Exchange Current Effects in the Deuteron*

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It is well known that the existence of charge carrying nuclear fields, resulting in an exchange of charge between nucleons, also gives rise to additional electromagnetic interaction terms for the nuclear system. It is commonly assumed that these effects are zero for the deuteron ground state, on account of the symmetry properties of the exchange current. The present investigation aims to draw attention to the fact that this conclusion is only true in the adiabatic approximation, which neglects the recoil of the nucleons. In the expression for the electromagnetic interaction of charge exchanging nucleons in motion, nuclear "two-particle currents" appear, in addition to the meson currents encountered in the adiabatic approximation. Whereas the latter are antisymmetric under charge exchange of a nucleon pair, the former are not, and give rise to electromagnetic exchange effects for the deuteron ground state.

I. INTRODUCTION

THE existence of electromagnetic exchange effects, accompanying nuclear charge exchange interaction, has been known for a considerable time.[1] The appearance of additional electromagnetic coupling terms ("exchange" moments) is a straightforward result of a (meson-) field-theoretical treatment of the nuclear interaction problem, since then the total charge-current density automatically satisfies the continuity equation.[2] In a phenomenological treatment of the problem, these extra terms must be added more or less *ad hoc*, guided by the requirement of gauge invariance as an expression for the differential form of the charge conservation relation.[3] Although this latter method lacks uniqueness, it is highly satisfactory inasmuch as it is free from the inherent deficiencies of a meson-field-theoretical model, and also independent of the limitation of the perturbation-calculation approach implied in the former. In the realm of nuclear problems, perturbation methods will always prevent a proper numerical interpretation of the results obtained. Nevertheless, as far as the structure of the obtained expression is concerned, the method provides (or may provide) valuable information, and possibly serve as a basis for the formulation of a more sophisticated phenomenological approach. So far, all field theoretical treatments have been based on the adiabatic approximation (infinitely heavy nucleons); in such an approximation, however, all nucleon *current* effects, resulting from the recoil of the nucleons are dropped, leaving the meson current effects, as exchange corrections of the nuclear *charge*. The well-known result, that the electromagnetic exchange moments (in so far as they are due to the meson currents) have zero diagonal elements for all deuteron states, does not apply to the recoil terms. The former matrix elements are easily seen to be antisymmetric under the substitution $N\rightleftarrows P$, since this implies that the role of a positive meson is taken over by a negative one, and vice versa, and thus reverses the sign of the matrix-element; but of course the same substitution $N\leftrightarrows P$ does not reverse the sign of the electromagnetic interaction of the two nucleons.

In a phenomenological treatment, the limitation of the adiabatic approach may be overcome by extending the discussion to velocity dependent forces.[4] Since such interactions (e.g., the $(\mathbf{L}\cdot\mathbf{S})$ coupling) have been advocated recently,[5] an investigation of its implications on the electromagnetic properties of the deuteron seems desirable, particularly in view of their importance in the problem of fitting the parameters of the static nuclear two-body potential.[6]

A phenomenological approach, however, based on a single time two-particle equation, seems to contain some limitations, of which a field theoretical approach is free; this concerns mainly the introduction of local interaction operators in the former case, and the apparent lack of covariance of the single time formalism.

We present, therefore, in the following pages, an approach to the problem, based on a specific model, introducing pseudoscalar charged mesons (neutral mesons were not included, since they do not contribute to the typical two-particle currents). Pseudoscalar coupling was chosen, but within the approximations of the present calculation the equivalence theorem holds. Pseudovector coupling only provides a different splitting of the electromagnetic exchange terms into such terms resulting from meson currents and nucleon currents.[7] Pseudoscalar coupling, however, was chosen on account of its somewhat greater formal simplicity.

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[1] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).
[2] F. Villars, Helv. Phys. Acta **20**, 476 (1947).
[3] R. G. Sachs, Phys. Rev. **74**, 433 (1948); R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951); R. K. Osborn and L. L. Foldy, Phys. Rev. **79**, 795 (1951).
[4] Blanchard, Avery, and Sachs, Phys. Rev. **78**, 292 (1950).
[5] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).
[6] See in this connection Feshbach, Schwinger, and Harr, *Effect of Tensor Range in Nuclear Two-Body Problems* (Computing Laboratory, Harvard University, 1949). This paper includes tables of

$$Q=(\sqrt{2}/10)\int_0^\infty r^2(uw-2^{-\frac{3}{2}}w^2)dr \quad \text{and} \quad p_D=\int_0^\infty drw^2$$

for various values of the parameters of the two-body interaction.
[7] Ch. Møller and L. Rosenfeld, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **20**, No. 12, 1943; S. T. Ma and F. C. Yu, Phys. Rev. **61**, 138 (1942).

Other types of mesons seem now to be definitely excluded.[8]

This calculation shows that indeed charge symmetric exchange terms exist and give contributions to the magnetic dipole and to the electric quadrupole moment of the deuteron ground state. The former is distinct from the terms calculated by Breit[9] and Sachs,[10] which however might be included, if the appropriate relativistic refinements of the deuteron wave functions were included. But in the present paper only the nonrelativistic deuteron wave function shall be used, since no numerical precision is aimed at. The order of magnitude of the effects discussed may be inferred from the results:

$$q_{\mathrm{exch}}=\left(\frac{g^2}{4\pi}\right)\left(\frac{\hbar}{mc}\right)\left(\frac{\hbar}{\mu c}\right)I_1\cong\frac{g^2}{4\pi}3\times10^{-27}\ \mathrm{cm}^2 I_1,$$

$$\mu_{\mathrm{exch}}=\left(\frac{g^2}{4\pi}\right)\left(\frac{\mu}{m}\right)\left(\frac{e\hbar}{2mc}\right)I_2=\frac{g^2}{4\pi}\left(\frac{\mu}{m}\right)I_2$$

$$\times\text{nuclear magnetons},$$

where I_1 and I_2 are dimensionless matrix elements, whose order of magnitude is estimated to $10^{-1}-10^{-2}$. The constant g is the dimensionless coupling constant of the pseudoscalar meson-nucleon coupling. It is obvious that the existence of a exchange quadrupole moment destroys the simple (kinematic) relation, expressing the quadrupole moment in terms of the radial S- and D-state wave functions, $u(r)$ and $w(r)$,

$$Q=(\sqrt{2}/10)\int_0^\infty dr(uw-2^{-\frac{3}{2}}w^2).$$

This brings about an additional uncertainty in the interpretation of the deuteron data in terms of a phenomenological nuclear interaction.

II. THE ELECTROMAGNETIC INTERACTION OF A PROTON-NEUTRON SYSTEM

Let ψ_P and ψ_N be the quantized proton- and neutron-fields, respectively. We assume then, to define our model, a charged pseudoscalar meson field φ responsible for the nuclear interaction. In a time dependent, "free-particle" representation, the nuclear interaction energy is given by

$$H_I(t)=ig\int dv(\bar{\psi}_P\gamma^5\psi_N\varphi+\bar{\psi}_N\gamma^5\psi_P\varphi^*), \quad (1)$$

g being the dimensionless coupling constant. In the following, we shall use units defined by $c=\hbar=1$. The interaction of the proton and meson fields with an external electromagnetic field A^{ext} is

$$H_{\mathrm{El}}(t)=ie\int dv[\varphi^*\partial\varphi/\partial x_\mu-\partial\varphi^*/\partial x_\mu\varphi-\bar{\psi}_P\gamma^\mu\psi_P]. \quad (2)$$

The state vector Ψ of the system will satisfy the Schrödinger equation,

$$i\partial\Psi/\partial t=(H_I+H_{\mathrm{El}})\Psi. \quad (3)$$

The solution of the field free problem ($A^{\mathrm{ext}}=0$) will be

$$\Psi(t)=U(t)\Psi_0,$$

and hence, the expectation value of the electromagnetic term,

$$\langle H_{\mathrm{El}}\rangle=(\Psi_0,U^{-1}H_{\mathrm{El}}U\Psi_0). \quad (4)$$

In the g^2-approximation

$$U(t)=\exp[-iS_1(t)]\exp[-iS_2(t)],$$

with

$$S_1(t)=\tfrac{1}{2}\int_{-\infty}^{+\infty}dt'\epsilon(t,t')H_I(t'), \quad (5a)$$

$$S_2(t)=\frac{i}{8}\iint_{-\infty}^{+\infty}dt'dt''\epsilon(t,t')\epsilon(t',t'')[H_I(t''),H_I(t')]. \quad (5b)$$

$U^{-1}H_{\mathrm{El}}U$ may then be expanded

$$U^{-1}H_{\mathrm{El}}U=H_{\mathrm{El}}+H_{\mathrm{El}}^{(1)}+H_{\mathrm{El}}^{(2)},$$

but only $H_{\mathrm{El}}^{(2)}$ will be needed in the present case

$$H_{\mathrm{El}}^{(2)}=-\frac{1}{4}\iint_{-\infty}^{+\infty}dt'dt''\epsilon(t,t')\epsilon(t',t'')$$

$$\times[H_I(t''),[H_I(t'),H_{\mathrm{El}}(t)]]. \quad (6)$$

Equation (6) is expressed in terms of free particle operators, and accordingly its two-particle matrix elements will be

$$(p_1',p_2'|H_{\mathrm{El}}^{(2)}|p_1,p_2).$$

Let then $\Phi_D(p_1,p_2)$ be the deuteron wave function in momentum space. Φ_D is introduced *ad hoc* as that state which satisfies certain requirements imposed by our previous knowledge of the properties of the deuteron. This is a possible inconsistency, inasmuch as the actual model does not provide an adequate solution of the problem of bound proton-neutron states. Nevertheless, the use and interpretation of

$$\langle H_{\mathrm{El}}^{(2)}\rangle=\iint\Phi_D^*(p_1',p_2')$$

$$\times(p_1',p_2'|H_{\mathrm{El}}^{(2)}|p_1,p_2)\phi_D(p_1,p_2) \quad (7)$$

[8] K. A. Brueckner, Phys. Rev. **79**, 641 (1950); Marshak, Tamor, and Wightman, Phys. Rev. **80**, 765 (1950); S. Tamor and R. E. Marshak, Phys. Rev. **80**, 766 (1950).

[9] G. Breit, Phys. Rev. **71**, 400 (1947); G. Breit and I. Bloch, Phys. Rev. **72**, 135 (1947).

[10] R. G. Sachs, Phys. Rev. **72**, 91 (1947).

is, in the writer's opinion, the straightforward relativistic formulation of the electromagnetic exchange effects, calculated to the same approximation in g^2.[11] The momenta p_1p_2 involved in (6) will actually be assumed to be small, and no attempt will be made to include in (7) the kinematic relativistic corrections. On the other hand it is important that the actual form of (5a, b) guarantees a proper relativistic treatment of the intermediate states.

$H_{\mathrm{El}}^{(2)}$ may be written in the form $-\int dv J_\mu(x)A_\mu^{\mathrm{ext}}(x)$. J_μ is the sum of two terms, corresponding to the meson- and nucleon-currents in (2). From (6) it follows at once,

$$J_\mu^{\mathrm{Mes}}(x)=+ieg^2\int\int d^4\xi d^4\eta\bar{\psi}_P(\xi)\gamma^5\psi_N(\xi)\times\left(\bar{D}(\xi-x)\frac{\partial\bar{D}(x-\eta)}{\partial x_\mu}-\frac{\partial\bar{D}(\xi-x)}{\partial x_\mu}\bar{D}(x-\eta)\right)\times\bar{\psi}_N(\eta)\gamma^5\psi_P(\eta),\quad\text{(8a)}$$

$$\begin{aligned}J_\mu^{\mathrm{Nucl}}(x)=+ieg^2\int\int d^4\xi d^4\eta&\times(\bar{\psi}_P(x)\gamma^\mu\bar{S}(x-\xi)\gamma^5\psi_N(\xi)\\&\times\bar{D}(\xi-\eta)\bar{\psi}_N(\eta)\gamma^5\psi_P(\eta)\\&+\bar{\psi}_P(\xi)\gamma^5\psi_N(\xi)\bar{D}(\xi-\eta)\\&\times\bar{\psi}_N(\eta)\gamma^5\bar{S}(\eta-x)\gamma^\mu\psi_P(x)).\quad\text{(8b)}\end{aligned}$$

It is easily verified that $J_\mu(x)$ satisfies the continuity equation

$$\partial J_\mu(x)/\partial x_\mu=0.$$

An additional property of J_μ is the antisymmetry of the total "exchange charge"

$$Q=-i\int dvJ_4(x),$$

under the substitution $P\rightleftarrows N$. Q therefore has zero matrix elements between states of equal charge symmetry. This antisymmetry is obvious for the term arising from J_μ^{Mes} (8a), since J_μ^{Mes} itself has this property. But (8b) has no symmetry property, and an explicit calculation is needed. Calling

$$-i\int dvJ_4^{\mathrm{Nucl}}(x)=(2\pi)^{-1}\int d\omega\exp(-i\omega t)Q(\omega),$$

we get,

$$Q(\omega)=eg^2\int\int d^4\xi d^4\eta\rho_{PN}(\xi)\bar{D}(\xi-\eta)\rho_{NP}(\eta)\times\{\exp(i\omega\xi_0)-\exp(i\omega\eta_0)\}\omega^{-1};$$

the Hermitian operator $\{Q(\omega)+Q(-\omega)\}$ therefore has the desired antisymmetry under the substitution $P\rightleftarrows N$.

Since J_μ^{Mes} will not affect the properties of the deuteron ground state, we shall give only a very brief discussion of this term: To calculate its matrix elements on the energy shell, we calculate actually

$$\bar{H}'^{(2)}=H'^{(2)}\delta(E_f-E_i)=(2\pi)^{-1}\int_{-\infty}^{+\infty}dt\int dv(-J_\mu^{\mathrm{Mes}}A_\mu^{\mathrm{ext}}).\quad(8)$$

The evaluation of (8) is straightforward and yields

$$\bar{H}'^{(2)}=\frac{ieg^2}{2\pi}\int_{-1}^{+1}dv\int\int d^4\xi d^4\eta\rho_{PN}(\xi)\times A_\mu\left(\frac{1+v}{2}\xi+\frac{1-v}{2}\eta\right)\frac{\partial F(\xi-\eta)}{\partial\xi_\mu}\rho_{NP}(\eta),\quad(9)$$

where

$$\rho_{PN}(x)=\bar{\psi}_P(x)\gamma^5\psi_N(x),$$

and

$$F(x)=(2\pi)^{-4}\int d^4k\exp(ikx)(k^2+\mu^2)^{-2},$$

provided $A^{\mathrm{ext}}(x)$ satisfies $\Box^2A^{\mathrm{ext}}(x)=0$. For a homogeneous magnetic field,

$$A_4=0;\quad \mathbf{A}(x)=\tfrac{1}{2}(\mathbf{H}\times\mathbf{x})$$

and a nonrelativistic momentum spectrum of the final and initial states, the matrix elements of $H'^{(2)}$ reduce to the matrix elements of the operator,

$$\frac{1}{8}\left(\frac{e}{2m}\right)\left(\frac{g^2}{4\pi}\right)\frac{\mu}{m}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_3(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}_1)(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla}_2)\times[(\mathbf{H}\cdot\mathbf{x}_1\times\mathbf{x}_2)(e^{-\mu r_{12}}/\mu r_{12})],\quad(10)^7$$

which is just the result of Møller.[11] In a similar way, we now calculate

$$\bar{H}''^{(2)}=(2\pi)^{-1}\int_{-\infty}^{+\infty}dt\int dv(-J_\mu^{\mathrm{Nucl}}A_\mu^{\mathrm{ext}}).$$

Carrying out the ξ-integration in the first, the η-integration in the second term of (8b), we get

$$\begin{aligned}\bar{H}''^{(2)}=(ieg^2/2\pi)\int\int d^4x_1d^4x_2\Big\{&\bar{\psi}_P(x_1)\gamma^5\gamma^\mu A_\mu(x_1)\\&\times\int_0^1dz\{\gamma\partial/\partial x_1+m(1-z)\}G(x_1-x_2;z)\\&\times\psi_N(zx_1+(1-z)x_2)\cdot\rho_{NP}(x_2)\\&+\rho_{PN}(x_1)\cdot\int_0^1dz\bar{\psi}_N(zx_2+(1-z)x_1)\\&\times\{\gamma\partial/\partial x_1+m(1-z)\}G(x_1-x_2;z)\\&\times\gamma^\mu A_\mu(x_2)\gamma^5\psi_P(x_2)\Big\},\quad(11)\end{aligned}$$

[11] Note that Møller's derivation is based on a model using pseudovector coupling of the mesons to the nucleons. His result therefore includes a term (due to the mixed interaction $\propto eg$), which, according to the pseudoscalar coupling scheme, is due to the nucleon currents. This additional term in Møller's result is (in our notation)

$$\tfrac{1}{8}(e/2m)(g^2/4\pi)(\mu/m)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_3\{(\mathbf{H}\cdot\mathbf{r}_1\times\boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{r}_{12})+(\mathbf{H}\cdot\mathbf{r}_2\times\boldsymbol{\sigma}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{r}_{12})\}\{1+(1/\mu r_{12})\}\exp(-\mu r_{12})/r_{12}^2.$$

In the subsequent discussion of the nucleon current effects, we have not singled out this term, but rather focused our attention to the charge symmetric parts.

where G is defined by

$$G(x;z)=(2\pi)^{-4}\int d^4k\exp(ikx)\{k^2+\mu^2z+m^2(1-z)^2\}^{-2}. \tag{12}$$

We shall now carry out all time integrations, introducing $G(\mathbf{x},\omega)$ by

$$G(x;z)=(2\pi)^{-1}\int dt\exp(-i\omega t)G(\mathbf{x},\omega;z). \tag{13}$$

Let us call p_0', p_0; q_0', q_0 the energies associated with $\bar{\psi}_P$, ψ_N; $\bar{\psi}_N$, ψ_P, respectively (i.e., $\bar{\psi}_P\sim\bar{u}_P(p')\exp[ip_0't]$), and in addition assume a time-independent potential $A_\mu(x)$. Then time integration gives rise to

$$\delta(p_0'+q_0'-p_0-q_0)$$
$$\cdot\begin{cases}\delta(p_0'-zp_0-\omega) & \text{for the 1st term in (11)}\\ \delta(q_0'-zq_0-\omega) & \text{for the 2nd term in (11).}\end{cases}$$

The first δ-function expresses just all over energy conservation. To use the second δ-functions, we make the explicit assumption that the wave packet $\Phi_D(p_1,p_2)$(7), used to approximate the deuteron-state, shall contain only nonrelativistic momenta, and only positive frequency parts (no "small" components). Under these conditions, we may use the relations,

$$\begin{aligned}\omega&=p_0'-zp_0\cong m(1-z),\\ \omega&=q_0-zq_0'\cong m(1-z),\end{aligned} \tag{14}$$

(provided the center-of-mass motion is negligible). Corrective terms to (14) will be of order $\mathbf{p}^2$ and neglected. After substituting this into (11), (12), and (13), we get for the operator $H''^{(2)}$ on the energy surface,

$$\begin{aligned}H''^{(2)}=+ieg^2\int\int dv_1dv_2\int_0^1 dz\{&\bar{\psi}_P(x_1)\gamma^5\gamma^\mu A_\mu(x_1)\\ &\times(\boldsymbol{\gamma}\cdot\mathbf{x}_1-\mathbf{x}_2)\psi_N(zx_1+(1-z)x_2)\rho_{NP}(x_2)\\ &+\rho_{PN}(x_1)\bar{\psi}_N(zx_2+(1-z)x_1)(\boldsymbol{\gamma}\cdot\mathbf{x}_1-\mathbf{x}_2)\gamma^\mu\\ &\times A_\mu(x_2)\gamma^5\psi_P(x_2)\}\\ &\times\exp(-\mu z^{\frac{1}{2}}|\mathbf{x}_1-\mathbf{x}_2|)/8\pi|\mathbf{x}_1-\mathbf{x}_2|.\end{aligned} \tag{15}$$

III. THE MATRIX ELEMENTS OF $H''^{(2)}$ IN THE NONRELATIVISTIC LIMIT

In this chapter we propose to calculate the matrix element,

$$(p',q'|H''^{(2)}|p,q) \tag{15a}$$

of $H''^{(2)}$ on the energy surface. This matrix element will be expressed in terms of the one particle momentum eigenstates $u_P^*(p')\,u_N(p)$; $u_N^*(q')$, $u_P(q)$ and eventually in terms of the large components χ alone

$$u(p)=(\chi(p);[(\boldsymbol{\sigma}\cdot\mathbf{p})/2m]\chi(p)).$$

Simultaneously, we shall introduce center of mass and relative coordinates and momenta

$$\mathbf{r}=\mathbf{x}_1-\mathbf{x}_2,\quad \mathbf{R}=\tfrac{1}{2}(\mathbf{x}_1+\mathbf{x}_2),$$
$$\boldsymbol{\pi}=\tfrac{1}{2}(\mathbf{p}-\mathbf{q});\quad \mathbf{P}=\mathbf{p}+\mathbf{q};$$

$\boldsymbol{\pi}'$ and $\mathbf{P}'$ being defined similarly. The phase factors in the matrix element derived from (15) are then

$$\exp[i(\mathbf{P}-\mathbf{P}')\cdot\mathbf{R}]$$
$$\times\begin{cases}\exp[i(\boldsymbol{\pi}^*-\boldsymbol{\pi}')\cdot\mathbf{r}] & \text{for the 1st term}\\ \exp[i(\boldsymbol{\pi}-\boldsymbol{\pi}^{*\prime})\cdot\mathbf{r}] & \text{for the 2nd term,}\end{cases} \tag{16}$$

with

$$\boldsymbol{\pi}^*=z\boldsymbol{\pi}-\tfrac{1}{2}(1-z)\mathbf{P},\quad \boldsymbol{\pi}^{*\prime}=z\boldsymbol{\pi}^*+\tfrac{1}{2}(1-z)\mathbf{P}'. \tag{17}$$

Let us, at this place, list the γ-matrix elements (in their nonrelativistic limit) that will occur in (15). We shall only give the terms arising from the 1st line of (15), but treat separately the cases of a magnetic field: $A_\mu=(\mathbf{A};0)$, and an electrostatic field: $A_\mu=(\mathbf{0};i\phi)$.

Magnetic field

$$u^*(p')\beta\gamma^5u(p)=(1/2m)\chi^*(p')(\boldsymbol{\sigma}\cdot\mathbf{p}'-\mathbf{p})\chi(p), \tag{18}$$

and

$$\begin{aligned}&u^*(p')\beta\gamma^5(\boldsymbol{\gamma}\cdot\mathbf{A}(\mathbf{x}_1))(\boldsymbol{\gamma}\cdot\mathbf{r})u(p)\\ &\quad=(1/2m)\chi^*(p')[(\boldsymbol{\sigma}\cdot\mathbf{p}'-\mathbf{p})(\mathbf{A}\cdot\mathbf{r})+i(\mathbf{p}'-\mathbf{p}\cdot\mathbf{A}\times\mathbf{r})\\ &\qquad+(\mathbf{p}'+\mathbf{p}\cdot\mathbf{A})(\boldsymbol{\sigma}\cdot\mathbf{r})-(\mathbf{p}'+\mathbf{p}\cdot\mathbf{r})(\boldsymbol{\sigma}\cdot\mathbf{A})]\chi(p).\end{aligned} \tag{19a}$$

Electrostatic field

$$u^*(p')\beta\gamma^5i\beta\phi(\mathbf{x}_1)(\boldsymbol{\gamma}\cdot\mathbf{r})u(p)=-\chi^*(p')\phi(\boldsymbol{\sigma}\cdot\mathbf{r})\chi(p). \tag{19b}$$

It is obvious from the form of the phase factors (16) that:

$$\mathbf{P}'-\mathbf{P}=-i\boldsymbol{\nabla}_R,\quad \boldsymbol{\pi}'-\boldsymbol{\pi}^*=-i\boldsymbol{\nabla}_r; \tag{20}$$

these gradients acting on that part of the integrand in (15), which is not included in the phase factors. As a result of (20), we have

$$\mathbf{p}'-\mathbf{p}=-\tfrac{1}{2}i\boldsymbol{\nabla}_R+(\boldsymbol{\pi}'-\boldsymbol{\pi}),\quad \mathbf{q}'-\mathbf{q}=-\tfrac{1}{2}i\boldsymbol{\nabla}_R-(\boldsymbol{\pi}'-\boldsymbol{\pi}).$$

A suitable decomposition of $(\boldsymbol{\pi}'-\boldsymbol{\pi})$ is then

$$\begin{aligned}\pi'-\pi&=(\pi'-\pi^*)+(\pi^*-\pi)\\ &=-i\nabla_r+(\pi^*+\tfrac{1}{2}P)(z-1)/z,\end{aligned} \tag{21a}$$

according to the 1st line of (16), and

$$\begin{aligned}(\pi'-\pi)&=(\pi^{*\prime}-\pi)+(\pi'-\pi^{*\prime})\\ &=-i\nabla_r+(\pi^{*\prime}-\tfrac{1}{2}P)(1-z)/z.\end{aligned} \tag{21b}$$

But we are only interested in the matrix elements for

$$P\sim 0,\quad P'\sim 0.$$

This enables us to use the relations,

$$\begin{aligned}p'-p&=-(i/2)\nabla_R-i\nabla_r+\pi^*(z-1)/z\\ &=-i\nabla_1+\pi^*(z-1)/z,\end{aligned} \tag{22}$$

$$q'-q=-(i/2)\nabla_R+i\nabla_r-\pi^*(z-1)/z$$
$$=-i\nabla_2-\pi^*(z-1)/z, \quad (23)$$

and similar relations derived from (21b). Note that the factor $\sim z^{-1}$ in (22) will not lead to a divergence, since it will eventually occur in the position

$$[(z-1)/z]\nabla\varphi(zr),$$

and thus cancel out. It may be helpful to write down the matrix element (15a) at this stage of development explicitly:

$$(p',q'|H''^{(2)}|p,q)$$
$$=+(ieg^2)\int d^3Re^{i(P-P')\cdot R}\int_0^1 dz\int d^3r$$
$$\times\{\chi_P^*(p')\chi_N^*(q')e^{-i(\pi'\cdot r)}\Omega\chi_N(p)\chi_P(q)e^{i(\pi^*\cdot r)}$$
$$+\chi_P^*(p')\chi_N^*(q')e^{-i(\pi^{*\prime}\cdot r)}\Omega^*\chi_N(p)\chi_P(q)e^{i(\pi\cdot r)}\}. \quad (24)$$

Ω is the following operator.

(a) Case of an electrostatic field, represented by a potential $\phi(\mathbf{x})$

$$\Omega=-(1/2m)\phi(\mathbf{x}_1)(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{q}'-\mathbf{q})\exp(-\mu z^{\frac{1}{2}}r)/8\pi r,$$
$$\Omega^*=+(1/2m)\phi(\mathbf{x}_2)(\boldsymbol{\sigma}_2\cdot\mathbf{r})(\boldsymbol{\sigma}_1\cdot\mathbf{p}'-\mathbf{p})$$
$$\times\exp(-\mu z^{\frac{1}{2}}r)/8\pi r. \quad (25)$$

(b) Case of a homogeneous magnetic field $\mathbf{H}$ represented by a vector potential

$$\mathbf{A}(\mathbf{x})=\tfrac{1}{2}(\mathbf{H}\times\mathbf{x}).$$

Ω and Ω^* are then of the form $-(\mathbf{M}\cdot\mathbf{H})$ and $-(\mathbf{M}^*\cdot\mathbf{H})$ respectively; and

$$\begin{aligned}\mathbf{M}=&(1/8m^2)(\boldsymbol{\sigma}_2\cdot\mathbf{q}'-\mathbf{q})[(\mathbf{x}_1\times\mathbf{x}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{p}'-\mathbf{p})\\&-i\mathbf{r}(\mathbf{p}'-\mathbf{p}\cdot\mathbf{x}_1)+i(\mathbf{p}'-\mathbf{p})(\mathbf{r}\cdot\mathbf{x}_1)\\&-(\mathbf{x}_1\times\mathbf{p}'+\mathbf{p})(\boldsymbol{\sigma}_1\cdot\mathbf{r})+(\mathbf{x}_1\times\boldsymbol{\sigma}_1)(\mathbf{p}'+\mathbf{p}\cdot\mathbf{r})]\\&\times\exp(-\mu z^{\frac{1}{2}}r)/8\pi r.\\\mathbf{M}^*=&(1/8m^2)(\boldsymbol{\sigma}_1\cdot\mathbf{p}'-\mathbf{p})[(\mathbf{x}_1\times\mathbf{x}_2)(\boldsymbol{\sigma}_2\cdot\mathbf{q}'-\mathbf{q})\\&+i\mathbf{r}(\mathbf{q}'-\mathbf{q}\cdot\mathbf{x}_2)-i(\mathbf{q}'-\mathbf{q})(\mathbf{r}\cdot\mathbf{x}_2)\\&+(\mathbf{x}_2\times\mathbf{q}'+\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{r})-(\mathbf{x}_2\times\boldsymbol{\sigma}_2)(\mathbf{q}'+\mathbf{q}\cdot\mathbf{r})]\\&\times\exp(-\mu z^{\frac{1}{2}}r)/8\pi r.\end{aligned} \quad (26)$$

We shall now evaluate the electric quadrupole moment resulting from (25) and the magnetic moment (26) for the deuteron ground state. Note that in terms of an isotopic spin notation, the two operators (25) and (26) are proportional to $\tau_+^{(1)}\tau_-^{(2)}$, which is equal to

$$\tfrac{1}{4}(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}-\tau_3^{(1)}\tau_3^{(2)})-\tfrac{1}{4}i(\boldsymbol{\tau}^{(1)}\times\boldsymbol{\tau}^{(2)})_3;$$

this gives us a factor (-1) for the deuteron ground state. In addition, we have to pick out of Ω the terms even in $(\mathbf{r},\boldsymbol{\pi})$, and symmetric in $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$.

The Electric Quadrupole Moment

We expand ϕ about the center of mass R, and define a tensor Ω_{ik} as the coefficient of $\partial^2\phi(R)/\partial R_i\partial R_k$, in (25). If we define, as usual,[12] a quantity q as 3 times the 33-component of the traceless tensor Q_{ik}, appearing in

$$H_{\text{quadr}}=\tfrac{1}{2}e\Sigma(Q_{ik}\partial^2/\partial R_i\partial R_k),$$

we get from (24)

$$q=-2ig^2\int_0^1 dz\int d^3r\{\varphi_D^*(r)(3\Omega_{33}-\mathrm{Tr}\Omega)\varphi_D(zr)$$
$$+\varphi_D^*(zr)(3\Omega_{33}^*-\mathrm{Tr}\Omega^*)\varphi_D(r)\}, \quad (27)$$

where φ_D represents the charge-singlet deuteron ground state, obtained from the momentum eigenstates involved in (24) by a suitable build up of a wave packet. From

$$\Omega_{ik}=\frac{-1}{2m}\left(\frac{r_ir_k}{8}\right)\left\{\left(\frac{1-z}{z}\right)(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\pi}^*)\right.$$
$$\left.+i\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}(1+\mu z^{\frac{1}{2}}r)\right]\right\}$$
$$\times\exp(-\mu z^{\frac{1}{2}}r)/8\pi r,$$

we get, by dropping terms $\sim(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)$ and assuming $S=1$:

$$(3\Omega_{33}-\mathrm{Tr}\Omega)=\left(\frac{-1}{2m}\right)\left(\frac{3r_3^2-r^2}{8}\right)\left\{\left(\frac{1-z}{z}\right)\right.$$
$$\times[(\mathbf{S}\cdot\mathbf{r})(\mathbf{S}\cdot\boldsymbol{\pi}^*)+(\mathbf{S}\cdot\boldsymbol{\pi}^*)(\mathbf{S}\cdot\mathbf{r})-(\mathbf{r}\cdot\boldsymbol{\pi}^*)]$$
$$\left.+i[1-(2(\mathbf{S}\cdot\mathbf{r})^2/r^2-1)(1+\mu z^{\frac{1}{2}}r)]\right\}$$
$$\times\exp(-\mu z^{\frac{1}{2}}r)/8\pi r, \quad (28)$$

and a similar expression for $(3\Omega_{33}^*-\mathrm{Tr}\Omega^*)_D$. Remembering that

$$i\pi^*=\nabla_r \text{ on } \varphi_D \text{ alone}, \quad -i\pi^{*\prime}=+\overleftarrow{\nabla}_r \text{ on } \varphi_D^*,$$

and introducing dimensionless integrals by taking $(1/\mu)$ as unit of length, we may write:

$$q=(+\tfrac{1}{8})(g^2/4\pi)(2\mu m)^{-1}\int_0^1 dz\int d^3r(3r_3^2-r^2)$$
$$\times\exp(-z^{\frac{1}{2}}r)/r\times\{\varphi_D^*(r)[\{(\mathbf{S}\cdot\mathbf{r})(\mathbf{S}\cdot\boldsymbol{\nabla})$$
$$+(\mathbf{S}\cdot\boldsymbol{\nabla})(\mathbf{S}\cdot\mathbf{r})-(\mathbf{r}\cdot\boldsymbol{\nabla})\}-(1-[2(\mathbf{S}\cdot\mathbf{r})^2/r^2-1]$$
$$\times(1+z^{\frac{1}{2}}r))]\varphi_D(zr)+\text{c.c.}\}. \quad (29)$$

The Magnetic Dipole Moment

After performing the substitutions (22) and (23), the operator $\mathbf{M}$ will assume the form

$$\mathbf{M}=\mathbf{M}'+\mathbf{R}\mathbf{M}'',$$

[12] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1947), Appendix II.

and it is only $\mathbf{M}'$ that contributes to the internal magnetic moment. Eventually the projection of $\mathbf{M}'$ on $\mathbf{J}$, the total angular momentum, will be taken,

$$\mathbf{M}'_J=\tfrac{1}{2}(\mathbf{M}'\cdot\mathbf{J})\mathbf{J},$$

($J=1$ for deuteron ground state). $(\mathbf{M}'\cdot\mathbf{J})$ will be calculated as $(\mathbf{M}'\cdot\mathbf{L})+(\mathbf{M}'\cdot\mathbf{S})$, and written in the form

$$\tfrac{1}{2}(\mathbf{M}'\cdot\mathbf{J})=(e/2m)\mu_D, \tag{30}$$

μ_D being the value in nuclear magnetons of the "exchange" contribution to the magnetic moment of the deuteron ground state. The evaluation of the two projections $(\mathbf{M}'\cdot\mathbf{L})$ and $(\mathbf{M}'\cdot\mathbf{S})$ is straightforward, but tedious; the result will again be written in terms of a dimensionless matrix element, taking μ^{-1} as unit of length. Then

$$\mu_D=-\tfrac{1}{8}(g^2/4\pi)(\mu/2m)\int_0^1 dz\int dv \times[\varphi_D^*(r)\,\mathfrak{D}_{\mathrm{op}}\varphi_D(zr)+\text{c.c.}]. \tag{31}$$

$$\begin{aligned}\mathfrak{D}_{\mathrm{op}}=&-4[S^2+(\mathbf{L}\cdot\mathbf{S})]V(r)\\&+[5(T-Y)+2(\mathbf{n}\cdot\mathbf{S})^2\{5(\mathbf{r}\cdot\nabla)-2\}]rdV/dr\\&+[T-Y+2(\mathbf{n}\cdot\mathbf{S})^2(\mathbf{r}\cdot\nabla)]r^3(d/dr)(r^{-1}dV/dr)\\&+[3\{Y-(\mathbf{L}\cdot\mathbf{S})\}(2z-1)/z-4(\mathbf{r}\cdot\nabla)(3z-2)/z\\&-\{Z-Y(\mathbf{r}\cdot\nabla)+(\mathbf{r}\cdot\nabla)^2\}(5z-3)/2z\\&-(\mathbf{L}\cdot\mathbf{S})(\mathbf{r}\cdot\nabla)(z-1)/z-L^2(z+1)/z]V(r)\\&+[\{-2Y+Z+2(\mathbf{r}\cdot\nabla)^2+(\mathbf{L}\cdot\mathbf{S})\\&+((\mathbf{n}\cdot\mathbf{S})^2+2)(\mathbf{r}\cdot\nabla)\}(1-z)/z\\&+\{[1+2(\mathbf{n}\cdot\mathbf{S})^2](\mathbf{L}\cdot\mathbf{S})+T(\mathbf{r}\cdot\nabla)\\&+2(\mathbf{n}\cdot\mathbf{S})^2(\mathbf{r}\cdot\nabla)^2+[1-2(\mathbf{n}\cdot\mathbf{S})^2]L^2\}(1+z)/z\\&-(2/z)Y(\mathbf{r}\cdot\nabla)]rdV/dr\\&+[-Y(\mathbf{r}\cdot\nabla)(2-z-z^2)/z^2+\{[Y-(\mathbf{r}\cdot\nabla)]\\&\times[L^2+(\mathbf{L}\cdot\mathbf{S})-(\mathbf{r}\cdot\nabla)^2]+2(\mathbf{r}\cdot\nabla)^2\\&+r^2(\mathbf{S}\cdot\nabla)^2+Z(\mathbf{r}\cdot\nabla)\}(1-z^2)/z^2]V(r),\end{aligned} \tag{32}$$

where the following abbreviations have been introduced:

$$T=3(\mathbf{S}\cdot\mathbf{n})^2-S^2,\quad Y=\{(\mathbf{S}\cdot\nabla),(\mathbf{S}\cdot\mathbf{r})\}_+,\quad Z=r^2[2(\mathbf{S}\cdot\nabla)^2-\nabla^2], \tag{32a}$$

and

$$\mathbf{n}=\mathbf{r}/r;\quad V(r)=\exp(-z^{\frac{1}{2}}r)/r.$$

The next step consists in expressing the expectation values (29) and (31) in terms of the radial deuteron S- and D-state wave functions $u(r)$ and $w(r)$:

$$\varphi_D(\mathbf{r})=[u(r)/r]\phi_0+[w(r)/r]\phi_2.$$

The calculation of the matrix elements with respect to L, S, J of the operators appearing in (29) and (31) is most conveniently done following the technique outlined by Rarita and Schwinger.[13] We shall actually restrict the subsequent analysis to (29), the electric quadrupole moment. A fair estimate of (31) seems too involved; the effect is likely to be of the order of magnitude of the uncertain kinematic relativistic corrections,[9,10] and its evaluation would therefore not improve our knowledge of the magnetic moment problem. On the other hand, the discussion of (29) is comparatively simple.

The matrix elements of the operators in (29) are

(a) $\{(\mathbf{S}\cdot\nabla),(\mathbf{S}\cdot\mathbf{r})\}_+-(\mathbf{r}\cdot\nabla)$:

JLS	101	121
101	$\frac{1}{3}(rd/dr-1)$	$\frac{2}{3}\sqrt{2}(rd/dr+2)$
121	$\frac{2}{3}\sqrt{2}(rd/dr-1)$	$-\frac{1}{3}(rd/dr+2)$

(b) $(2(\mathbf{S}\cdot\mathbf{n})^2-1)$:

JLS	101	121
101	1/3	$2\sqrt{2}/3$
121	$2\sqrt{2}/3$	$-1/3$

(c) $(3r_3^2/r^2-1)_{J_3=1}$:

JLS	101	121
101	0	$\sqrt{2}/5$
121	$\sqrt{2}/5$	$-1/5$

Let us now introduce the notations: $s=zr$,

$$u'(s)=du/ds;\ w'(s)=dw/ds;$$

we may then write (29) in terms of u and w as follows:

$$\begin{aligned}q=\tfrac{1}{4}(g^2/4\pi)(2\mu m)^{-1}\int_0^1 dz/z\int_0^\infty drr\exp(-z^{\frac{1}{2}}r)\\ \times\{(4/15)u(r)[(1+z^{\frac{1}{2}}r)u(s)+\{(1-z)/z\}\\ \times(su'(s)-u(s))]+(\sqrt{2}/15)u(r)[(4+z^{\frac{1}{2}}r)w(s)\\ +\{(1-z)/z\}(sw'(s)+2w(s)]-(\sqrt{2}/15)w(r)\\ \times[(4+z^{\frac{1}{2}}r)u(s)+\{(1-z)/z\}\{su'(s)-u(s)\}]\\ +(1/15)w(r)[(8+5z^{\frac{1}{2}})w(s)\\ +3\{(1-z)/z\}\{sw'(s)+2w(s)\}]\}.\end{aligned} \tag{33}$$

The value of (33) is estimated by calculating numerically the r-integral for a set of values of z,

$$z=0,\ 0.075,\ 0.10,\ 0.25,\ 0.5,\ 1.0.$$

The functions $u(r)$ and $w(r)$ are those calculated by the Harvard Computation Laboratory,[2] and the values of the interaction parameters adjusted to give the correct triton binding-energy.[14]

According to (33), q is then proportional to

$$I=\int_0^1(dz/z)F(z), \tag{34}$$

$F(z)$ being the r-integral in question. A plot of $F(z)$ *vs* z (Fig. 1) reveals that the major contributions to I arise

[13] W. Rarita and J. Schwinger, Phys. Rev. **59**, 556 (1941), Appendix I.

[14] R. L. Pease and H. Feshbach, Phys. Rev. **81**, 142 (1951).

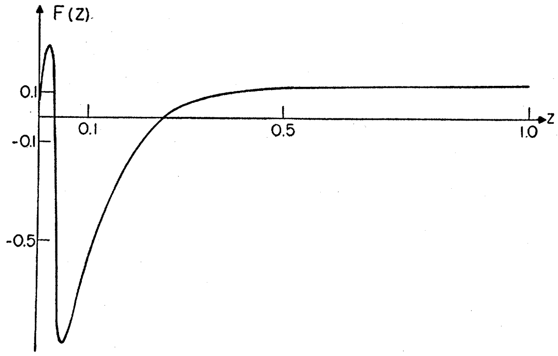

FIG. 1. The r-integral of Eq. (33), $F(z)$, plotted *vs* z.

from values $z<1/5$, and are therefore extremely sensitive to the behavior of the wave function at small values of r. The approximate value of I (34), is -0.1, and leads to a value of

$$q=-(\tfrac{3}{8})(g^2/4\pi)\times 10^{-28}\ \mathrm{cm}^2.$$

Plausible values for $g^2/4\pi$ are as high as 5–6;[15] the electric exchange quadrupole moment q is therefore, on the basis of our model, expected to be approximately -10^{-28} cm^2, corresponding to 7 percent contribution to the actual value[16] of Q:

$$Q=(2.738\pm 0.016)\times 10^{-27}\ \mathrm{cm}^2.$$

IV. DISCUSSION

In this section, we shall rely on the results of a phenomenological description of the deuteron,[6,17] with the interaction

$$V(r)=-V_0\{[1+\tfrac{1}{2}g(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-1)]\exp(-\beta r)/\beta r \\ +\gamma S_{12}\exp(-\tau r)/\tau r\}; \quad (35)$$

S_{12} is the tensor force operator: $S_{12}=2T$ (32a); g, γ, β, τ are dimensionless parameters. The quantity r is measured in units μ^{-1} ($=1.4\times 10^{-13}$ cm for $\mu=276m_{el}$).

In presence of exchange effects, the relation

$$Q^*=Q_{\exp}-q=(\sqrt{2}/10)\int_0^\infty drr^2(uw-2^{-\frac{3}{2}}w^2), \quad (36)$$

should be used; it appears therefore as indicated to estimate the effect of a variation of Q on the parameter of (35), especially on γ. Keeping these parameters adjusted to reproduce the deuteron data, except Q^*, the following approximate relations hold,

$$\gamma V_0/Q\cong\mathrm{const}, \quad (37a)$$

$$(1+\gamma\beta/\tau)V_0\cong\mathrm{const} \quad (37b)$$

(β and τ being kept constant). According to reference 14, a good choise is $\beta=2.33$, $\tau=1.8$. For $Q=2.74\times 10^{-27}$ cm^2, we derive with the help of (37a, b) and the tables of reference 6 a ratio,

$$\Delta\gamma/\gamma\cong 3\Delta Q/Q.$$

Thus, considering modestly our value of q as the source of an uncertainty in Q^* (36) of the relative order of 5 percent, an uncertainty in γ of about 15 percent. The effect is still much more pronounced, if we assume short-range tensor forces. In both cases, however, the expected D-state admixture is not affected very much by the change in Q of a few percent.

Some general remarks may be added: The actual fact that the behavior of the wave function near the origin (and therefore the high momenta in $\varphi_D(\pi)$) is of critical importance, suggests that our nonrelativistic calculation of the expectation value of q is altogether not on a too firm basis. We should, in this connection, keep in mind that we have, in a way, artificially reduced the effect of high momenta in the deuteron state by the choice of our wave function $\varphi_D(\pi)$, which corresponds to a far less singular interaction than the interaction associated with our actual field theoretical model.[18] It is easily seen that (29) and especially (32) are strongly affected by wave functions more irregular near the origin, and will probably assume values much larger than our actual result. (quite apart from the fact that these formulas themselves become inaccurate, since in deriving (15) we dropped all terms $\mathbf{p}^2/M^2$, assuming $\pi\sim\mu c$ at most). It is hoped to take up this problem again, as soon as a perturbation treatment, which rests upon the covariant wave function for bound states,[19] will be developed. This will also provide a basis for a discussion of the magnetic moment problem. Here, the already known phenomenological relativistic corrections are largely sufficient to "explain" any discrepancy between the observed magnetic moment and the value one expects on the basis of the calculated D-state admixture.

Actually, the astonishingly good internal consistency of the electric and magnetic deuteron data obtainable on the basis of a completely nonrelativistic description, seem to indicate that all additional effects will have to be small. This applies to the kinematic relativistic effects, as well as to the phenomena discussed in the present paper. Their common feature is to depend strongly on the high momentum tail in $\varphi_D(\pi)$, or on the behavior of the wave function near the origin in the configuration space of the relative motion. [See for instance our discussion of Eq. (33)]. The magnitude of the exchange effect discussed in this paper agrees essentially with this condition of smallness. Since we

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[16] Kolsky, Phipps, Jr., Ramsey, and Silsbee, Phys. Rev. **81**, 1061 (1951).

[17] Meanwhile some of this material has been published by H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951).

[18] M. Levy, Phys. Rev. **84**, 441 (1951).

[19] M. Gell-Mann and F. Low, Phys. Rev. **84**, 350 (1951); E. E. Salpeter and H. A. Bethe, Phys, Rev. **84**, 1232 (1951).

have actually calculated the expectation value of a too singular operator, we expect that in a consistent theory the exchange effects will be even smaller than our present results. It remains to be seen, however, whether a completely covariant description of the two nucleon system[19] will corroborate the point of view advocated here.

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The Total Neutron Cross Section of Nitrogen*

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The total cross section of nitrogen has been measured for neutrons in the energy range 200 to 1800 kev, using scatterers of liquid nitrogen and of lithium azide. Eleven resonances were found, corresponding to excited states in the compound nucleus N^{15}, with natural widths of from 3 to 54 kev. Nine of these can be identified with resonances previously known in the reactions $N^{14}(n,p)C^{14}$, $N^{14}(n,\alpha)B^{11}$, or $C^{14}(p,n)N^{14}$. From a comparison of the measured and computed elastic scattering cross sections, J-values can be assigned to most of the observed states. In some cases the parity also can be determined and the effective level spacing D_J computed for decay by neutron, proton, or α-particle emission. At least for proton emission, it must be concluded that the quantities D_J, which contain implicitly the matrix elements for the transition, can vary by a factor of at least 100.

I. INTRODUCTION

THE virtual excited states of N^{15} offer an excellent opportunity to study, with present experimental techniques, competitive reactions in a light nucleus. Three different reactions, $n+N^{14}$, $p+C^{14}$ and $\alpha+B^{11}$, lead to virtual levels of N^{15}. Furthermore, the levels of N^{15} become virtual with respect to proton, neutron, and α-particle emission at roughly the same excitation energies: 10.21, 10.83 and 10.99 Mev, respectively.[1] Consequently, with electrostatic generators capable of 3- or 4-million volts, one is able to study the virtual levels in the region above 11 Mev by observing twelve different reactions. Of these, the (p,α), (α,p), (p,p), (α,α), (α,γ), and (p,γ) have thus far not been investigated. The reactions (n,p), (p,n), (n,α), (α,n), (n,γ), and (n,n) have been studied with different degrees of resolution and over various ranges of excitation energy.[2]

In particular, the two inverse reactions, $N^{14}(n,p)C^{14}$ and $C^{14}(p,n)N^{14}$, have been studied with fairly good resolution. Johnson and Barschall[3] measured the absolute cross section for the (n,p) and (n,α) processes and found three strong resonances, with indications of several weaker ones, in the neutron energy range 0.2 to 2.0 Mev. The relative neutron yield from the inverse (p,n) reaction has been measured over this same range of excitation energies by Roseborough *et al.*[4] Using somewhat better resolution, they found nine clearly defined resonances. In general these two investigations are in good agreement, the existing differences being ascribed to the degree of resolution employed.

With these data available, it was thought that a measurement of the total neutron cross section with good resolution would be of considerable interest. From the total neutron cross section and the (n,p) and (n,α) cross sections, one can deduce the elastic neutron scattering cross section, a quantity which is interpreted with some reliability by present nuclear theory. One can then hope to obtain a more detailed picture of the process by assigning total angular momentum and parity values to the virtual states and angular momentum values to the reacting particles.

II. EXPERIMENTAL METHOD

The $Li^7(p,n)Be^7$ reaction was used to produce monoenergetic neutrons of variable energy. Protons of well-defined energy were obtained from the Rockefeller electrostatic generator. After acceleration the protons are analyzed by a ninety-degree deflection in a magnetic field that is both controlled and measured by a proton magnetic moment resonance device. The generator voltage is stabilized by a variable corona load which is driven by an error signal produced by the proton beam

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1 Li, Whaling, Fowler, and Lauritsen, Phys. Rev. 83, 512 (1951).

2 For a summary see Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. 22, 291 (1950).

3 C. H. Johnson and H. H. Barschall, Phys. Rev. 80, 818 (1950).

4 Roseborough, McCue, Preston, and Goodman, Phys. Rev. 83, 1133 (1951).