The mean value of the coordinate of the electron taken over the Wannier localized function $\varphi_{ln}(x)$ is

$$(\varphi_{ln}(x),\ x^\mu\varphi_{ln}(x))=x_l{}^\mu+X_n{}^\mu. \tag{1}$$

Thus $X_n{}^\mu$ is the mean displacement of the center of charge from the origin in unit cell.

It was shown in (I) that for any choice of phase of the Bloch functions ψ_{kn} so that they are periodic in p space, there exists a function $\eta_n(p)$ such that

$$\xi_{nn}{}^\mu=\frac{\partial}{\partial p^\mu}\eta_n(p). \tag{2}$$

The error made in (I) was to treat η_n as a periodic function of p_μ whereas it had been proven only that $\xi_{nn}{}^\mu$ was a periodic function. Since $\xi_{nn}{}^\mu$ is periodic, however, the most general form of η_n is

$$\eta_n(p)=\eta_n'(p)+X_n{}^\mu p^\mu. \tag{3}$$

In a band n for which some of the $X_n{}^\mu$ are different from zero, the treatment of (I) requires that across some plane in the Brillouin zone the phase of the wave function ψ_{kn} jumps by the increment of $X_n{}^\mu p^\mu$ across the zone. The difficulty noted in the footnotes to (I) is that derivatives of the ψ_{kn} by k do not converge on this plane.

There is, of course, an arbitrariness in the definition of the Bloch functions ψ_{kn} on which the theory is based, since it has been assumed of them only that they are eigenfunctions of H_0 and are continuous periodic functions of k. An equally suitable set of basic functions $\bar{\psi}_{kn}$ are obtained from the ψ_{kn} by multiplying them by a factor $U(\hbar k, n)$ which is any continuous and periodic function of k with modulus unity. If $U(\hbar k, n)$ is chosen to be

$$U(\hbar k, n)=\exp[-i\eta_n'/\hbar], \tag{4}$$

and the $\bar{\psi}_{kn}$ used as the basic functions one gets

$$\xi_{nn}{}^\mu=X_n{}^\mu. \tag{5}$$

Further, by a suitable choice of $U(\hbar k, n)$, $X_n{}^\mu$ can always be made so small that if plotted from the center of unit cell it does not extend past the surface of the cell. The change of phase of basic functions is accomplished by transforming all operators in the Wannier theory with the unitary operator

$$\mathbf{U}=\Sigma_n\mathfrak{v}_{nn}U(p, n). \tag{6}$$

As a consequence of the argument above, one can now show that the treatment of the problems discussed in (I) is essentially correct. In particular, the chief modification in the equation of motion in a magnetic field is that Luttinger's Hamiltonian should involve $E_n(p+(e/c)\mathfrak{A}(x+X_n))$ rather than $E_n(p+(e/c)\mathfrak{A}(x))$. By proper choice of the origin in unit cell one could always arrange that for any one band $X^\mu=0$. Therefore, the conclusions originally drawn as to the validity of Luttinger's form of Wannier's one-band theory remain unchanged.

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The Saturation Property of Nuclear Forces

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IN the Fermi gas model of heavy nuclei all the lowest states of neutrons and protons are occupied up to those with a maximum momentum. On account of the Pauli principle the interaction potentials between nucleons in heavy nuclei are expected to be different from those between free nucleons.

The matrix element for the transition from an initial state in which two nucleons have momenta $\mathbf{p}_1$, $\mathbf{p}_2$ to a final state with the momenta $\mathbf{p}_1-\mathbf{k}$, $\mathbf{p}_2+\mathbf{k}$ *via* a neutral scalar meson field is given by

$$g^2f(p_1)f(p_2)[1-f(p_1-k)][1-f(p_2+k)]\left\{\frac{1}{E(p_1)-E(p_1-k)-\omega_k}+\frac{1}{E(p_2)-E(p_2+k)-\omega_k}\right\}\exp i(\mathbf{k},\ \mathbf{x}_2-\mathbf{x}_1)(\psi_1'{}^*\beta\psi_1)(\psi_2'{}^*\beta\psi_2).$$

Here $f(\mathbf{p})$ is the probability that the state $\mathbf{p}$ is occupied and E, ω are the energies of the nucleon and the meson, respectively. The interaction potential between two nucleons having momenta $\mathbf{p}_1$, $\mathbf{p}_2$ within the Fermi sphere is given by

$$V(1, 2)=g^2\beta^{(1)}\beta^{(2)}\int[1-f(p_1-k)][1-f(p_2+k)]\times\left\{\frac{1}{E(p_1)-E(p_1-k)-\omega_k}+\frac{1}{E(p_2)-E(p_2+k)-\omega_k}\right\}\times\exp i(\mathbf{k},\ \mathbf{x}_2-\mathbf{x}_1)\frac{d\mathbf{k}}{(2\pi)^3}.$$

If we apply the energy law, two nucleons within the Fermi sphere cannot go over to the states outside the fermi sphere. However, according to Araki and Huzinaga,[1] the energy law cannot be made use of in general when forces between fermions in bound states are to be derived. In nonrelativistic approximation we get the following potential

$$V(1, 2)=-g^2\int[1-f(p_1-k)][1-f(p_2+k)]\frac{\exp i(\mathbf{k},\ \mathbf{x}_2-\mathbf{x}_1)}{\omega_k{}^2}\frac{d\mathbf{k}}{(2\pi)^3}.$$

We now calculate the mean energy of $V=\frac{1}{2}\Sigma_{i,k}V(i, k)$ using the Fermi gas model.

$$\bar{V}=\bar{V}_{\rm ord}+\bar{V}_{\rm exc}$$

$$\bar{V}_{\rm ord}=\frac{1}{2}\Big\{\sum_{p_1,p_2}^{(+)}\int\psi_{p_1}{}^*(x_1)\psi_{p_2}{}^*(x_2)V(1, 2)\psi_{p_1}(x_1)\psi_{p_2}(x_2)dx_1dx_2+\sum_{p_1,p_2}^{(-)}\int\psi_{p_1}{}^*(x_1)\psi_{p_2}{}^*(x_2)V(1, 2)\psi_{p_1}(x_1)\psi_{p_2}(x_2)dx_1dx_2+2\sum_{p_1}^{(+)}\sum_{p_2}^{(-)}\int\psi_{p_1}{}^*(x_1)\psi_{p_2}{}^*(x_2)V(1, 2)\psi_{p_1}(x_1)\psi_{p_2}(x_2)dx_1dx_2\Big\},$$

$$\bar{V}_{\rm exc}=\frac{-1}{2}\Big\{\sum_{p_1,p_2}^{(+)}\int\psi_{p_1}{}^*(x_1)\psi_{p_2}{}^*(x_2)V(1, 2)\psi_{p_2}(x_1)\psi_{p_1}(x_2)dx_1dx_2+\sum_{p_1,p_2}^{(-)}\int\psi_{p_1}{}^*(x_1)\psi_{p_2}{}^*(x_2)V(1, 2)\psi_{p_2}(x_1)\psi_{p_1}(x_2)dx_1dx_2\Big\},$$

where $\psi_p(x)=\Omega^{-\frac{1}{2}}\exp i(\mathbf{p},\ \mathbf{x})$ and $\Sigma^{(+)}$, $\Sigma^{(-)}$ represent summations extending over the occupied neutron states and proton states, respectively. If we first perform the integration over x, we find that only $k=0$ contributes to $\bar{V}_{\rm ord}$, but $[1-f(p_1)][1-f(p_2)]$ vanishes because of $f(p_1)=f(p_2)=1$. So the nonsaturating $\bar{V}_{\rm ord}$ vanishes, but $\bar{V}_{\rm exc}$ does not.

The details will be published elsewhere.

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β-Decay of Mirror Nuclei and the Shell Model

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SPIN and parity predictions from the nuclear shell model are in good agreement with nearly all experimental β-decay data.[1] For mirror nuclei the ft-values agree within a factor two with the theoretical nuclear matrix elements for suitably chosen β-decay coupling constants.[2-3]

It is the intention in this note to base a determination of these coupling constants on those mirror nuclei which have closed shells (0, 2, 8, 20) in both neutrons and protons±one particle. For these