This corresponds to the classical virial theorem, since in the presence of a magnetic field the right-hand side of (1) is replaced by $\langle\boldsymbol{\alpha}\cdot\boldsymbol{\pi}\rangle$ in the quantum case.

Perhaps the most useful form of the virial theorem is the case corresponding to no magnetic field,

$$-\langle\mathbf{r}\cdot\mathbf{F}\rangle=\langle\boldsymbol{\alpha}\cdot\mathbf{p}\rangle=\langle E-e\phi-\beta\rangle. \tag{6}$$

It is clear that (5) and (6) do not apply for continuum states. For example, for a free particle $\langle\boldsymbol{\alpha}\cdot\mathbf{p}\rangle=\langle p^2/E\rangle$.

Alternatively, for a static potential ϕ, we consider the expectation value for any stationary state of the anticommutator of β and H to obtain

$$\langle\beta(E-e\phi)\rangle=1, \tag{7}$$

(that is, $\langle\beta\boldsymbol{\alpha}\cdot\mathbf{p}\rangle=0$) which gives the well-known result $\langle\beta\rangle=1/E$ for a free particle. However, for a bound state in the Coulomb field, it follows from (6) that

$$\langle\beta\rangle=E, \tag{7a}$$

so that in this case,

$$e\langle\beta\phi\rangle=E^2-1. \tag{7b}$$

The nonrelativistic limit $e\langle\phi\rangle=-2\langle T\rangle$, T the nonrelativistic kinetic energy operator, follows from (7b); and more generally from (6) the usual limiting form $e\langle\mathbf{r}\cdot\nabla\phi\rangle=2\langle T\rangle$ results.

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Soft and Hard F-Centers

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OBERLY[1] has suggested that two types of F-centers exist. The soft type can be bleached easily by light, while the hard type is relatively unaffected. The suggestion is based on his photoconductivity measurements and work by Petroff.[2] The author would like to explain the observations in an alternate manner which does not require two types of centers and agrees with the conventional model.[3]

Using x-rayed KBr, Oberly measured at room temperature the changes of $\eta\omega$ (η is the quantum yield and ω is the electron range) caused by irradiation with F-light. He measured $\eta\omega$ as a function of wavelength after a 7- and a 23-minute exposure to F-light. Between 550 mμ and 1000 mμ, $\eta\omega$ decreases between the 7- and the 23-minute measurements while the reverse is true in the violet beyond the F-band.

One interpretation is that η changes because the soft centers disappear. This requires, in the author's opinion, soft and hard F, R, M, and N centers.

The author would like to suggest that ω changes. During the exposure to F-light negative-ion vacancies are formed from those centers which release electrons. We shall assume that negative-ion vacancies remain as incipient vacancies during the course of the experiments. Thus when the crystal is exposed to F-light, the concentration of negative-ion vacancies and the corresponding macroscopic cross section for an electron to be captured by a vacancy increases; the latter in turn decreases ω. Oberly's data is explained as follows: (1) The photoconductivity between 550 mμ and 1200 mμ is due to electrons whose ω decreases during F-light exposure. (2) The photoconductivity between 470 mμ and 550 mμ is due to holes whose ω is not affected by the concentration of negative-ion vacancies. (3) Oberly observed that an exposure to F-light affects photoconductivity measurements more than absorption measurements. The reason seems to be that absorption depends only on the concentration of F-centers, while photoconductivity depends critically on the concentration of negative-ion vacancies as well as the concentration of F-centers.

Another reason which might cause one to believe that there are two types of F-centers is that the half-width of the band varies experimentally.[4] Some F-centers may be surrounded by a perfect lattice, while other F-centers may be near vacancy clusters or dislocations. The centers near a perturbation in the lattice probably have a different half-width than those in a nearly perfect lattice. The method of excitation and the past history of the crystal would seem to determine the location of the F-centers and their half-width. The variation in the F-centers seems to be a matter of degree rather than a matter of having two types with distinct η.

Petroff has suggested two types of F-centers to explain the rate of growth of the M-band. Seitz[3] has proposed that an M-center is an electron attached to two negative and one positive-ion vacancy. It seems probable that the M-band is formed by breaking up vacancy clusters and that the intermediate state may be a vacancy complex which gives no absorption rather than a center hidden under the F-band.

There seems to be the possibility of explaining Oberly's data without changing the present picture of color centers. The author will not attempt to explain some secondary effects observed by Oberly which need further experimental exploration. At present there seems to be no reason to believe that these secondary effects will require a more elaborate model.

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Optical Focusing in Constant Radius Accelerators

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THAT the focusing forces are weak at low energies and large radii in electron accelerators with magnetic fields characterized by the usual parameter n can be readily deduced from the definition of this quantity, namely,

$$n\equiv-(R/H_z)(\partial H_z/\partial r)\equiv-(R/H_z)(\partial H_r/\partial z),$$

where R is the equilibrium orbit radius, H_z is the vertical field at the orbit, and H_r is the radial field at the orbit. We find in fact that

$$\Delta z/\Delta H_r=n^{-1}R^2/(RH_z),\quad \Delta r/\Delta H_z=(1-n)^{-1}R^2/(RH_z).$$

The quantity on the left is a direct measure of the orbit shift due to a stray field ΔH_r or ΔH_z and thus of the amount the particle will go astray due to this field. RH_z depends only on the particle energy, which must be considered at its lowest value, namely, the injection energy. The value of n is of course restricted to lie between 0 and 1.

It is evident that for machines of large radius a stronger focusing force at injection is very desirable. Such a force can be supplied by a sequence of magnetic lenses regularly spaced around the acceleration tube. Because of the axial symmetry of this arrangement there is no theoretical limit, short of the fact that the lenses must be discrete, on the amount of focusing force available. It will be assumed that the field strength of the lenses is constant in the time, at least after a certain time. This condition must be fulfilled because of the size limitation of the lenses, which must fit into the available aperture. The calculations show that there is no adiabatic variation of oscillation amplitude with particle acceleration if the lens fields are constant. It may thus be desirable to supply a small increase in the lens field during the early part of the acceleration cycle to insure that the orbits will lie within the acceleration tube.

The separation of the lenses should be considerably less than four focal lengths, the theoretical maximum. The individual lens separations and focal lengths should be held constant with respect to the average to a tolerance of about 1 percent for an average machine to avoid catastrophic resonance effects as the high transverse oscillation frequency corresponding to the relatively strong low-energy focusing force is gradually reduced with increase in the particle energy. It is of course necessary to provide the usual transverse field focusing at high energies.

Preliminary tests of twelve lenses equally spaced on the Naval Research Laboratory small aperture accelerator (radius $2\frac{1}{2}$ feet, aperture cross section 30-mm diameter) gave excellent focusing and evidence of many electron circuits through the tube on a fluorescent screen inserted into the path of the particles. This behavior was not possible at the injection energy of about 30 kv prior to the use of lens focusing.

A paper dealing with the theoretical aspects of the problem as well as with experimental results using more accurately spaced and wound lenses will be issued when the latter information has been obtained.

The Electron Neutron Interaction

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A WEAK, attractive interaction between neutrons and electrons has been observed experimentally.[1-4] From a qualitative point of view, such an effect is in accord with meson theory, for if a neutron can dissociate into charged particles,

$$N \leftrightarrow P + \pi^-,$$

it should have some interaction with an electric field. However, although several authors[5-8] have treated this problem in detail, there has been some confusion as to just what value meson theory gives for this interaction. For instance, using the same type of theory—pseudoscalar mesons, pseudoscalar coupling, with the usual weak coupling perturbation theory carried to second order in the meson-nucleon coupling—different methods of calculation have produced different answers. Thus, Slotnick and Heitler, and Dancoff and Drell obtain values about four times larger than do Case and Borowitz and Kohn. Moreover, Foldy's phenomenological treatment[9] gives a value lying between these two results.

Of course, meson theory weak coupling calculations of such effects cannot be taken too seriously—for instance, they do not even give the correct neutron to proton magnetic moment ratio. Nevertheless, in view of recent interest in the electron-neutron interaction[10,11] it seems desirable to state definitely just what value pseudoscalar meson theory does predict for this effect and to explain the relation of the various results referred to above. This note summarizes the results of an investigation aimed at settling these questions. A calculation of the electromagnetic properties of neutrons has been carried out with special attention to the matter of interpretation of the terms obtained from field theory. A more detailed account will be submitted for publication shortly.

As in previous calculations, the meson-nucleon coupling term in the Hamiltonian density is taken to be

$$\mathcal{H}' = g\sqrt{2}(\psi_N{}^\dagger\gamma^5\psi_P\phi^* + \psi_P{}^\dagger\gamma^5\psi_N\phi),$$

where ϕ is the charged meson field and ψ_P, ψ_N are the nucleon fields. (The electromagnetic properties of neutrons are the same for both charged and symmetric meson theories.) With the usual Feynman-Dyson techniques it can be shown that as a consequence of this coupling, the (first-quantized) Dirac equation for a single neutron becomes

$$(m\beta + \boldsymbol{\alpha}\cdot\mathbf{p} + H_1)\psi = i\dot{\psi}, \tag{1}$$

where, dropping terms which contain higher order derivatives of the external potential A_μ or higher powers of g,

$$H_1 = -\tfrac{1}{2}\mu\beta F_{\mu\nu}\sigma_{\mu\nu} - i\lambda\beta\Box^2 A_\mu\gamma^\mu,$$

with

$$\begin{aligned} \mu &= -(eg^2/8\pi^2 M)f_0(\eta), \quad \lambda = (eg^2/8\pi^2M^2)f_1(\eta),\\ f_0(\eta) &= 1-\eta\log\eta^{\frac{1}{2}} + (\eta-2)\eta^{\frac{1}{2}}(4-\eta)^{-\frac{1}{2}}\cos^{-1}(\eta^{\frac{1}{2}}/2),\\ f_1(\eta) &= (13-4\eta)(12-3\eta)^{-1} + (\tfrac{1}{4}-\tfrac{2}{3}\eta)\log\eta\\ &\quad + \eta^{\frac{1}{2}}(4-\eta)^{-\frac{3}{2}}(-35/3+17\eta/2-4\eta^2/3)\cos^{-1}(\eta^{\frac{1}{2}}/2). \end{aligned} \tag{2}$$

M is the nucleon mass, and η is the square of the meson to nucleon mass ratio.

An unambiguous interpretation of the additional term H_1 can best be obtained by considering the two-component Pauli equation to which (1) reduces in the nonrelativistic limit. (The experiments always use slow neutrons.) This is

$$[M + p^2/2M - \mu\mathbf{H}\cdot\boldsymbol{\sigma} - (\mu/2M + \lambda)\,\mathrm{div}\mathbf{E}]\psi = i\dot{\psi}, \tag{3}$$

showing that while the first term of H_1 alone is responsible for the magnetic moment, both terms contribute to the interaction with an electric field—in fact, as shown below, the two contributions are of the same order of magnitude. (An additional Thomas type term has been omitted from (3) since it cannot contribute to the observed scattering effects.[1])

To express these results in conventional form, take the source of $\mathbf{E}$ to be an electron, so that $\mathrm{div}\mathbf{E} = -4\pi e\rho_0$, where ρ_0 is the electron particle density, $\int d\tau\rho_0 = 1$. Then the effective potential which the neutron sees is

$$V = (e^2g^2/2\pi M^2)(-\tfrac{1}{2}f_0 + f_1).$$

The conventional "equivalent well depth," V_0, defined in terms of the classical electron radius $r_e = e^2/m$, is

$$V_0 = \int d\tau V \bigg/ \frac{4\pi}{3}r_e{}^3 = \frac{3}{2\pi}\frac{g^2}{4\pi}\left(\frac{m}{M}\right)^2\frac{m}{e^4}(-\tfrac{1}{2}f_0 + f_1). \tag{4}$$

Using $\sqrt{\eta} = 0.151$[12] for the meson-nucleon mass ratio gives $f_0 = 0.820$; $f_1 = -0.131$; $(-\frac{1}{2}f_0 + f_1) = -0.541$. Adjusting g so that the neutron magnetic moment μ is given correctly[13] by (2) leads to

$$g^2/4\pi\hbar c = 7.33, \tag{5}$$

so that

$$V_0 = -5.38 \text{ kev}, \tag{6}$$

in fair agreement with the recent experimental figures of (5300 ± 1000)[3] and (4100 ± 1000)[4] electron volts.

The relation of these results to those of other authors can be expressed quite simply. Case and Borowitz and Kohn neglect the contribution of the first term of H_1 to the electron-neutron interaction. This is equivalent to replacing $(-\frac{1}{2}f_0 + f_1)$ in (4) by just f_1, which, with the choice (5) for the coupling constant, gives

$$V_{C;B,K} = -1.30 \text{ kev}. \tag{7}$$

On the other hand, a phenomenological treatment like Foldy's, in which the magnitude of the electron-neutron interaction is deduced from the value of the magnetic moment only, corresponds to using just $\frac{1}{2}f_0$ in (4), leading to

$$V_F = -4.08 \text{ kev}. \tag{8}$$

Finally, the calculations by Slotnick and Heitler, and Dancoff and Drell of the matrix element for neutron scattering in an electric field in nonrelativistic approximation give, with the present choice of coupling constant,

$$V_{S,H} = -4.99 \text{ kev}, \quad V_{D,D} = -6.10 \text{ kev}. \tag{9}$$

These nearly agree with the value (6), thus supporting Foldy's suggestion that neglect of the first term in H_1 is largely responsible for the discrepancy between (7) and (9).

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