf{Y}_\eta)|T_\xi LSI)+(1-\alpha^2)(T_\xi' L'SI|\mathbf{I}\cdot(\mathbf{Y}_\xi\pm i\mathbf{Y}_\xi)|T_\xi L'SI)|^2, \quad \text{(A2)}$$

where only the parameters on which a matrix element depends have been retained. But

$$\sum_{m'}|(Im'|\mathbf{I}|Im)|^2=(Im|\mathbf{I}^2|Im)=I(I+1),$$

so Eq. (A2) becomes

$$\left|\int\sigma\right|^2=\frac{1}{I(I+1)}|\alpha^2(T_\xi' LSI|\mathbf{I}\cdot(\mathbf{Y}_\xi\pm i\mathbf{Y}_\eta)|T_\xi LSI) +(1-\alpha^2)(T_\xi' L'SI|\mathbf{I}\cdot(\mathbf{Y}_\xi\pm i\mathbf{Y}_\eta)|T_\xi L'SI)|^2. \quad \text{(A3)}$$

It is now possible to make use of an important feature of the supermultiplet formalism, namely, that the values of the matrix elements of any operator connecting only states within a supermultiplet depend only on the quantum numbers of the states involved and the character of the supermultiplet. It is thus possible to calculate these matrix elements using functions and operators appropriate to the simplest configuration capable of generating the supermultiplet. For the nuclei involved in the image transitions, these are either one- or three-particle functions. In the one-particle case, the calculation can be carried out without recourse to explicit functions, as shown below; the final results are the same for the three-particle case, as has been verified by direct calculation.

For one particle, the operator $\mathbf{I}\cdot(\mathbf{Y}_\xi\pm i\mathbf{Y}_\eta)$ takes the form

$$\begin{aligned}\mathbf{I}\cdot(\mathbf{Y}_\xi\pm i\mathbf{Y}_\eta)&=(\mathbf{L}+\mathbf{S})\cdot\mathbf{S}(\tau_\xi\pm i\tau_\eta)\\&=(\mathbf{L}\cdot\mathbf{S}+\mathbf{S}^2)(\tau_\xi\pm i\tau_\eta).\end{aligned} \quad \text{(A4)}$$

But

$$\mathbf{L}\cdot\mathbf{S}=1/2(\mathbf{I}^2-\mathbf{L}^2+\mathbf{S}^2),$$

so Eq. (A4) becomes

$$\mathbf{I}\cdot(\mathbf{Y}_\xi\pm i\mathbf{Y}_\eta)=(T_\xi\pm iT_\eta)(\mathbf{I}^2-\mathbf{L}^2+\mathbf{S}^2).$$

Insertion of this into Eq. (A3) gives

$$\left|\int\sigma\right|^2=\frac{1}{I(I+1)}|(T_\xi'|T_\xi \pm iT_\eta|T_\xi)|^2|\alpha^2(LSI|\mathbf{I}^2-\mathbf{L}^2+\mathbf{S}^2|LSI) +(1-\alpha^2)(L'SI|\mathbf{I}^2-\mathbf{L}^2+\mathbf{S}^2|L'SI)|^2. \quad \text{(A5)}$$

Now for the mirror transitions,

$$|(T_\xi'|T_\xi\pm iT_\eta|T_\xi)|^2=1;$$

moreover, the partial wave functions are chosen to be eigenfunctions of $\mathbf{I}^2$, $\mathbf{L}^2$, and $\mathbf{S}^2$. Thus Eq. (A5) becomes

$$\left|\int\sigma\right|^2=\frac{1}{I(I+1)}|\alpha^2(I(I+1)-L(L+1)+S(S+1)) +(1-\alpha^2)(I(I+1)-L'(L'+1)+S(S+1))|^2.$$

Insertion into this of the assumed values $S=\frac{1}{2}$, $L=I-S$, $L'=I+S$ gives just Eq. (13).

[18] S. A. Moszkowski, Phys. Rev. **82**, 118 (1951).
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[20] L. M. Langer and R. J. D. Moffat, Phys. Rev. **82**, 635 (1951).
[21] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1935), p. 61.