An Investigation of the Magnetic Properties of Liquid He^3†

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CONTINUED interest in the magnetic properties of liquid He^3 [1–3] has prompted the authors to place on record their results already briefly communicated.[4] Although Goldstein[1] showed that the possibility of nuclear ferromagnetism in liquid He^3 was very slight, experimental examination of its magnetic properties seemed desirable. Two methods were employed. The first was a heterodyne beat method. A coil of inductance L surrounding the sample formed part of an oscillating circuit, the frequency of which was essentially $\nu=[2\pi(LC)^{\frac{1}{2}}]^{-1}$. Since the inductance of the coil is proportional to the permeability of the medium surrounding it, condensation of any material within the coil results in a frequency shift $-d\nu/\nu=dL/2L=fd\mu/2\mu=2\pi f\kappa$, where f is the filling factor for the coil equal to 0.098 for the coil used, μ is the permeability of the sample, and κ is its volume susceptibility. The oscillator was operated at 1.47 Mc. The system was calibrated with liquid oxygen ($d\nu=272\pm3$ cycles). He^4 condensed inside the coil gave no sudden shift larger than ±5 cycles although a gradual drift in the frequency occurred, probably due to change in bath level. With He^3 at 2.08°K and at 1.27°K there was no frequency shift larger than 5 cycles. Hence the volume susceptibility (κ) of liquid He^3 at these temperatures is less than 5×10^{-6}. If liquid He^3 were a nuclear ferromagnetic, long relaxation times for the domains would give a null result, as would magnetic saturation of the sample by the earth's field. The second method employed was, therefore, a low frequency (60 cycles) method which avoided these objections, unless the relaxation time was of the order of hours. Two matched 6000-turn coils were mounted above one another and connected in opposition. A Pyrex sample bulb several cubic millimeters in volume was situated in the upper search coil, and the whole assembly was located in a cryostat around which was wound a Helmholtz coil. The earth's field in the vicinity of the search coils plus that from the permanent magnetization of the associated equipment yielded a resultant field almost horizontal equal to 0.65 oersted. By operating the Helmholtz coil to give a 60-cycle oscillating field of 0.65-oersted maximum, the resultant field oscillating over a 90° arc, beginning 45° from the vertical. The signal from the search coils was taken through a phase adjusting and amplitude balancing network, and then through amplifiers to the Y-plates of an oscilloscope. The X-plates were connected across the Helmholtz coil input. With the sample bulb evacuated, the phase and amplitude of any remaining signal from the search coils was balanced to give only a horizontal line on the oscilloscope. Introduction of a substance of high permeability in the upper coil would then have transformed the oscilloscope trace into an ellipse.

Calibration of the system indicated that a 7-μv peak-to-peak signal from the search coils could be observed. The effective linkage of the number of turns of the search coil with the sample was determined to lie between 5 and 22 percent as follows: The initial and constant (for very low fields) permeability of a bundle of pure iron wire was measured. A sample of this iron was placed in a glass capillary (similar to the one used with the He^3), inserted in the search coil and the signal measured. From this the ratio of the effective number of turns to the total number was calculated to be 22 percent. Measurement of the paramagnetic susceptibility of oxygen using the same coil system but a higher gain amplifier and improved phase shifting network gave a filling factor of 5 percent.

If liquid He^3 were ferromagnetic, the induced emf for the above mentioned apparatus would be $e_{\text{ind}}=dF/dt=-\omega(4\pi\eta\mu)NAf\sin\omega t$ $=2.75f$ millivolts peak-to-peak, where $F=\Delta BANf\cos\omega t$, $\Delta B=$ difference in flux through sample area with and without sample, $A=$area of sample, $N=$number of turns in search coil, $f=$filling factor, $\eta=$ the number of He^3 atoms/cc$=1.6\times10^{22}$,[5] and $\mu=$magnetic moment of $He^3=1.07\times10^{-23}$ cgs units.[6] Using the above values for filling factor, the observed signal should have been $140<e_{\text{obs}}<600$ μv (peak-to-peak).

Two experiments were carried out with this apparatus. In the first, the sample bulb was flushed with O_2 gas before cooling the assembly and condensing the He^3 in an attempt to decrease the relaxation time.[7] The temperature, measured by the vapor pressure of the He^3, was 0.904°K. No change in the oscilloscope signal was observed upon condensation of the He^3. The sample was held at this temperature for about 40 minutes. In the second experiment, the He^3 was condensed at 2.51°K. The temperature was then gradually reduced to 1.35°K over a 45 minute period. Again, no change in the oscilloscope signal was observed.

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The Virial Theorem for a Dirac Particle*

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THE authors have recently had occasion to use a number of identities, satisfied by Dirac wave functions, which take the form of a virial theorem in the relativistic quantum case. In the hope that these results may prove useful to others they are communicated herewith.

The well-known relativistic virial theorem in the classical case is

$$-\overline{\mathbf{r}\cdot\mathbf{F}}=\bar{T}+\bar{L}_0, \tag{1}$$

where T and L_0 are the kinetic energy and Lagrangian for a free particle, the bar indicates a time average and $\mathbf{F}$ is the Lorentz force and in the quantum mechanical case we obtain the corresponding result by considering the expectation value

$$\left\langle\frac{d}{dt}\mathbf{r}\cdot\mathbf{p}\right\rangle=i\langle\mathbf{r}\cdot(H,\mathbf{p})+(H,\mathbf{r})\cdot\mathbf{p}\rangle=0, \tag{2}$$

where the round brackets in Eq. (2) designate commutators and the single particle Hamiltonian is

$$H=e\phi+\boldsymbol{\alpha}\cdot(\mathbf{p}-e\mathbf{A})+\beta. \tag{3}$$

We use $\hbar=m=c=1$. From (2) and (3) we obtain

$$\langle\boldsymbol{\alpha}\cdot\mathbf{p}\rangle=e\langle\mathbf{r}\cdot\nabla(\phi-\boldsymbol{\alpha}\cdot\mathbf{A})\rangle. \tag{4}$$

In addition, we may consider the expectation value of the operator with $\boldsymbol{\pi}=\mathbf{p}-e\mathbf{A}$ replacing $\mathbf{p}$ in (2). Then it follows that

$$\left\langle\frac{d}{dt}\mathbf{r}\cdot\mathbf{A}\right\rangle=\langle\boldsymbol{\alpha}\cdot\mathbf{A}\rangle+\langle\mathbf{r}\cdot(\boldsymbol{\alpha}\cdot\nabla)\mathbf{A}\rangle=0.$$

Using

$$\nabla\boldsymbol{\alpha}\cdot\mathbf{A}=\boldsymbol{\alpha}\times\text{curl}\mathbf{A}+(\boldsymbol{\alpha}\cdot\nabla)\mathbf{A},$$

the relativistic quantum-mechanical virial theorem may be written in the form

$$-\langle\mathbf{r}\cdot\mathbf{F}\rangle=\langle\boldsymbol{\alpha}\cdot(\mathbf{p}-e\mathbf{A})\rangle=\langle E-e\phi-\beta\rangle, \tag{5}$$

where E is the energy including rest energy and the Lorentz force operator is interpreted as follows,

$$\mathbf{F}=e(-\nabla\phi+\boldsymbol{\alpha}\times\text{curl}\mathbf{A}).$$