We have also calculated $|F|^2$ and $|G|^2$ for mirror nuclei having 6 protons and neutrons±one nucleon and 16 protons and neutrons±one nucleon, since the numbers 6 and 16 are assumed to represent closed configurations. The data are listed in Table II and shown in Fig. 1. Especially for the nuclei of mass numbers 11 and 31 the fit is bad. This can hardly be explained without the assumption of a rather strong distortion of the nuclear wave functions. Also the magnetic moments of these nuclei show that one does not have pure single particle states. Estimates indicate that perturbations of the order of magnitude required may arise from the coupling between the single particle motion and nuclear surface deformations.[6,7]

For the more complicated mirror nuclei the shell model wavefunctions are more ambiguous. However, in a few cases included in Table III the states are given uniquely by the $j-j$ coupling shell model together with the charge symmetry requirements. The results for these nuclei are plotted in Fig. 1, and it is seen that the matrix elements are slightly too small.

A more detailed account of β-matrix elements derived from nuclear shell models will appear in Dan. Mat. Fys. Medd.

We wish to thank Professor N. Bohr for his interest in our work, and we are indebted to Dr. Ben Mottelson and Mr. Aage Bohr for many valuable discussions and suggestions.

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Tests of Charge Independence from Pion Production in Nuclear Collisions*

A. M. L. Messiah
University of Rochester, Rochester, New York
(Received March 6, 1952)

New possibilities of testing the hypothesis of charge independence[1] arise in connection with studies of pion production in nuclear collisons. Formally, charge independence is most conveniently expressed with the help of the isotopic spin formalism which can be extended to include pions; it amounts to assuming that the total isotopic spin is a good quantum number in reactions involving nucleons and pions only.[2] If this assumption is true, it turns out that the cross sections for the production of charged and neutral pions in nuclear collisions must satisfy rather stringent relations.

Thus, consider a production process,

$$N_1+N_2\rightarrow N+\pi, \tag{1}$$

where N_1 and N_2 designate the colliding nuclei and N the outgoing nuclear fragments (bound or unbound). We designate by t_1, t_2, t, ϑ the isotopic spins of N_1, N_2, N, and π, respectively, ($\vartheta=1$) and by T the total isotopic spin ($|t_1-t_2|\leqslant T\leqslant t_1+t_2$).

The simplest case is $t_1=0$, $t_2=0$ (e.g., $d+\alpha$, $\alpha+\alpha$, $d+C^{12}$, $\cdots$); then $T=0$ and the cross sections σ_+, σ_0, σ_-, for the production of π^+, π^0, π^- at a given solid angle and with a given energy must be equal since there exists no preferred direction in charge space:

$$\sigma_+=\sigma_0=\sigma_-. \tag{2}$$

The equality of σ_+ and σ_- is merely a consequence of charge symmetry; therefore one must measure the π^0 yield if one wishes to use (2) as a test of charge independence.

When $t_1=0$, $t_2=\frac{1}{2}$ (e.g., $p+d$, $p+\alpha$, $p+C^{12}$, $\cdots$, $d+Be^9$, $\cdots$), $T=\frac{1}{2}$; therefore t may assume two values: $\frac{1}{2}$ or $\frac{3}{2}$. For convenience, we treat the particular case of p-d collisions; it will become apparent that the general case leads to identical results. The 3-nucleon system leads to one quartet and two doublets; the corresponding charge functions will be designated by the symbols ξ_q, ξ'_q, ξ''_q, respectively, where the subscript q stands for the total charge of the nuclear system (with the convention that the proton has isotopic spin $+\frac{1}{2}$, $q=t_z+\frac{3}{2}$) and ϑ_+, ϑ_0, ϑ_- will designate the charge functions of the 3 types of pions. Only 3 mutually orthogonal charge states can be formed with 3 nucleons and 1 pion, having the proper values $T=\frac{1}{2}$, $T_z=\frac{1}{2}$:

$$\begin{aligned}\chi&=\sqrt{\tfrac{1}{6}}\xi_1\theta_+-\sqrt{\tfrac{1}{3}}\xi_2\theta_0+\sqrt{\tfrac{1}{2}}\xi_3\theta_-\\ \chi'&=\sqrt{\tfrac{2}{3}}\xi_1'\theta_+-\sqrt{\tfrac{1}{3}}\xi_2'\theta_0\\ \chi''&=\sqrt{\tfrac{2}{3}}\xi_1''\theta_+-\sqrt{\tfrac{1}{3}}\xi_2''\theta_0.\end{aligned} \tag{3}$$

Therefore, the scattered wave function assumes the form $\Psi_\chi+\Psi'_{\chi'}+\Psi''_{\chi''}$ where Ψ, Ψ', and Ψ'' depend on the nucleon spins, the relative coordinates of the nucleons and the coordinate $\mathbf{r}$ of the pion relative to the center of mass of the nucleons. Using Eq. (3), this scattered wave can be rewritten in order to separate the terms corresponding to mesons of different charge, namely

$$\begin{aligned}&[\sqrt{\tfrac{1}{6}}\Psi\xi_1+\sqrt{\tfrac{2}{3}}(\Psi'\xi_1'+\Psi''\xi_1'')]\theta_+\\ &\quad-[\sqrt{\tfrac{1}{3}}\Psi\xi_2+\sqrt{\tfrac{1}{3}}(\Psi'\xi_2'+\Psi''\xi_2'')]\theta_0+[\sqrt{\tfrac{1}{2}}\Psi\xi_3]\theta_-.\end{aligned} \tag{4}$$

Let us consider first the cross sections for producing π^++H^3 and π^0+He^3, respectively. In this case, $t=\frac{1}{2}$ and the nuclear wave functions in the final state assume the form

$$\begin{aligned}\Phi_{H^3}&=\phi'\xi_1'+\phi''\xi_1''\\ \Phi_{He^3}&=\phi'\xi_2'+\phi''\xi_2'',\end{aligned} \tag{5}$$

where ϕ', ϕ'' are functions of the space and spin coordinates of the nucleons. That the same ϕ', ϕ'' appear in both wave functions in the way indicated is a consequence of the assumption of charge independence of nuclear forces. To find the desired cross sections, we take the scalar products of the scattered wave (4) with $\Phi_{H^3}\theta_+$ and with $\Phi_{He^3}\theta_0$ obtaining,

$$\begin{aligned}F_+(\mathbf{r})&=\sqrt{\tfrac{2}{3}}[\langle\phi'|\Psi'\rangle+\langle\phi''|\Psi''\rangle]\\ F_0(\mathbf{r})&=-\sqrt{\tfrac{1}{3}}[\langle\phi'|\Psi'\rangle+\langle\phi''|\Psi''\rangle].\end{aligned} \tag{6}$$

Since, according to (6), $F_+(\mathbf{r})=-2F_0(\mathbf{r})$, the same relationship holds between the scattering amplitudes; taking then the absolute squares,

$$\sigma(p+d\rightarrow H^3+\pi^+)=2\sigma(p+d\rightarrow He^3+\pi_0). \tag{7}$$

It is clear that relation (7) would hold if one replaced the two final states considered above, H^3 and He^3, by any other charge doublet.[3] In the same way, one finds that the cross sections for the production of π^+, π^0, π^- leading to nuclei in charge quartet states go as 1:2:3, to be compared with the ratios 2:1:0 found for doublet transitions. It follows that the cross sections σ_+, σ_0, σ_- defined above satisfy the relations,

$$\sigma_+/(1+2\rho)=\sigma_0/(2+\rho)=\sigma_-/3, \tag{8}$$

where ρ is the ratio of the doublet to quartet contribution. Equation (8) leads to the (rather weak) inequality: $\sigma_-<3\sigma_+$ and, by elimination of ρ, to

$$\sigma_0=\tfrac{1}{2}[\sigma_-+\sigma_+]. \tag{9}$$

The type of derivation given above applies to the more complicated cases and relations similar to (9) can be derived for any value of t_1 and t_2. In particular, in nucleon-nucleon collisions one finds

$$\sigma_0^{np}+\sigma_0^{pp}=\tfrac{1}{2}\sigma_+^{pp}+\sigma_+^{np}, \tag{10}$$

where the superscripts indicate the charge of the colliding nucleons, the subscript the charge of the meson produced. Equation (10) is valid provided the initial beams (n or p) have the same energy, and the mesons are observed with the same energy and at the same angle.[4] An interesting particular case of Eq. (10) is obtained by considering π^0 and π^+ production with deuteron formation;[5] then

$$\sigma(n+p\rightarrow d+\pi^0)=\tfrac{1}{2}\sigma(p+p\rightarrow d+\pi^+). \tag{11}$$

Relations (2), (9), (10) and their particular exemplifications like (7) and (11) provide many possible tests of the charge independence hypothesis. In order to eliminate small uncertainties in the predictions associated with the mass difference between the charged and neutral pion and between the proton and neutron on one hand, and with Coulomb effects on the other, experiments should be conducted with light nuclei at the highest possible meson energies.

The role of the constancy of the isotopic spin in nucleon-pion problems was called to the author's attention by Professor Fermi's stimulating lectures on pion scattering at the University of Rochester last January. Thanks are due to Dr. Marshak for his interest in this research and for several helpful comments and criticisms.

Note added in proof:—Since this letter was sent in, Dr. J. M. Luttinger has informed me that he has independently derived Eqs. (9) and (10).

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1 As usual, "charge independence" denotes the equality of $n-p$, $p-p$, and $n-n$ forces for states of the same spin and partity; "charge symmetry" refers to the equality of $n-n$ and $p-p$ forces alone.

2 For a general discussion of the applications of charge independence to pion-nucleon problems and of the extension of the isotopic spin formalism to the pion field, the reader is referred to the treatment of K. M. Watson [Phys. Rev. **85**, 852 (1952)] and also to W. Heitler [Proc. Roy. Irish Acad. **51**, 33 (1946)] and K. M. Watson and K. K. Brueckner [Phys. Rev. **83**, 1 (1951)].

3 In particular for final states containing the deuteron, as was pointed out by Dr. Chew.

4 Equation (10) reduces to Eq. (21) of Watson and Brueckner (reference 2) when, in accordance with their assumptions, the production of π^0 in $p-p$ collisions is forbidden.

5 Equation (11) arose out of a discussion between Dr. Marshak, Mr. Petschek, and the author.

Regularities in the Total Cross Sections for Fast Neutrons*

H. H. Barschall
University of Wisconsin, Madison, Wisconsin,
and
Los Alamos Scientific Laboratory, Los Alamos, New Mexico
(Received March 17, 1952)

RECENTLY Miller, Adair, and others[1] have measured the total cross sections for fast neutrons of many of the heavier elements in the energy range from about 0.1 to 3 Mev. It was found that, disregarding the effect of individual resonances, neighboring elements show very similar variations of cross section with energy while there are marked differences in the shape of the cross section curves between elements of appreciably different atomic number. This behavior is shown in Fig. 1. In this figure the measured cross sections divided by the geometrical area of the nucleus are plotted against neutron energy. The nuclear area was calculated for a nuclear radius of $1.45\times A^{\frac{1}{3}}\times 10^{-13}$ cm. The elements are arranged according to their atomic weight A, since in the case of Te and I it was found that a smoother surface resulted from such an arrangement than if the elements were ordered according to their atomic number. No attempt has been made to include details of the fluctuations in cross section; in particular, the behavior at thermal and epithermal energies has been ignored, since the cross sections at the lowest energies depend primarily on the presence of individual resonances.

An interesting feature of the surface shown in Fig. 1 is the large value of the cross section at low energies for elements around Sr. This peak appears to shift to higher energies with increasing atomic weight. Furthermore, the cross section of the elements heavier than Ir exhibit a minimum at neutron energies around 1 Mev.

The behavior shown in Fig. 1 is in disagreement with the continuum theory proposed by Weisskopf and his collaborators,[2] since this theory predicts a monotonic decrease of the total cross section with energy. Following a suggestion by Wigner, Weisskopf[3] has more recently calculated the energy dependence of the cross section on the basis of a single particle interaction and finds that variations of the total cross section with energy similar to those shown in Fig. 1 may be obtained.

* Work performed under the auspices of the AEC.

1 Miller, Fields, and Bockelman, Phys. Rev. **85**, 704 (1952); more complete reports will be published later.

2 Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947); H. Feshbach and V. F. Weisskopf, Phys. Rev. **76**, 1550 (1949).

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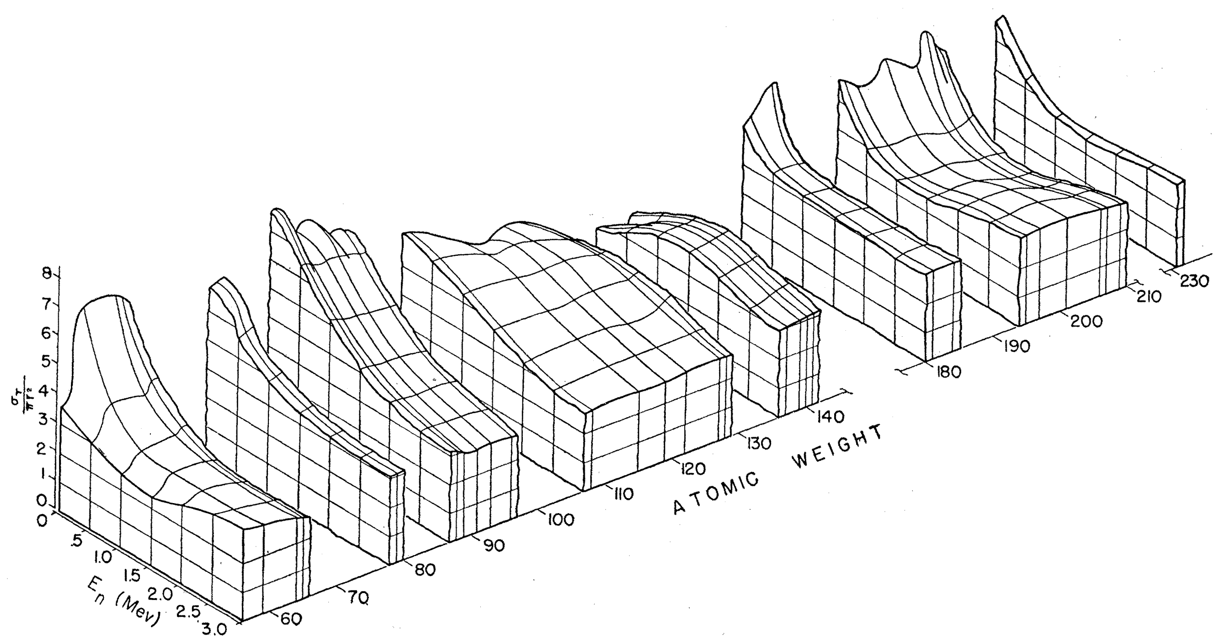

FIG. 1. Total neutron cross sections of elements heavier than Mn as a function of neutron energy. The surface is based on measurements for the atomic weights at which straight vertical lines appear in the figure.