Atomic Masses from Titanium through Zinc*

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The double focusing mass spectrometer previously described has been used to measure 27 atomic masses in the region from titanium to zinc. When supplemented by our earlier values and nuclear reaction energies the data give the masses of 81 nuclides between mass numbers 31 and 70. A fit of the semi-empirical mass formula of Wigner indicates the existence of magic numbers at 20 and 28 protons and neutrons associated with discontinuities in the binding energy surface. The discontinuities are primarily changes in slope of the surface.

IN an earlier communication[1] we reported on the determination of atomic masses in the region about mass 40. The study has now been extended through zinc with the result that values are now available for almost every stable nuclide from S^{32} through Zn^{70}. The measurements were made with a double focusing mass spectrometer. A brief description of the apparatus and procedure of measurement has already been given[1,2] and hence will not be repeated here.

DOUBLET MEASUREMENTS

Table I gives the mass doublets investigated together with the mass differences found. As in previous work a "run" consisted of 10 or more consecutive tracings of the mass spectrum with alternate forward and backward sweep. Since readings are taken from half-height to half-height of the peaks compared—and on each side of the peak—the number given for the mass difference in a single run is the average of at least 20 readings. Runs were taken on different days over a period of several months. The errors given are probable errors computed statistically. As in previous work a given hydrocarbon peak may have an unresolved satellite produced by ions containing one less hydrogen atom but a C^{13} atom in place of one of the C^{12} atoms. From a measurement of intensity of the lighter hydrocarbon fragment a suitable correction may be made to the peak in question. The last column gives values for the corresponding doublets as found by other investigators.

ATOMIC MASSES

Table II gives the atomic masses computed from the present data. In order to follow a consistent procedure we have employed the masses for H^1 and C^{12} found in this laboratory. In our earlier paper[1] we discuss the discrepancies in the mass values for these atoms as given by different investigators. Should our values turn out to be slightly in error, the computed masses for the atoms now investigated would not change enough to affect the validity of the conclusions reached later in the present paper.

Table III compares some of our mass differences with values computed from nuclear reaction data. In general, the agreement is excellent. However, a discrepancy in the value for $Cr^{52}-V^{51}$ appears to be present.

COMPARISON WITH THE WIGNER FORMULA

By combining our mass spectroscopic data with nuclear reaction energies we have constructed an extensive table of masses from $A=31$ to $A=66$ in order to search for irregularities which could be associated with nuclear shell structure. This region includes proposed "magic numbers" 20 and 28 for both neutrons and protons. The odd-even fluctuations characteristic of the lighter nuclei make it difficult to see discontinuities in the binding energy surface or packing fraction curve. Others[3,4] have attempted to remove these effects by comparing masses with those predicted by the Bohr-Wheeler formula. We believe a better approach is to fit the Wigner formula to the data.

The binding energy of a nucleus may be expressed as the sum of a Coulomb energy, a kinetic energy, and a potential energy from nuclear forces. Wigner[5] proposed approximations for each of these terms. He computes the Coulomb energy for a uniformly charged sphere with a radius proportional to $A^{\frac{1}{3}}$. The radius constant may be evaluated from mirror nuclei giving

$$\mathrm{CE}=0.635Z(Z-1)/A^{\frac{1}{3}} \text{ milli-mass units.} \quad (1)$$

He computes the kinetic energy for a degenerate Fermi gas occupying the same volume. Using the radius constant from the Coulomb energy we find

$$\mathrm{KE}=14.640A+32.53(T^2+\tfrac{1}{2}\delta)/A \text{ mMU} \quad (2)$$

where $T=(N-Z)/2=$isotopic spin.

$$\delta=\begin{cases}0 \text{ for even } Z, \text{ even } N.\\ 1 \text{ for even } Z, \text{ odd } N.\\ 1 \text{ for odd } Z, \text{ even } N.\\ 2 \text{ for odd } Z, \text{ odd } N.\end{cases}$$

* Research supported by the joint program of the ONR and AEC.

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[2] A. O. Nier and T. R. Roberts, Phys. Rev. **81**, 507 (1951).
[3] C. H. Townes and W. Low, Phys. Rev. **79**, 198 (1950).
[4] A. H. Wapstra, Phys. Rev. **84**, 837, 838 (1951).
[5] E. Wigner, University of Pennsylvania Bicentennial Conference (1941), University of Pennsylvania Press, Philadelphia. See also G. Gamow and C. L. Critchfield, *Atomic Nucleus and Nuclear Energy Sources* (Oxford Press, London, 1949).

Wigner expresses the potential energy in terms of two functions L' and L, where L' is the average magnitude of potential energy between pairs of nucleons from forces independent of symmetry. L is the average magnitude of the potential energy from forces which change sign with symmetry. Thus

$$\mathrm{PE}=-\tfrac{1}{2}A(A-1)L'-\Xi L \text{ mMU}, \quad (3)$$

where Ξ is the number of symmetric minus the number of antisymmetric couplings. In our region of A, Ξ is a negative number and may be expressed for our purpose as

$$\Xi=2A-\tfrac{1}{8}A^2-\tfrac{1}{2}T(T+4)-\tfrac{3}{4}\delta. \quad (4)$$

In the absence of spin orbit coupling AL and AL' should be approximately constant and should be smooth functions of A. The sum of terms (1) (2) (3) is the binding energy

$$\mathrm{BE}=\mathrm{CE}+\mathrm{KE}+\mathrm{PE}. \quad (5)$$

The binding energy and potential energy are negative quantities.

The binding energy of the nucleus may be computed from the atomic mass M, neglecting the electronic binding energies,

$$\mathrm{BE}=M-(ZM_{\mathrm{H}}+NM_n), \quad (6)$$

where M_{H} is the mass of the hydrogen atom and M_n is the mass of the neutron. In the absence of an adequate theory of nuclear forces we cannot compare the binding energies computed from expressions (5) and (6). Instead we combine the expressions and calculate the potential functions L and L'.

Table IV gives the data used to compute L and L'. The second column lists reaction energies giving the mass defects $(A-M)$ of unstable nuclei. If no reaction is listed the mass defect is from our mass spectroscopic data. The potential energy in the fourth column is computed by subtracting from the binding energy the Coulomb and kinetic energies. With 1.008146 for the mass of hydrogen and 1.008987 for the mass of the

TABLE I. Mass doublets.

Doublet	No. of runs	Δm in 10^{-4} amu	Previous work
CH_2S-Ti^{46}	3	354.0 ±0.4	349.0± 9.5[a]
CH_3S-Ti^{47}	4	438.3 ±0.9	444.2± 9.4[a]
C_4-Ti^{48}	6	522.0 ±0.6	521.6± 4.6[a]
C_4H-Ti^{49}	4	599.3 ±0.5	588.3± 5.1[a]
$C_4H_2-Ti^{50}$	6	708.92±0.29	694.6± 3.6[a]
$C_4H_3-V^{51}$	6	792.8 ±0.5	
$C_4H_2-Cr^{50}$	5	695.6 ±0.6	673.2± 3.7[b] 714.5± 2[c]
$C_4H_4-Cr^{52}$	4	908.8 ±0.9	920.3± 4.2[b] 908.4± 2[c]
$C_4H_5-Cr^{53}$	5	983.8 ±0.8	1008.7± 4.1[b]
$C_4H_6-Cr^{54}$	1	1079 ±2	1100.0± 4.6[b]
$C_4H_7-Mn^{55}$	5	1165.8 ±1.1	
$C_4H_6-Fe^{54}$	8	1072.0 ±0.5	1065.3± 4.7[b] 1075.1± 2.2[d]
$C_4H_8-Fe^{56}$	6	1278.2 ±1.0	1235 ±17[e] 1284.1± 3.9[f] 1271.3± 2.3[b]
$C_4H_9-Fe^{57}$	6	1350.9 ±0.9	1338.1± 5.0[b]
$C_4H_{10}-Fe^{58}$	1	1448 ±4	1458.8± 4.7[b]
$C_4H_{10}-Ni^{58}$	4	1433.8 ±0.9	1371.2± 3.9[g] 1434.3± 2.3[f] 1447.68[h]
C_5-Ni^{60}	5	702.0 ±2.9	695.9± 3.1[g] 714[h]
C_5H-Ni^{61}	4	782.9 ±2.3	735 ±15[g] 807.64[h]
$C_5H_2-Ni^{62}$	4	886.9 ±0.8	860.7±3.7[g] 913.88[h]
SO_2-Ni^{64}	3	346.9 ±0.7	
$C_5H_3-Cu^{63}$	6	943.9 ±0.5	
$C_5H_5-Cu^{65}$	7	1115.9 ±0.5	
SO_2-Zn^{64}	4	326.82±0.20	
$O_2-Zn^{64}/2$	4	252.46±0.22	
$C_5H_6-Zn^{66}$	4	1208.7 ±0.5	1213.8± 3.9[b]
$C_5H_7-Zn^{67}$	4	1280.8 ±0.5	1280.1± 6.3[b]
$C_5H_8-Zn^{68}$	4	1375.1 ±0.6	1355.5± 6.3[b]
$C_5H_{10}-Zn^{70}$	4	1528.8 ±0.5	1346 ±16[b]

[a] T. Okuda and K. Ogata, Phys. Rev. **60**, 690 (1941).
[b] K. Ogata, Phys. Rev. **75**, 200 (1949).
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[h] A. E. Shaw, Phys. Rev. **75**, 1011 (1949).

TABLE II. Atomic masses computed from data in Table I assuming $H^1=1.008146\pm3$ and $C^{12}=12.003842\pm4$.

Ti^{46}	45.96697± 5	Ni^{58}	57.95345±10
Ti^{47}	46.96668±10	Ni^{60}	59.94901±29
Ti^{48}	47.96317± 6	Ni^{61}	60.94907±23
Ti^{49}	48.96358± 5	Ni^{62}	61.94681± 9
Ti^{50}	49.96077± 4	Ni^{64}	63.94755± 7
V^{51}	50.96052± 5	Cu^{63}	62.94926± 6
Cr^{50}	49.96210± 7	Cu^{65}	64.94835± 6
Cr^{52}	51.95707± 9	Zn^{64}	63.94955± 2
Cr^{53}	52.95772± 8	Zn^{66}	65.94722± 6
Cr^{54}	53.9563 ± 2	Zn^{67}	66.94815± 6
Mn^{55}	54.95581±10	Zn^{68}	67.94686± 7
Fe^{54}	53.95704± 5	Zn^{70}	69.94779± 6
Fe^{56}	55.95272±10		
Fe^{57}	56.95359±10		
Fe^{58}	57.9520 ± 4		

TABLE III. Comparison with some nuclear reactions.

	From Table I	From nuclear reactions	Reactions used
$Ti^{47}-Ti^{46}$	1−0.00029±11	1−0.00028±11	$Ti^{46}(d,p)Ti^{47}$[a]
$Ti^{48}-Ti^{47}$	1−0.00351±12	1−0.00355±14	$Ti^{47}(d,p)Ti^{48}$[a]
$Ti^{49}-Ti^{48}$	1.00041± 8	1.00045± 8	$Ti^{48}(d,p)Ti^{49}$[a]
$Cr^{52}-V^{51}$	1−0.00345±10	1−0.00262	$V^{51}(n,\gamma)V^{52}$[b] $V^{52}(\beta)Cr^{52}$[c]
$Mn^{55}-Fe^{54}$	1−0.00123±11	1−0.00138± 4	$Mn^{55}(p,n)Fe^{55}$[d] $Fe^{54}(n,\gamma)Fe^{55}$[b]
$Fe^{57}-Fe^{56}$	1.00087±14	1.00098± 1	$Fe^{56}(n,\gamma)Fe^{57}$[b]
$Ni^{61}-Ni^{60}$	1.00006±37	1.00020± 3	$Ni^{60}(n,\gamma)Ni^{61}$[b]

[a] J. A. Harvey, Phys. Rev. **81**, 353 (1951).
[b] Kinsey, Bartholomew, and Walker, Phys. Rev. **78**, 481 (1950).
[c] A. C. G. Mitchell, Revs. Modern Phys. **22**, 36 (1950).
[d] H. T. Smith and R. V. Richards, Phys. Rev. **74**, 1275 (1948).

TABLE IV. Data used in comparing experimental results with Wigner formula.

Nuclide	Reaction	Mev	Mass defect mMU	−PE mMU	−Ξ
S^{31}	(γ,n)	−14.8 ±4[a]	10.9 ±4	779.15	60
P^{32}	β	1.712±8[a]	15.93±1	803.44	68
S^{32}			17.76±1	808.37	64
S^{33}			17.87±5	832.36	72
S^{34}			21.24±5	859.10	79
S^{35}	β	0.167[a]	19.78±5	882.42	88
Cl^{35}			19.96±5	886.11	85
Cl^{36}	(n,γ)	8.56 ±3[b]	20.15±6	910.55	94
A^{36}			21.00±3	915.28	90
Cl^{37}			22.34±5	936.49	102
A^{37}	(p,n)	−1.598±3[a]	21.46±5	939.49	99
Cl^{38}	(n,γ)	6.11 ±3[b]	19.91±6	959.09	112
A^{38}			25.09±4	966.43	107
K^{38}	(γ,n)	−13.2 ±2[a]	18.8 ±2	966.07	107
K^{39}			23.94±3	994.08	114
Ca^{39}	(γ,n)	−15.9 ±4[a]	16.5 ±4	992.89	114
A^{40}			24.87±3	1014.88	126
K^{40}	(n,γ)	7.76 ±5[b]	23.26±4	1017.49	124
Ca^{40}			24.55±9	1023.37	120
A^{41}	(d,p)	3.84 ±3[a]	22.39±4	1037.67	137
K^{41}			25.10±4	1042.99	133
Ca^{41}	(d,p)	6.09[a]	24.60±20	1047.07	130
Sc^{41}	β^+	4.94[a]	18.20±20	1047.19	130
K^{42}	(d,p)	5.12 ±10[a]	24.01±10	1066.72	144
Ca^{42}			27.84±4	1073.54	139
Ca^{43}			27.49±6	1097.59	150
Ca^{44}			30.76±6	1124.84	160
Ca^{45}	β	0.254[a]	29.63±6	1148.75	172
Sc^{45}			29.90±5	1152.42	168
Ti^{46}			33.03±5	1183.96	175
Ti^{47}			33.32±10	1208.48	187
Ca^{48}			32.22±10	1226.74	208
Sc^{48}	β	2.954[a]	33.71±6	1230.32	204
Ti^{48}			36.88±6	1235.90	198
V^{48}	β^+	3.03[a]	32.53±7	1237.05	196
Ti^{49}			36.42±5	1260.29	211
Cr^{49}	(γ,n)	−13.4 ±2[e]	32.5 ±2	1266.33	204
Ti^{50}			39.23±4	1287.57	223
V^{50}	(γ,n)	−11.15 ±20[d]	36.5 ±2	1288.98	220
Cr^{50}			37.90±7	1294.87	215
V^{51}			39.48±5	1316.06	232
Cr^{51}	(p,n)	−1.534±3[a]	38.67±5	1319.73	228
V^{52}	(n,γ)	7.30 ±3[b]	38.34±6	1339.94	246
Cr^{52}			42.93±9	1347.76	240
Mn^{52}	β^+	4.116[a]	37.41±9	1348.32	238
Cr^{53}			42.28±8	1371.79	254
Mn^{53}	(p,n)	−1.380±8[e]	41.64±8	1375.97	250
Fe^{53}	(γ,n)	−13.8 ±2[a]	37.1 ±2	1377.81	247
Cr^{54}			43.7 ±2	1397.53	267
Mn^{54}	(p,n)	−2.162±5[e]	42.2 ±2	1400.84	264
Fe^{54}			42.96±5	1406.75	259
Mn^{55}			44.19±11	1426.83	277
Fe^{55}	(n,γ)	9.28 ±3[b]	43.96±6	1431.71	273
Co^{55}	β^+	2.42[a]	40.24±6	1434.65	270
Mn^{56}	(n,γ)	7.25 ±3[b]	42.76±7	1450.25	292
Fe^{56}			47.28±10	1458.73	286
Co^{56}	β^+	3.59[a]	42.33±10	1460.38	284
Fe^{57}			46.41±9	1482.38	301
Ni^{57}	(γ,n)	−11.7 ±2[a]	43.0 ±2	1491.36	294
Fe^{58}			48.0 ±4	1508.21	315
Ni^{58}			46.55±10	1517.95	307
Fe^{59}	β	1.56[a]	47.0 ±3	1532.16	331
Co^{59}	(n,γ)	7.73 ±4[b]	48.7 ±3	1538.15	326
Ni^{59}	(n,γ)	9.01 ±3[b]	47.24±11	1542.52	322
Co^{60}	β	2.811[a]	47.97±29	1562.15	342
Ni^{60}			50.99±29	1569.85	336
Co^{61}	β	1.42[f]	49.51±23	1588.06	357
Ni^{61}			50.93±23	1594.18	352
Ni^{62}			53.19±9	1620.53	367
Ni^{63}	β	0.063[a]	50.67±6	1642.85	384
Cu^{63}			50.74±6	1647.92	379
Zn^{63}	β^+	2.36[a]	47.11±6	1650.64	375
Ni^{64}			52.45±7	1669.18	400
Zn^{64}			50.45±2	1677.49	390
Ni^{65}	β	2.10[a]	49.39±6	1691.35	418
Cu^{65}			51.65±6	1697.64	412
Zn^{65}	(p,n)	2.17 ±1[a]	50.16±6	1701.47	407
Zn^{66}			52.78±6	1728.07	423

[a] *Nuclear Data*, Natl. Bur. Standards Circular 499 (1950).
[b] Kinsey, Bartholomew, and Walker, Phys. Rev. **78**, 481 (1950).
[c] W. E. Ogle and R. E. England, Phys. Rev. **78**, 63 (1950).
[d] Sher, Halpern, and Mann, Phys. Rev. **84**, 387 (1951).
[e] Lovington, McCue, and Preston, Technical Report No. 54, Laboratory of Nuclear Science and Engineering, M.I.T. (September, 1951).
[f] Smith, Haslam, and Taylor, Phys. Rev. **84**, 843 (1951).

neutron the expression is

$$-\mathrm{PE}=\text{mass defect}+8.146Z+8.987N+\mathrm{CE}+\mathrm{KE}\ \text{mMU}. \quad (7)$$

The final column gives the value of the symmetry function Ξ which is an integer from its definition.

From Eq. (3) one sees that the potential energy difference of isobars does not depend upon L'. Thus, using the data of Table IV, we have calculated L for many mass numbers from

$$L=\Delta(-\mathrm{PE})/\Delta\Xi, \quad (8)$$

excluding pairs which jump over the possible magic numbers 20 and 28. In the cases of three or more isobars, we computed the best value of L by least squares. Figure 1 shows the values of L (and AL) plotted against A. Contrary to expectation, L is not a smooth function of A but shows large discontinuities which are definitely associated with magic numbers 20 and 28. Region I contains nuclei with 20 or less neutrons, region II nuclei with 20 or more neutrons and 20 or less protons, and so on. The jumps are associated with 20 neutrons, 20 protons, 28 neutrons, 28 protons in order with increasing A. L increases on crossing a neutron number and decreases on crossing a proton number. The energy involved is surprisingly large, for example $\Delta L\Xi$ for the 28 neutron jump is 75 mMU. Between the discontinuities we have simply approximated L (or AL) by straight lines as shown. We assume AL is constant in regions I and II, but L is constant in regions III and IV. Region V does not have sufficient data to permit any approximation.

By substituting the average values of L or AL in the potential energy formula (3) we have computed L'. Figure 2 shows AL' plotted against A. Because we do not observe jumps of 75 mMU in the masses we expect L' to show discontinuities which almost cancel the effect of the discontinuities in L. Figure 2 shows that AL' can be represented in each region by a straight line with a small slope. In this way we arrive at the empirical

equations

$$(-\mathrm{PE})=(A-1)(a_1+a_2A)+(\Xi/A)a_3 \text{ regions I, II,} \quad (9)$$

$$(-\mathrm{PE})=(A-1)(a_1+a_2A)+\Xi a_3 \text{ regions III, IV,} \quad (10)$$

where a_1, a_2, and a_3 are constants characteristic of each region.

By means of the empirical equations we can investigate the effect of magic numbers upon the binding energy with due allowance for symmetry. The first step is to determine by least square the constants which best fit all nuclei of each region excluding magic number nuclei. The constants in mMU are:

region I, $AL=40.31$, $\frac{1}{2}AL'=25.963+0.0848A$;
region II, $AL=55.52$, $\frac{1}{2}AL'=27.363+0.0785A$;
region III, $L=0.8989$, $\frac{1}{2}AL'=23.316+0.1407A$;
region IV, $L=1.2316$, $\frac{1}{2}AL'=23.106+0.1752A$.

Table V (a), (b), (c), and (d) give the differences between the observed and computed binding energies in 10^{-4} atomic mass units. A positive value indicates that the nucleus has greater stability (more negative binding energy) than given by the empirical fit. The entries in heavy type are the residuals for the fitted region. The fit is satisfactory except, perhaps, in region III.

The next step is to extrapolate the empirical equations to the magic number nuclei and beyond. Table V also gives the differences between these computed and observed binding energies. Let us consider nuclei with 20 neutrons. Table V (a) shows that they can be grouped

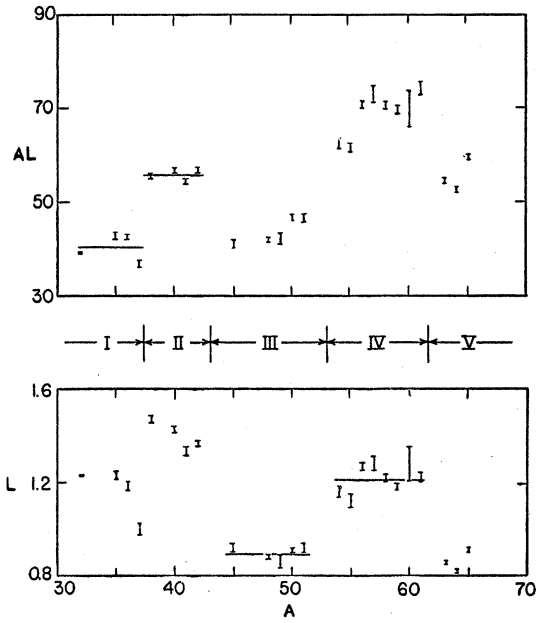

FIG. 1. Variation of the Wigner constant L (and AL) in mMU with mass number A for nuclides in region near magic numbers 20 and 28. Region *I* contains nuclides having 20 or less neutrons; region *II*, 20 or more neutrons and 20 or less protons; region *III*, 28 or less neutrons and 20 or more protons; region *IV*, 28 or more neutrons and 28 or less protons; and region *V*, 28 or more protons.

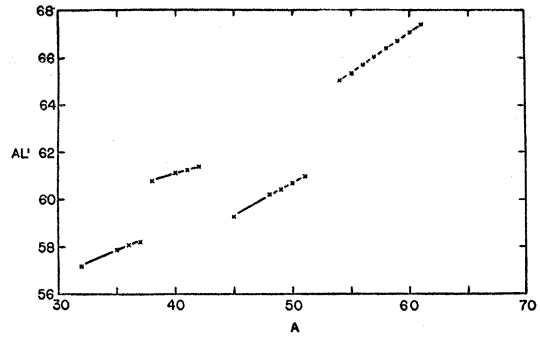

FIG. 2. Variation of the Wigner constant L' in mMU with mass number A computed from formula (3) with average values of L given in Fig. 1.

with nuclei having less than 20 neutrons, Table V (b) shows that they also can be grouped with nuclei having more than 20 neutrons but less than 20 protons. Each magic number shows similar properties. The errors become large when we extrapolate beyond the boundaries of the regions. From this we conclude that the effect of magic numbers is, primarily, changes of slope in the binding energy surface (however, with this form of empirical equation not more than two or possibly three nuclei could fit both ways exactly).

In our empirical equations we have attributed all of the effect of the magic numbers to the potential energy. The kinetic energy should require some modification and it is very possible that the assumptions of uniform charge and uniformly increasing radius used to compute the Coulomb energy should also be modified. These considerations do not invalidate the conclusions of the last paragraph but they do point out the doubtful theoretical significance of the empirical constants. Even as a method of predicting reaction energies, the empirical equations are limited. The residuals in the least square fits exceed the probable errors in the measurements and more accurate masses probably would not reduce them, so that we can expect predictions of individual masses to be in error by as much as 1 Mev.

From this attempt to fit a Wigner formula to our mass data we have reached the following conclusions. Both 20 and 28 are "magic" numbers associated with discontinuities in the binding energy surface. These discontinuities are primarily changes in the slope of the surface rather than discrete jumps in the binding energy. The regions between the magic numbers do not all have the same shape; the two regions below 20 protons require a different form of empirical equation from the two regions above 20 protons. Finally we believe the accuracy of mass measurements in this region now requires a considerable modification of the semi-empricial mass formulas before an adequate fit can be obtained.

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TABLE V. Difference in 10^{-4} amu between a fitted Wigner formula and experimental masses. The Wigner constants L and L' are obtained in each case by a least squares fit of the nuclides indicated by heavy type.

(a) Constants fitted to nuclides with less than 20 neutrons

Neutrons	P	S	Cl	A	K	Ca	Sc
24						−55	−75
23				−53	−57	−55	
22				−31	−39	−36	
21			−20		−25	−28	
20			−1	0	−4	−6	−27
19		**1**	**2**	**−3**	**−3**	−16	
18		**8**	**3**	**5**			
17	**1**	**−1**					
16		**0**					
15		**−6**					
	15	16	17	18	19	20	21
				Protons			

(b) Constants fitted to nuclides with more than 20 neutrons and less than 20 protrons

Neutrons	S	Cl	A	K	Ca	Sc	Ti
24					16	3	14
23			**0**	**0**	3		
22			2	**−1**	3		
21		**0**		**0**	−1		
20		−1	0	3	3	0	
19	−17	−11	−16	−4	−9		
18	−29	−28	−25				
	16	17	18	19	20	21	22
			Protons				

(c) Constants fitted to nuclides with less than 28 neutrons and more than 20 protons

Neutrons	Cl	A	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni
33										−41		−34
32										−28	−27	−19
31									−30	−19		−19
30								−9	−12	0		−3
29							−12	−1	−3	1	−1	8
28				4		8	0	12	5	11	3	
27					4	**−2**	**−5**	**1**	**0**	**−4**		
26						**6**		**9**				
25				−12		**−2**	**−6**	**−4**				
24				−2	**−11**	**8**						
23		−26	−21	−10								
22		−7	−9	2								
21	−8		1	5								
20	14	21	20	24	6							
19	17	17	17	18								
18		23	28									
	17	18	19	20	21	22	23	24	25	26	27	28
						Protons						

(d) Constants fitted to nuclides with more than 28 neutrons and less than 28 protons

Neutrons	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
36									−3	−26	−45
35									−11		−49
34								**2**	5	−22	−43
33							**1**	**−1**	1		−44
32							**−1**	**0**	2		
31						**−6**	**−1**		−6		
30					**3**	**−1**	**5**		−2		
29				**−1**	**3**	**−1**	**−1**	**−3**	2		
28	17		8	−3	3	−5	−3	−9			
27		3	−10	−15	−15	−16	−24				
26			−15		−18						
25	−30		−29	−28	−36						
	20	21	22	23	24	25	26	27	28	29	30
						Protons					

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