have actually calculated the expectation value of a too singular operator, we expect that in a consistent theory the exchange effects will be even smaller than our present results. It remains to be seen, however, whether a completely covariant description of the two nucleon system[19] will corroborate the point of view advocated here.

In concluding this paper, the author wishes to thank Dr. Henry Primakoff, and also Dr. H. Snyder and Dr. M. Newman of the Brookhaven National Laboratory, for interesting discussions. It is also a pleasure to acknowledge the hospitality of the Brookhaven National Laboratory, where most of this manuscript was prepared during a visit in August, 1951.

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The Total Neutron Cross Section of Nitrogen*

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The total cross section of nitrogen has been measured for neutrons in the energy range 200 to 1800 kev, using scatterers of liquid nitrogen and of lithium azide. Eleven resonances were found, corresponding to excited states in the compound nucleus N^{15}, with natural widths of from 3 to 54 kev. Nine of these can be identified with resonances previously known in the reactions $N^{14}(n,p)C^{14}$, $N^{14}(n,\alpha)B^{11}$, or $C^{14}(p,n)N^{14}$. From a comparison of the measured and computed elastic scattering cross sections, J-values can be assigned to most of the observed states. In some cases the parity also can be determined and the effective level spacing D_J computed for decay by neutron, proton, or α-particle emission. At least for proton emission, it must be concluded that the quantities D_J, which contain implicitly the matrix elements for the transition, can vary by a factor of at least 100.

I. INTRODUCTION

The virtual excited states of N^{15} offer an excellent opportunity to study, with present experimental techniques, competitive reactions in a light nucleus. Three different reactions, $n+N^{14}$, $p+C^{14}$ and $\alpha+B^{11}$, lead to virtual levels of N^{15}. Furthermore, the levels of N^{15} become virtual with respect to proton, neutron, and α-particle emission at roughly the same excitation energies: 10.21, 10.83 and 10.99 Mev, respectively.[1] Consequently, with electrostatic generators capable of 3- or 4-million volts, one is able to study the virtual levels in the region above 11 Mev by observing twelve different reactions. Of these, the (p,α), (α,p), (p,p), (α,α), (α,γ), and (p,γ) have thus far not been investigated. The reactions (n,p), (p,n), (n,α), (α,n), (n,γ), and (n,n) have been studied with different degrees of resolution and over various ranges of excitation energy.[2]

In particular, the two inverse reactions, $N^{14}(n,p)C^{14}$ and $C^{14}(p,n)N^{14}$, have been studied with fairly good resolution. Johnson and Barschall[3] measured the absolute cross section for the (n,p) and (n,α) processes and found three strong resonances, with indications of several weaker ones, in the neutron energy range 0.2 to 2.0 Mev. The relative neutron yield from the inverse (p,n) reaction has been measured over this same range of excitation energies by Roseborough *et al.*[4] Using somewhat better resolution, they found nine clearly defined resonances. In general these two investigations are in good agreement, the existing differences being ascribed to the degree of resolution employed.

With these data available, it was thought that a measurement of the total neutron cross section with good resolution would be of considerable interest. From the total neutron cross section and the (n,p) and (n,α) cross sections, one can deduce the elastic neutron scattering cross section, a quantity which is interpreted with some reliability by present nuclear theory. One can then hope to obtain a more detailed picture of the process by assigning total angular momentum and parity values to the virtual states and angular momentum values to the reacting particles.

II. EXPERIMENTAL METHOD

The $Li^7(p,n)Be^7$ reaction was used to produce monoenergetic neutrons of variable energy. Protons of well-defined energy were obtained from the Rockefeller electrostatic generator. After acceleration the protons are analyzed by a ninety-degree deflection in a magnetic field that is both controlled and measured by a proton magnetic moment resonance device. The generator voltage is stabilized by a variable corona load which is driven by an error signal produced by the proton beam

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[1] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).

[2] For a summary see Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).

[3] C. H. Johnson and H. H. Barschall, Phys. Rev. **80**, 818 (1950).

[4] Roseborough, McCue, Preston, and Goodman, Phys. Rev. **83**, 1133 (1951).

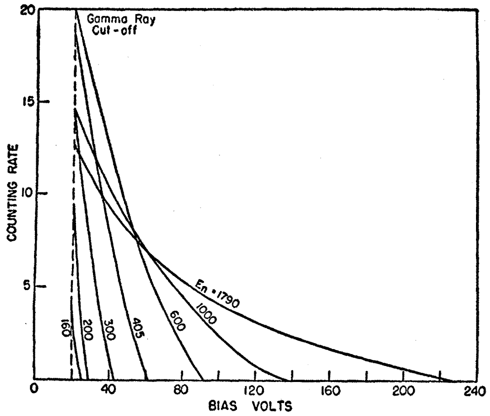

FIG. 1. Relative response of the propane counter to neutrons of energy E_n, as a function of the discriminator bias voltage.

at the exit slits of the magnetic analyzer. Tests have shown that with the entrance and exit slits of the analyzer set at 1.0-mm width, the effective energy spread is approximately 0.075 percent. With this energy definition several microamperes of beam current are available. By narrowing the slits better energy definition is obtained but at the expense of beam current. The voltage scale of the generator is based on the $Li^7(p,n)$ threshold which is taken to be 1.8822 ± 0.0019 Mev as determined by Herb *et al.*[5]

Thin targets of lithium were prepared by evaporation of the metal in vacuum onto 10-mil tantalum disks. Target preparation was carried out in the target assembly of the generator. To prevent deterioration under proton bombardment the targets were rotated eccentrically to the proton beam and cooled by an air jet sprayed on the outside of the tantalum backing. Targets of approximately 10-kev thickness were measured by the previously described "rise" method.[6] The thickness of thinner targets was estimated from the ratio of yields, as measured with a BF_3 long counter, at a proton energy where the $Li^7(p,n)$ cross section is not changing rapidly (3.0 Mev). This method is subject to some uncertainty because of the possibility of target oxidation. Therefore, for the thinnest targets, the overall resolution was established by measuring the width of the sharp resonance at 585 kev in the total neutron cross section of sulfur.

It is now known that the residual nucleus, Be^7, has an excited state at 434 kev which gives rise to a second group of neutrons from the $Li^7(p,n)$ reaction when the energy of the ground-state group is greater than 640 kev. Hence, for the region above 640 kev, the value of the total cross section obtained is a weighted average of the cross section at two different neutron energies which depends both on the relative intensities of the two groups and the efficiency of the detector for the two neutron energies. Fortunately, the intensity of the second group is low; several investigations of the neutron spectrum have shown that for ground-state neutron energies ranging from 900 to 2250 kev the relative intensity of the second group is about 10 percent.[7]

We have endeavored to eliminate the ambiguity resulting from the presence of the two groups by the use of a neutron detector which is made insensitive to the lower energy group. This detector, a proton recoil proportional counter, was constructed by filling a brass cylinder one inch in diameter and four inches effective length with propane gas to a pressure of 55 cm. The counter was operated with 2700 volts on the 4-mil center wire. Pulses were amplified on a four-tube preamplifier and Model 204-B (A.I.C.) amplifier. A typical set of integral bias curves at different neutron energies is shown in Fig. 1. With recourse to these curves, the bias was adjusted to discriminate against the second, lower energy group. The disadvantage of this practice is the reduction in the counter efficiency, especially at higher neutron energies where the relative difference in energy of the two groups decreases. The background counting rate of the counter, which was determined by cutting off the direct neutron flux with a long, thin paraffin cone, varied from $\frac{1}{2}$ to 2 percent of the counting rate with the cone removed.

The first cross-section measurements were carried out with a liquid nitrogen scatterer. The experimental arrangement is shown in Fig. 2. A Dewar flask constructed of glass of 1 mm thickness served as container for the liquid nitrogen. To reduce the rate of evaporation, the flask was silvered except for a sight-line along the axis. The liquid nitrogen used in the experiment is judged by its vendors[8] to have a purity of 99 percent or better. The scatterer thickness, nt, is 0.521×10^{24} nuclei/cm^2, where the density is taken to be 0.808 g/cm^3.

A certain amount of complication results from the fact that neutrons are scattered by the glass of the Dewar flask. The following procedure was used to obtain the cross section. First the counting rate, n_0, with the flask in place but empty was measured. Next, the counting rate, n_b, with the flask full of water was obtained. This thickness of water transmits essentially no neutrons and therefore n_b is interpreted as the counting rate caused by the neutron background plus neutrons scattered into the detector by the glass of the flask. The n_b ranged from 3 to 5 percent of n_0. Finally, the counting rate, n, with liquid nitrogen in the flask was taken. The cross section and its statistical error

[5] Herb, Snowden, and Sala, Phys. Rev. **75**, 246 (1949).
[6] Hanson, Taschek, and Williams, Revs. Modern Phys. **21**, 635 (1949).
[7] Johnson, Laubenstein, and Richards, Phys. Rev. **77**, 413 (1950). Freier, Rosen, and Stratton, Phys. Rev. **79**, 721 (1950). B. Hamermesh and V. Hummel, Phys. Rev. **78**, 73 (1950). P. H. Stelson, Ph.D. thesis, MIT (1950).
[8] Supplied by the Low Temperature Laboratory, MIT.

were calculated as

$$\sigma=\frac{1}{nt}\ln\left(\frac{1}{T}\right),\quad \frac{\Delta\sigma}{\sigma}=\frac{(\Delta T/T)}{\ln(1/T)},$$

where $T=(n-n_b)/(n_0-n_b)$. The neutron flux was monitored with a BF_3 long counter placed 2 meters away from the target at 90° to the proton beam.

A second, independent measurement of the neutron cross section of nitrogen was carried out using as a scatterer the compound lithium azide (LiN_3). This compound, although difficult to prepare and handle, is quite satisfactory from a nuclear viewpoint since the relative percentage of nitrogen nuclei is favorable and the neutron cross section of lithium is particularly simple. The lithium azide was prepared by Dr. T. T. Magel of MIT and his analysis indicated that it had a purity of at least 98.5 percent. Since lithium azide is hydroscopic, the scatterer was made by sealing the compound in a thin-walled (5-mil) steel cylinder one inch in diameter. A scatterer thickness of 0.116×10^{24} molecules/cm² was used. The scatterer and the propane counter were placed at mean distances of 12 and 28 cm, respectively, from the target.

A calculation of the correction to the cross section resulting from neutrons scattered into the detector by the scatterer indicates that it is less than 1 percent. This correction was not applied since, in general, the statistical error in the cross section is about 3 percent and since there are uncertainties in its calculation because of the unknown angular distribution of the scattered neutrons and the effect of multiple scattering.

The total neutron cross section of lithium was measured using metallic lithium scatterers. Our values agree well with those of Adair,[9] except that we find the resonance at 256 kev to be somewhat larger. This experiment has been reported.[10]

The total neutron cross section of nitrogen, measured respectively with scatterers of liquid nitrogen and lithium azide, is shown in Figs. 3 and 4. The agreement is in general close. With the azide a very narrow resonance, 5A, was discovered which had been overlooked in the work with liquid nitrogen. Data for each resonance, derived from these curves, have been entered in Table I. In this table, columns 2 and 3 give the resonance energies in the laboratory (E_R) and center of mass (E_R') systems, corrected for target thickness. Wherever the position of the maximum is shifted appreciably by interference with potential scattering, the resonance energy was determined by fitting a theoretical curve.

Column 4 of Table I gives values of the (n,p) and (n,α) cross sections at each resonance based essentially on the absolute measurements of Johnson and Barschall,[3] but with corrections and additional estimates for the weak resonances based on Roseborough *et al.*[4]

[9] R. K. Adair, Phys. Rev. **79**, 1018 (1950).
[10] P. H. Stelson and W. M. Preston, Phys. Rev. **84**, 162 (1951).

Column 4 shows also S_t, the total resonant cross section for neutrons measured in the present work. Experimentally, the definition of S_t is somewhat arbitrary. In the energy range under consideration, resonances produced by s-neutrons show a pronounced interference "dip" at energies below resonance, and for these S_t(exp) has been defined as the difference between the maximum and minimum values of the total cross section. For p-neutrons the "dip" is much smaller and it is negligible for higher angular momenta. In these cases, S_t(exp) has been taken as the height of the maximum value of σ_t, above the estimated "background" level. The values given are in some cases averages of the data from both liquid nitrogen and LiN_3 scatterers. Corrections have been applied to the measured values of S_t to take account of the finite value of the resolution employed.

Column 5 of Table I gives $S_{n,n}$(exp), the experimental value of the resonant elastic scattering cross section obtained by subtracting from S_t(exp) the appropriate values of the (n,p) and (n,α) cross sections.

Column 6 gives the natural resonance width, Γ, in the laboratory reference system, calculated from the relation $\Gamma^2=\Gamma_m^2-\Delta^2$, in which Γ_m is the measured width of a resonance and Δ is the resolution width. Including the effects of target thickness, proton energy spread, and variation of neutron energy with angle, the resolution width was 3.8 kev for the measurement of resonance No. 1 and about 5 kev for the remainder of the work with LiN_3; it was 11 kev at resonances Nos. 9 and 10, covered only with the liquid nitrogen scatterer. In cases where it was deemed more accurate, Γ was taken from the work on the $C^{14}(p,n)$ reaction.[4] There is a discrepancy in the data for the narrow resonance No. 1 with the two scatterers. We believe the results with LiN_3 are more reliable, since in this case the experimental resolution was determined directly by an independent measurement of the even sharper resonance at 585 kev in sulfur, for which $\Gamma=1.3$ to 1.5 kev.[11] In the earlier work with liquid nitrogen the resolution width, the result largely of the target thickness, was estimated by the neutron yield method. Oxidation or diffusion, after running for some hours,

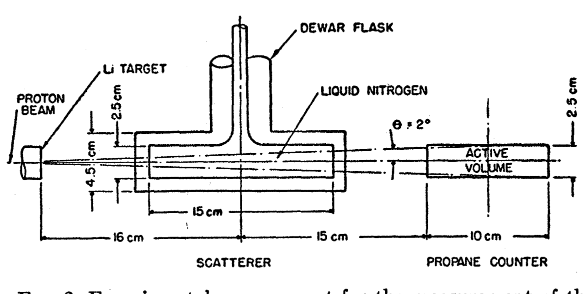

FIG. 2. Experimental arrangement for the measurement of the total neutron cross section of nitrogen. The scatterer is liquid nitrogen in a special Dewar flask.

[11] Peterson, Barschall, and Bockelman, Phys. Rev. **79**, 593 (1950).

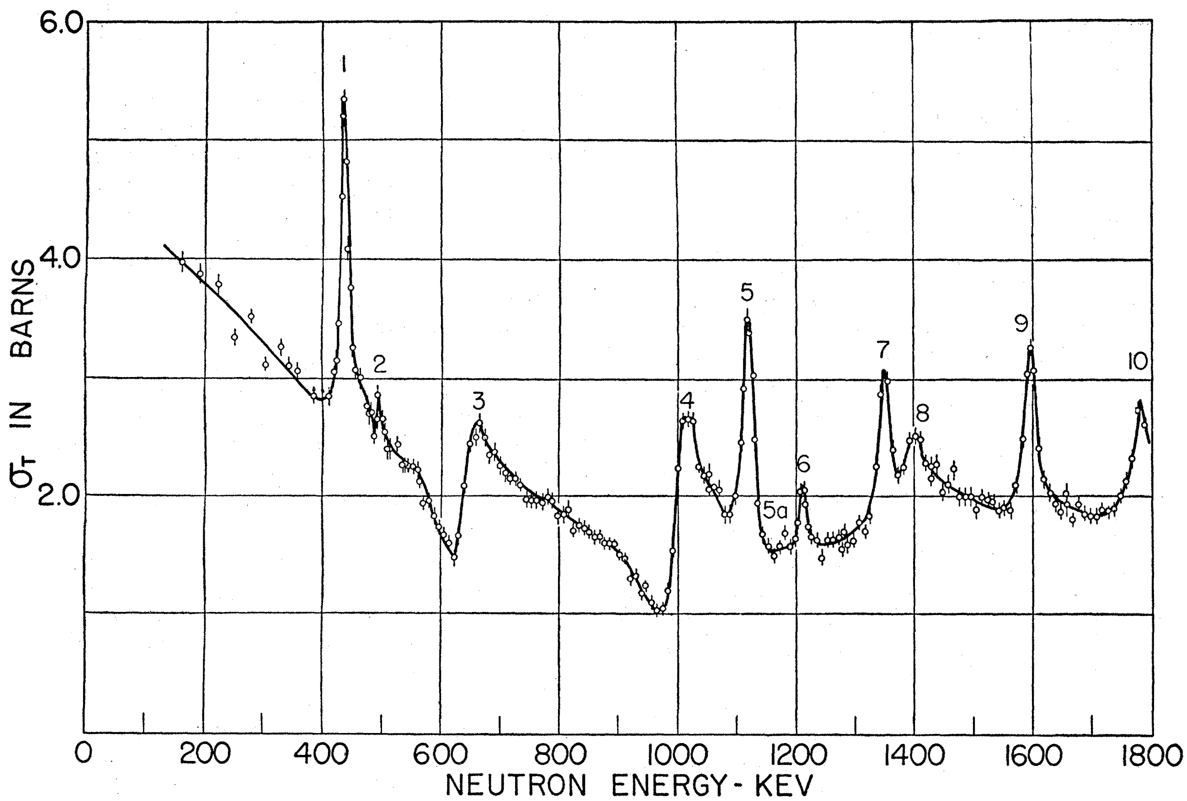

FIG. 3. Total neutron cross section of nitrogen *vs* incident neutron energy in the laboratory coordinate system. Scatterer: liquid nitrogen.

may have significantly increased the effective target thickness.

III. THEORETICAL CROSS SECTION CALCULATIONS

We have used the theory of nuclear reactions as formulated by Feshbach, Peaslee, and Weisskopf,[12] by Blatt and Weisskopf,[13] and by Feld *et al.*[14] The single-level Breit-Wigner formula gives

$$S_J{}^l(a,b)=4\pi\lambdabar_a{}^2(2l+1)g_J{}^l\Gamma_a{}^l\Gamma_b{}^{l'}/\Gamma^2, \quad (1)$$

where $S_J{}^l(a,b)$ is the maximum value of the cross section for the reaction: particle a (angular momentum l, spin i, wavelength $2\pi\lambdabar_a$ in the center-of-mass coordinate system)+nucleus (spin I)$\rightarrow$compound nucleus (in a state of spin J)$\rightarrow$particle b (angular momentum l') +residual nucleus (state with spin I'). $\Gamma_a{}^l$ and $\Gamma_b{}^{l'}$ are the partial widths for emission of particles a and b, respectively, from the compound nucleus, and Γ is the total width of the resonance J. The statistical weight factor is

$$g_J{}^l=(2J+1)/[(2i+1)(2I+1)(2l+1)]. \quad (2)$$

It is evident that the important information concerning the nucleus is contained in the partial widths. Theory suggests that these be broken down into two factors,

$$\Gamma_a{}^l=T_a{}^l(D_J{}^a/2\pi), \quad \Gamma_b{}^{l'}=T_b{}^{l'}(D_J{}^b/2\pi), \quad (3)$$

where $T_a{}^l$ is the probability that an incoming particle will penetrate the centrifugal and Coulomb potential barriers and enter the nucleus. In principle, it is a calculable quantity for neutrons and charged particles. The quantity D_J represents the basic nuclear factor and is related to Wigner's "reduced" width, "γ^2".[15] The statistical theory indicates that D_J should be approximately equal to the average spacing between levels of the compound nucleus, in the neighborhood of the resonance under consideration, and of the same spin J and parity π. However, little is known about the variations in D_J.

[12] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).

[13] J. M. Blatt and V. F. Weisskopf, Laboratory for Nuclear Science and Engineering, MIT, Cambridge, Massachusetts, Technical Report No. 42.

[14] Feld, Feshbach, Goldberger, Goldstein, and Weisskopf, "Final report of the fast neutron data project," USAEC Report No. NYO-636, January 31, 1951.

[15] E. P. Wigner, Am. J. Phys. **17**, 99 (1949). Our $D_J=(\pi/\lambdabar_0)\gamma^2$, where $\lambdabar_0$ is the particle wavelength inside the nucleus.

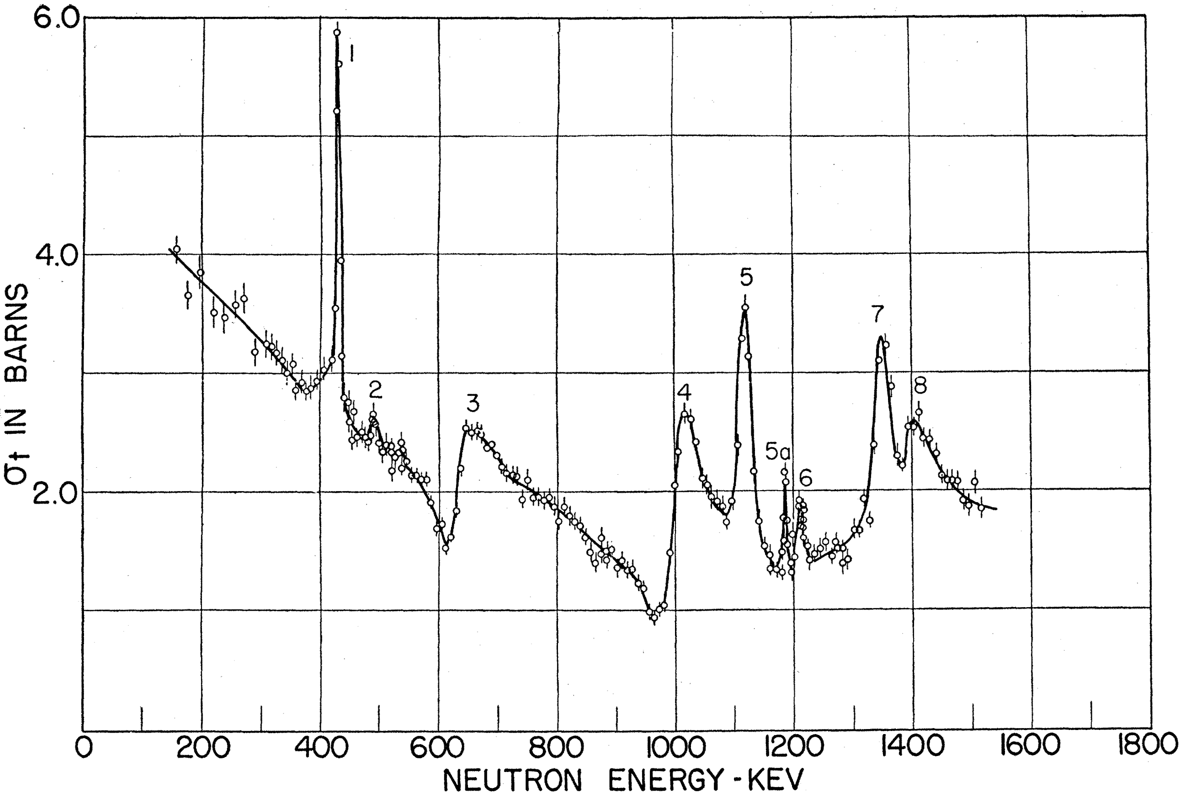

FIG. 4. Total neutron cross section of nitrogen *vs* incident neutron energy in the laboratory coordinate system. Scatterer: lithium azide.

The quantities $T_n{}^l$ for neutrons are computed from the formula

$$T_n{}^l = \frac{4xx_0}{x_0{}^2|v_l|^2 + x^2|v_l'|^2 + 2xx_0} \tag{4}$$

$$\begin{aligned} |v_0|^2 &= 1 & |v_0'|^2 &= 1 \\ |v_1|^2 &= (1+x^2)/x^2 & |v_1'|^2 &= (1-x^{-2})^2 + x^{-2} \\ |v_2|^2 &= (9+3x^2+x^4)/x^4 & |v_2'|^2 &= (1-6x^{-2})^2 + (6x^{-3}-3x^{-2})^2 \\ |v_3|^2 &= (225+45x^3+6x^4+x^6)/x^6 & |v_3'|^2 &= (1-21x^{-2}+45x^{-4})^2 + (45x^{-3}-6x^{-1})^2. \end{aligned} \tag{5}$$

The quantity $x = R/\lambda_n$, and except where noted we have used the simple relation for the radius of nuclear interaction, $R = 1.5A^{\frac{1}{3}} \times 10^{-13}$ cm, for calculations involving neutrons or protons (A is the atomic weight). $x_0 = R(1/\lambda_n{}^2 + 1/\lambda_0{}^2)^{\frac{1}{2}}$ where λ_0 is the wavelength of a neutron in the nucleus. We have used the value $\lambda_0 = 1 \times 10^{-13}$ cm, corresponding to a kinetic energy in the nucleus of about 20 Mev.[12] In Fig. 5 we have plotted $T_n{}^0$, $T_n{}^1$, and $T_n{}^2$ as functions of neutron energy in the center-of-mass system, for N^{15}.

The quantities $T_p{}^l$ for protons have been calculated using a formula identical with Eq. (4). The quantities $|v_l|^2$ and $|v_l'|^2$ are defined for charged particles by

$$|v_l|^2 = F_l^2 + G_l^2, \quad |v_l'|^2 = F_l'^2 + G_l'^2. \tag{6}$$

The definitions of the functions F_l, F_l', G_l, and G_l' are given in Bloch *et al.*,[16] and they have been calculated, using tables of Coulomb wave functions,[17] for the case of protons incident on C^{14}, and for values of l up to 3. The corresponding values of $T_p{}^l$ are plotted in Fig. 5.

The cross section for potential scattering of neutrons is given by

$$\sigma_{\text{pot}}{}^l = 4\pi\lambda_n{}^2(2l+1)\sin^2\delta_l \tag{7}$$

and the phase constants δ_l are given by

$$\begin{aligned} \delta_0 &= x, \\ \delta_1 &= x - \tfrac{1}{2}\pi + \cot^{-1}x, \\ \delta_2 &= x - \pi + \cot^{-1}[(x^2-3)/3x]. \end{aligned} \tag{8}$$

[16] I. Bloch, *et al.*, Phys. Rev. **80**, 553 (1950).
[17] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).

TABLE I. Summary of data relating to N^{15} resonances, derived from neutron reactions with N^{14}. Energies are expressed in kev and cross sections in barns. Columns 2 and 3 give the resonance energies, in the laboratory and center-of-mass systems, respectively. Columns 4 and 5 give experimental values of the maximum cross sections, and column 6 the observed resonance width. For each assumed spin and parity assignment, J, π in column 8, the succeeding columns list: calculated values of $\gamma=\Gamma_n/\Gamma$ and of the resonant scattering cross section $S_{n,n}$; required values for the orbital angular momenta l_n, l_p, and l_α for particle emission; and the corresponding effective levels pacings D^n, D^p, and D^α. If the parity of the ground state of N^{14} is taken as odd, instead of even, the parity of each state in N^{15} (formed by neutrons of a given l_n) is reversed. The corresponding values of l_p, D^p, l_α, and D^α are shown in italics.

(1) No.	(2) E_R	(3) E_R'	(4) Experimental cross sections	(5) Exptal. $S_{n,n}$	(6) Γ	(7) $4\pi\lambda\!\!\!^{-}{}_n^2$	(8) J, π	(9) γ	(10) Calc. $S_{n,n}$	(11) l_n	(12) D^n	(13) l_p	(14) D^p	(15) l_α	(16) D^α
1	430	401	$S_{n,p}<0.002$	4.8	3.5	6.52	1/2, o	1.0	2.17	1	250	1	<0.4		
							e					*0*	*<0.1*		
			$S_t=4.8\pm?$				3/2, o	1.0	4.34	1	250	1	<0.4		
							e					*2*	*<7*		
							5/2, o	1.0	6.5	1	250	3	<350		
							e					*2*	*<7*		
							5/2, e	1.0	6.5	2	7300	2	<7		
							o					*3*	*<350*		
2	492	459	$S_{n,p}=0.28$	<0.1	7.5	5.69	1/2, o	0.18	0.06	1	76	1	880		
							e					*0*	*210*		
			$S_t=0.26\pm0.10$				3/2, o	0.08	0.024	1	34	1	990		
							e					*2*	*17,000*		
							5/2, o	0.052	0.015	1	22	3	9×10^5		
							e					*2*	*18,000*		
							5/2, e	0.052	0.015	2	600	2	18,000		
							o					*3*	*9×10^5*		
3	639	596	$S_{n,p}=0.20$	1.06	43	4.37	*A* 1/2, e	0.836	1.15	0	440	0	200		
							o					*1*	*720*		
4	998	931	$S_{n,p}=0.020$	1.68	46	2.81	*A* 3/2, e	0.989	1.82	0	480	2	280		
							o					*1*	*23*		
5	1120	1045	$S_{n,p}=0.010\pm0.005$	2.00	19	2.50	1/2, o	1.0	0.81	1	400	1	4		
							e					*0*	*2*		
			$S_t=2.01$				*B* 3/2, o	1.0	1.62	1	400	1	4		
							e					*2*	*50*		
							*A** 5/2, o	1.0	2.43	1	400	3	1600		
							e					*2*	*50*		
							A 5/2, e	1.0	2.48	2	4300	2	50		
							o					*3*	*1600*		
5*A*	1188	1108	$S_{n,p}<0.01$	$\geqslant1.07$	$\leqslant3.2$	2.36	1/2, o	1.0	0.79	1	65	1	<1		
							e					*0*	*<0.5*		
			$S_t\geqslant1.1$				3/2, o	1.0	1.57	1	65	1	<1		
							e					*2*			
							5/2, o	1.0	2.36	1	65	3	<530		
							e					*2*	*<14*		
							5/2, e	1.0	2.36	2	650	2	<14		
							o					*3*	*<530*		
6	1211	1130	$S_{n,p}=0.010\pm0.005$	0.61	13	2.31	*A* 1/2, o	0.987	0.75	1	260	1	10		
							e					*0*	*4*		
			$S_t=0.62$												
7	1350	1260	$S_{n,p}=0.089$	1.70	22	2.07	1/2, o	0.847	0.48	1	330	1	120		
							e					*0*	*51*		
			$S_t=1.79$				3/2, o	0.931	1.16	1	360	1	53		
							e					*2*	*500*		
							A 5/2, o	0.955	1.83	1	370	3	9300		
							e					*2*	*330*		
							*A** 5/2, e	0.955	1.89	2	3100	2	330		
							o					*3*	*9300*		
8	1401	1308	$S_{n,p}=0.19$	0.81	54	2.00	1/2, o	*i*	—	—	—	—	—		
							e								
			$S_{n,\alpha}=0.03$				*A* 3/2, o	0.792	0.80	1	750	1	330	0	600
							e					*2*	*3000*	*1*	*2300*
			$S_t=1.03$				5/2, o	0.874	1.53	1	820	3	49,000	2	15,000
							e					*2*	*1760*	*1*	*1400*
9	1595	1489	$S_{n,p}=0.020\pm0.010$	1.54	22	1.75	1/2, o	0.964	0.52	1	340	1	22		
							e					*0*	*11*		
			$S_{n,\alpha}\leqslant0.01$				3/2, o	0.976	1.05	1	340	1	15		
							e					*2*	*100*		
			$S_t=1.56$				*A** 5/2, o	0.988	1.64	1	340	3	1500		
							e					*2*	*65*		
							A 5/2, e	0.988	1.72	2	2300	2	65		
							o					*3*	*1500*		

TABLE I.—(*Continued*).

(1) No.	(2) E_R	(3) E_R'	(4) Experimental cross sections	(5) Exptal. $S_{n,n}$	(6) Γ	(7) $4\pi\lambda_n^2$	(8) J, π	(9) γ	(10) Calc. $S_{n,n}$	(11) l_n	(12) D^n	(13) l_p	(14) D^p	(15) l_α	(16) D^α
10	1779	1661	$S_{n,p}=0.010$	0.74	24	1.57	1/2, o	i	—	—	—	—	—	—	—
							e								
			$S_{n,\alpha}=0.27\pm0.10$				3/2, o	i	—	—	—	—	—	—	—
							e								
			$S_t=1.02$				A 5/2, o	0.768	0.85	1	260	3	810	2	13,000
							e					*2*	*86*	*1*	*1700*
							A^* 5/2, e	0.768	0.93	2	1500	2	36	1	1700
							o					*3*	*810*	*2*	*13,000*

Near a resonance for neutron scattering, the potential and resonance scattering interfere. In general, it is necessary to add to the (complex) amplitude of the resonance cross section a fraction g_J^l of the potential scattering coming from neutrons of angular momentum l; the remainder of the potential scattering is incoherent with the resonance scattering.[18] The total elastic scattering cross section in the neighborhood of a resonance of a state of spin J is

$$\sigma_J=4\pi\lambda_n^2\left\{\sum_l(2l+1)\sin^2\delta_l+\frac{2J+1}{2(2I+1)}\frac{\gamma}{\epsilon^2+1}\left[\gamma+2\sin\delta_l(\epsilon\cos\delta_l-\sin\delta_l)\right]\right\}, \quad (9)$$

where $\gamma=\Gamma_n^l/\Gamma$ and $\epsilon=(E_n-E_r)/\frac{1}{2}\Gamma$, E_n is the neutron energy, and E_r is the resonance energy. The first term is the potential scattering; it is summed over all values of l. The second term is expressed as a function of ϵ, the distance "off resonance" in units of $\Gamma/2$. In general, it has both positive and negative values and it approaches zero as $\epsilon\rightarrow\pm\infty$. It is evaluated for the lowest value of l which can contribute to the state J.[19]

The maximum and minimum values of the second term of Eq. (9) can be found by differentiation with respect to ϵ, assuming δ_l and γ are constant in the region of interest. (This is a good approximation if $E_R\gg\Gamma$.) They occur at the values ϵ_+ and ϵ_- which are the solutions of the equation,

$$\epsilon^2+\epsilon\left[\frac{\gamma+2\sin^2\delta_l}{\sin\delta_l\cos\delta_l}\right]-1=0. \quad (10)$$

By substitution of ϵ_+ and ϵ_- in Eq. (9), the maximum and minimum values of the cross section can be determined.

If only elastic scattering can occur, then $\gamma=1$ and the equations reduce to much simpler forms, equivalent to those given by Adair.[16] The maximum and minimum occur respectively at the energies:

$$E_{\max}=E_r+\tfrac{1}{2}\Gamma\tan\delta_l;\quad E_{\min}=E_r-\tfrac{1}{2}\Gamma\cot\delta_l \quad (11)$$

$$\sigma_J(\min)=4\pi\lambda_n^2\left\{\sum_l(2l+1)\sin^2\delta_l-\frac{(2J+1)}{2(2I+1)}\sin^2\delta_l\right\} \quad (12)$$

$$\sigma_J(\max)-\sigma_J(\min)=4\pi\lambda_n^2\frac{(2J+1)}{2(2I+1)}. \quad (13)$$

For low neutron energies, potential scattering except for $l=0$ is often negligible. In that case the maximum value of the resonant scattering cross section alone, for $l>0$, is given by

$$S_J^l(n,n)=4\pi\lambda_n^2\frac{(2J+1)}{2(2I+1)}\gamma^2. \quad (14)$$

IV. APPLICATION OF THE THEORY TO NITROGEN

Neutrons incident on N^{14} form virtual excited states of the compound nucleus N^{15}, each of which is characterized by a definite value of total angular momentum, J, and parity, π. The states which can be formed by neutrons with $l_n=0$, 1, 2, and 3 are listed in Table II. At excitation energies of the compound nucleus above

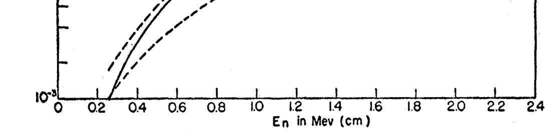

FIG. 5. Calculated barrier penetration factors, T_n^l and T_p^l, for the emission of neutrons and protons of angular momentum l from the nucleus N^{15}. The abscissa gives the neutron energy E_n in the center-of-mass coordinate system and hence the excitation energy of N^{15} above the neutron binding energy. The corresponding proton energy is $E_p'=E_n+Q$, where $Q=626$ kev.

[18] R. K. Adair, Revs. Modern Phys. **22**, 249 (1950). The treatment by Adair assumes that $\Gamma\approx\Gamma_n^l$; we have generalized it to include cases, where $\Gamma=\Gamma_n^l+\Gamma_p^l+\cdots$.

[19] Because of parity restrictions, if l is the lowest value of the angular momentum of neutrons which can form the state J, then only $l+2$, $l+4$, $\cdots$ neutrons can contribute also. At moderate energies, the contribution of $l+2$ neutrons is often negligible relative to that of l neutrons.

TABLE II. Total angular momentum J and parity π (o=odd parity, e=even) of the states of N^{15}. For a given target nucleus in a state of spin I, parity π_t, the table lists the states J, π which can be formed by incident particles of spin i, parity π_i, for various values of the orbital angular momentum l of the incident particle. The parity of N^{14} is assumed to be even.

Target	I, π_t	Incident particle	i, π_i	J, π of the states of N^{15}: $l=0$	$l=1$	$l=2$	$l=3$
N^{14}	1, e	n	1/2, e	1/2, e	1/2, o	1/2, e	3/2, o
				3/2, e	3/2, o	3/2, e	5/2, o
					5/2, o	5/2, e	7/2, o
						7/2, e	9/2, o
C^{14}	0, e	p	1/2, e	1/2, e	1/2, o	3/2, e	5/2, o
					3/2, o	5/2, e	7/2, o
B^{11}	3/2, o	α	0, e	3/2, o	1/2, e	1/2, o	
					3/2, e	3/2, o	
					5/2, e	5/2, o	
						7/2, o	

10.99 Mev, these states are energetically able to decay by emission of a neutron, a proton (leaving C^{14}), an α-particle (leaving B^{11}), or a γ-ray. Analysis is simplified by the fact that N^{14}, C^{14} and B^{11} must be left in their ground states over the energy range considered in the present experiment, since the first excited states of these nuclei lie 2.3, 5.6, and 2.14 Mev, respectively, above the ground states. Also the γ-ray emission probability is small relative to that for particles, and can be neglected.

Table II shows also the states of N^{15} formed by $p+C^{14}$ and by $\alpha+B^{11}$, for various values of l.[20] Since formation and decay are inverse processes, we see from Table II that the state (5/2, o), for example, can be formed by $l=1$ neutrons on N^{14}, and can decay by the emission of $l=3$ protons or $l=2$ α-particles.[21]

Returning now to Table I, column (9) lists, for each observed resonance, values of $\gamma=\Gamma_n/\Gamma$ calculated as follows: we assume that the total level width $\Gamma=\Gamma_n+\Gamma_\alpha+\Gamma_p$. Then from Eq. (1) the sum of the maximum values at resonance of the n,p and n,α cross sections is

$$S_J(n,p)+S_J(n,\alpha)=4\pi\lambda\!\!\!^{-}{}_n{}^2\frac{(2J+1)}{6}\left[\frac{\Gamma_n\Gamma_p}{\Gamma^2}+\frac{\Gamma_n\Gamma_\alpha}{\Gamma^2}\right].$$

The quantity in brackets can be written as $\gamma(1-\gamma)$, from which

$$\gamma(1-\gamma)=\frac{S_J(n,p)+S_J(n,\alpha)}{4\pi\lambda\!\!\!^{-}{}_n{}^2\cdot\frac{1}{6}(2J+1)}. \quad (15)$$

For each resonance, we substitute in Eq. (15) the experimental values of $S_J(n,p)$ and $S_J(n,\alpha)$. For each value assumed for J, in column (8), Eq. (15) yields a quadratic in γ. Of the two solutions, one can be eliminated by comparison with the observed cross sections, the other is entered in column (9). In column (10) of Table I we list the calculated value of the resonance scattering cross section, $S_{n,n}$(calc), for each of the possible assignments (J,π).

Using Table II, we can determine the lowest angular momenta l_n, l_p, and l_α of the neutron, proton, or α-particle which can be emitted from each state J,π. These values are listed in columns (11), (13), and (15) of Table I. In the adjacent columns are the corresponding values of the level spacing, D_J, for each particle and state, calculated from Eq. (3). (The partial widths are obtained from the ratios of the partial cross sections and the relation $\Gamma_p+\Gamma_\alpha+\Gamma_n=\Gamma$, where Γ is the total level width observed experimentally.)

V. DISCUSSION OF RESULTS

A. Assignment of J-Values

As shown in the last section, the assignment of J-values to the resonance levels of N^{15} depends primarily on a comparison of the experimental and computed elastic scattering cross sections at resonance. However, the experimental values for the (n,p) and (n,α) cross sections enter into the calculation of $S_{n,n}$(calc) through the quantity $\gamma=\Gamma_n/\Gamma$. In many cases, $\gamma\approx1$, but it is an important correction for resonances Nos. 3, 8, and 10. As indicated by the symbol "i" in column (9) of Table I, γ is imaginary for $J=1/2$ or 3/2 at No. 10.[22] Resonance No. 2 is exceptional; it is the only one of the eleven resonances for which γ has the lower value of the two solutions of Eq. (15), corresponding to a scattering cross section smaller than the reaction cross section.

Referring to Table I, $S_{n,n}$(exp) is in no case greater than the value computed for $J=5/2$. Thus, there is no direct evidence for states in N^{15} formed by neutrons of $l_n>1$. The most reasonable J-value, based on a comparison of $S_{n,n}$(exp) and $S_{n,n}$(calc), is indicated for each resonance by the letter A. B indicates a less probable assignment, but one judged to be within the range of possible experimental error. No assignment is possible for No. 2, since $S_{n,n}$ was too small to measure. Nos. 1 and 5A are so narrow that the correction for resolution width is uncertain, but $J>1/2$ in both cases. For No. 5, the value of $S_{n,n}$(exp) lies about halfway between those computed for $J=3/2$ and $J=5/2$, an unexpectedly large disagreement. The remainder agree with the A-assignments within 0.2 barn, averaging 0.1 barn lower than the calculated values.

B. Assignment of Parity

The assignment of parity to a state of N^{15} is equivalent to determining the orbital angular momentum l_n

[20] We assume here even parity for N^{14}, although the evidence is conflicting. This is discussed below.

[21] The (5/2, o) state can also be formed by, or can emit, neutrons of $l=3$. The great difference in barrier penetration factors allows us to neglect this process at moderate energies.

[22] The reaction cross section cannot be larger than $\pi\lambda\!\!\!^{-}{}^2(2J+1)/6$; this sets a lower limit on J. The large correction which we have applied to Johnson and Barschall's value of $S_{n,\alpha}$ for resonance No. 10, because of poor resolution, makes the conclusion that $J>3/2$ very tentative. We have assumed a resolution width of 45 kev. (C. H. Johnson, private communication.)

of the neutron by which it is formed from N^{14}. In the present energy range the S-wave potential scattering is considerably larger than that resulting from neutrons of $l_n>0$. The resulting large "interference dip" at (1/2, e) and (3/2, e) resonances ($l_n=0$) distinguishes them at once from the (1/2, o) and (3/2, o) states ($l_n=1$). A state $J=5/2$, however, can be formed by neutrons of $l_n=1$ or 2. At resonance No. 9, the interference dip is at most 0.08 barn for $J=5/2$, $l_n=1$, while it is negligible for $l_n=2$. The difference is experimentally indistinguishable, so that the parity cannot be determined in this manner.

C. The Level Spaicng D

We may call the quantities $D_{J,\pi}{}^n$, $D_{J,\pi}{}^p$, and $D_{J,\pi}{}^\alpha$ the effective level spacings, for a particular resonance, for neutrons, protons, and α-particles. Whatever their connection with the actual separation in energy of resonance levels of the same type, the D's contain implicitly the unknown matrix elements associated with each transition. They are subject to the upper limit imposed on Wigner's reduced width[15]

$$D \leqslant 3\pi\hbar^2/2MR\lambda\!\!\!^{-}_0,$$

where M is the particle mass and $\lambda\!\!\!^{-}_0$ its wavelength in the nucleus. For N^{15}, this requires that D^n and $D^p \leqslant 53$ Mev, $D^\alpha<13$ Mev, if we set $\lambda\!\!\!^{-}_0=1\times10^{13}$ cm for all three particles. These upper limits are not of much help, in the present case, in assigning parities. For resonance No. 10, D^α has its limiting value for $J=5/2$ and parity odd, but the uncertainty both in the (n,α) cross section and in the barrier penetration calculations makes this indecisive.

We have observed 11 resonances altogether in a range of 1600 kev, or an average level spacing $\bar{D}=145$ kev. Most of these resonances can be reasonably assigned to 5, or, at most, 6 states. We may therefore suppose that an order of magnitude estimate of the average value of $D_{J,\pi}$ will be given by $\bar{D}_{J,\pi}=5\times145=725$ kev.

In Table I, in the four cases for which the most reasonable assignment is $J=5/2$, let us choose the parity which makes the values of $D_{J,\pi}$ nearer to 700 kev, as indicated by A^*. Omitting the data for resonances Nos. 1, 2, and 5A, the remaining eight then give as an estimate of the minimum range of variation of the D's:

D^n	260–3100 kev, average	900 kev	(8 resonances)
D^p	9–1600 kev, average	540 kev	(8 resonances)
D^α	600–1700 kev, average	1150 kev	(2 resonances).

Of the three resonances omitted, Nos. 1 and 5A either have very low values of D^p, or $S_{n,n}$(exp) has been considerably underestimated; nothing definite can be said of No. 2.

D. The Parity of N^{14}

The shell model predicts that the ground states of N^{14} and C^{14} both have even parity, but this has been questioned by Gerjuoy[23] and others. In Table I, we have indicated in italics the parity assignments which apply if the ground state of N^{14} is assumed to have odd parity, and the corresponding values of l_p, D^p, l_α, and D^α. Because of the large variation of D^p in either case, there seems to be little reason to favor one parity or the other. However, it may be pointed out that the small (n,p) cross section of resonance No. 4, relative to No. 3, is perhaps more naturally explained by the difference in barrier penetration factors, which follows if C^{14} and N^{14} have the same parity, rather than by attributing it to a large difference in the D-values, as in the case of opposite parity.

E. The Interaction Radius R

There is considerable uncertainty about the magnitude of the interaction radius R. For neutrons and protons, we have used the relation $R=1.5A^{\frac{1}{3}}\times10^{-13}$ cm, which gives $R=3.7\times10^{-13}$ cm for $A=15$. We have taken $R=5.35\times10^{-13}$ cm for calculations involving α-particles. To illustrate the dependence of the barrier penetration factors, T, on the interaction radius, we have recalculated them for neutrons and protons using $R=4.9\times10^{-13}$ cm.[24] The ratios of the penetration factors for the two radii are plotted in Fig. 6 as a function of neutron energy; they are large for higher angular momenta. Use of the larger radius would lower the computed D-values and their minimum range of variation.

Figure 7 shows the result of subtracting off from the total cross section of nitrogen the contributions of the known resonances. The calculated potential scattering is shown for comparison. The large residual cross section below 1 Mev presumably comes in part from an S-

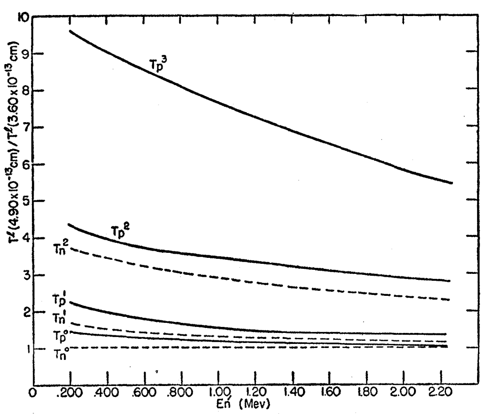

FIG. 6. The ratio of the calculated values of the barrier penetration factor, T, for N^{15}, for two assumed values of the interaction radius R (4.90×10^{-13} cm and 3.60×10^{-13} cm) plotted as a function of the incident neutron energy in the center of mass coordinate system. The subscripts p and n refer to protons and neutrons, respectively, the superscripts give the angular momentum l.

[23] E. Gerjuoy, Phys. Rev. **81**, 62 (1951).

[24] This value is given by $R=1.4(A^{\frac{1}{3}}+1)\times10^{-13}$ cm, $A=14$, as used by R. F. Christy and R. Latter, Revs. Modern Phys. **20**, 185 (1948).

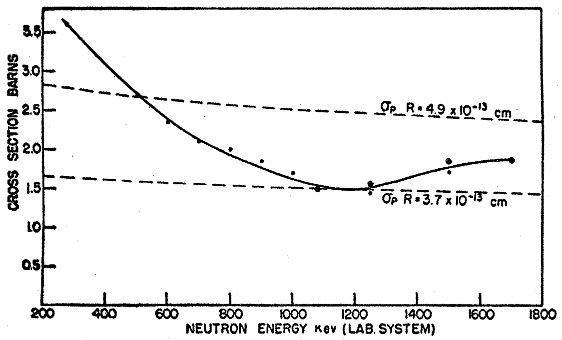

FIG. 7. The solid line is the residual cross section of nitrogen, after subtracting from the total cross section the contributions of the observed resonances. Circled points were computed from data taken with the liquid nitrogen scatterer, the remainder from LiN_3 data. The dashed curves show the computed potential scattering cross section for two values of the interaction radius R.

resonance below 0.2 Mev, since both its magnitude and slope are too great to be accounted for by potential scattering alone. The rise beyond 1.2 Mev seems real, but may be caused by some unknown systematic error. A radius $R=3.7\times10^{-13}$ cm gives a potential scattering cross section in good agreement with the observed value near $E_n=1200$ kev.

F. Comparison with Other Work

After this article was written, a paper on the total cross section of nitrogen by Johnson, Petree, and Adair[25] was published. Their measurements agree with ours very closely within the range covered by both experiments, 0.2 to 1.4 Mev, but with somewhat poorer resolution they did not observe the resonances which we have numbered 2, 5A, and 6. The agreement at both of the S-resonances (our Nos. 3 and 4) is within 0.1 barn for the measured cross sections and 1 kev for the resonance energies.

In conclusion, we wish to express our appreciation to Dr. Victor F. Weisskopf for many helpful discussions, to Dr. Herman Feshbach and Miss Mida Karakashian for aid in the computation of barrier penetration factors, and to Mr. Donald Thompson and Mr. I. E. Slawson for their invaluable help with the Rockefeller generator.

[25] Johnson, Petree, and Adair, Phys. Rev. **84**, 775 (1951).