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Photodisintegration of the Deuteron*

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The angular distribution of the protons from the $d(\gamma,n)p$ reaction has been investigated for 5–13-Mev gamma-rays by means of the D_2O loaded emulsion technique with the Case betatron serving as gamma-ray source. The results have been compared to a differential cross section of the form $d\sigma=\{a+(b+c\cos\theta)\sin^2\theta\}d\Omega$ in the center-of-mass system and the isotropy coefficient found consistent with the mean values: $a/b=0.04\pm0.03$ from 5 to 11 Mev and 0.24 ± 0.07 from 11 to 13 Mev. The forward asymmetry coefficient was determined for energies above 8 Mev and found to be $c/b=0.24\pm0.09$. These results are in agreement with the theoretical calculations of Marshall and Guth except for the isotropy coefficient observed above 11 Mev where the theory predicts $a/b\sim0.01$.

I. INTRODUCTION

THE determination of the angular distribution of protons arising from the photodisintegration of the deuteron is of interest in that it provides a test of the theory of nuclear forces as applied to this two-body problem. Considerable work has been done on the determination of the angular distribution at energies near the threshold for the process[1] and a number of recent experiments have been performed in the 4- to 20-Mev range.[2–8] The sources used in these latter works were the 6.1- and 7.0-Mev quanta from the $F^{19}+H^1$ reaction;[2,5–8] the 14.8- and 17.6-Mev quanta from the Li^7+H^1 reaction;[3,6] and 20-Mev bremsstrahlung.[4] The detecting systems employed were nuclear track plates loaded with calcium nitrate containing heavy water of crystallization;[2] track plates loaded by soaking in heavy water;[3,6–8] a scattering chamber containing D_2 gas with track plates as detectors;[4] a cloud chamber containing CD_4 gas;[5] and deuterated paraffin adjacent to a track plate.[6] The work to be described in this paper employs D_2O-loaded nuclear track plates and 14-, 17-, and 20-Mev bremsstrahlung.

An angular distribution function in the c.m. system of the form

$$d\sigma=\{a+(b+c\cos\theta)\sin^2\theta\}d\Omega$$

has been suggested by Marshall and Guth.[9] The coefficients a/b and c/b represent a measure of isotropy and forward asymmetry in the center-of-mass system, respectively. These quantities can be treated as parameters in a quantitative comparison of theory and experiment. The values of these parameters obtained by the various investigators have been collected in Table I.

* Work supported by the AEC.
1 Bishop, Beghian, and Halban, Phys. Rev. **83**, 1052 (1951), and references.
2 Gibson, Green, and Livesy, Nature **160**, 534 (1947).
3 H. Wäffler and S. Younis, Helv. Phys. Acta **22**, 414 (1949).
4 E. G. Fuller, Phys. Rev. **79**, 303 (1950).
5 Phillips, Lawson, and Kruger, Phys. Rev. **80**, 326 (1950).
6 P. V. C. Hough, Phys. Rev. **80**, 1069 (1950).
7 G. Goldhaber, Phys. Rev. **81**, 930 (1951).
8 Gibson, Grotdal, Orlin, and Trumpy, Phil. Mag. **42**, 555 (1951).
9 J. F. Marshall and E. Guth, Phys. Rev. **78**, 738 (1950).

II. EXPERIMENTAL

In this investigation the D_2O-loaded emulsion technique was used along with 14-, 17-, and 20-Mev bremsstrahlung from the Case betatron. A background plate loaded with H_2O was exposed with each D_2O plate and corresponding areas on the two plates were searched. About 85 percent of the data came from 200μ Kodak NTB plates while the remainder was from 150μ and 100μ NTB plates. The 200μ plates were exposed 3 m from the betatron target in a plastic microscope slide box. The beam was collimated to a diameter of about 2 cm and passed through the box along the length of the plates without touching the top, bottom, or sides of the box. This arrangement gave relatively less fog than earlier exposures with less collimation and smaller plate containers. All plates were exposed with the gamma-ray beam almost parallel to the surface of the emulsions. An incidence angle of 2° was used to reduce absorption of gamma-rays in the wet emulsions.

The plates were soaked for about two hours at room temperature in order to obtain loadings near saturation and were processed in the usual way immediately after the brief exposures. Track densities of order ten tracks per square millimeter were obtained. Work with the

TABLE I. Results of others as indicated by references.

Energy (Mev)	Isotropy coefficient a/b	Forward asymmetry coefficient c/b
5.	-0.07[a]	0.2 ± 0.2[a]
6.	0.09 ± 0.07[b]	
6.5	$0.02^{+0.04}_{-0.02}$[c]	$0.4^{+0.2}_{-0.3}$[b]
	~0[d–f]	
7.	0.17 ± 0.07[b]	0.2 ± 0.2[a]
	0.17[a]	
9.	-0.12[a]	0.21 ± 0.14[a]
11.	-0.06[a]	0.25 ± 0.14[a]
13.	0.05[a]	0.35 ± 0.14[a]
17.	0.05 ± 0.15[g]	0.33 ± 0.22[a]
	$0.02^{+0.14}_{-0.02}$[c]	
	0.22[a]	

a E. G. Fuller, reference 4.
b G. Goldhaber, reference 7.
c P. V. C. Hough, reference 6.
d Gibson *et al.*, reference 2.
e Phillips *et al.*, reference 5.
f Gibson *et al.*, reference 8.
g H. Wäffler and S. Younis, reference 3.

microscope included the standard determination of the orientation and sense of the tracks in space by measurement of the track depth normal to the plane of the emulsion and of the projections in the plane of the emulsion of the track length and angle from the gamma-ray beam. In addition, the distances between the track ends and the emulsion surfaces were measured and used to eliminate tracks which escaped or nearly escaped from the emulsion. Tracks were accepted for analysis if they had less than an arbitrary 45° dip in the wet emulsion and both ends cleared the surfaces of the processed plate by at least 7μ.

The total shrinkage factor varied from 5.7 to 7.0 depending on the water content of the plates. This factor was obtained by measuring the ordinary processing shrinkage factor and the way in which volumes of emulsion and water added. These measurements gave $S=2.4\pm0.1$ for the processing shrinkage factor and $P=1.00\pm0.06$ for the ratio of the swelling of the emulsion to the volume of water absorbed. The values of S and P obtained for each batch of plates were used in the calculations for all plates of the batch, the volume of water in each plate being determined by weighings. In addition, determination of the total shrinkage requires a knowledge of the amount of water absorbed by the gelatin base of the NTB plates (see reference 7). The thicknesses of the gelatin bases were determined by subtracting the processed emulsion thicknesses measured with a microscope from the total gelatin thicknesses measured with a micrometer. The total shrinkage factors used depend upon the assumption that the gelatin in the emulsion and the gelatin in the base absorb water in the ratio of their volumes. The error in the parameter a/b resulting from a possible 8 percent systematic error in the shrinkage factors has been calculated and a consequent uncertainty of ±0.02 has been included in our results for a/b listed in Table II.

III. CALCULATION

(A) Range *vs* Energy

Proton range-energy curves were obtained from calculations based on the experimental range-energy curves of Lattes *et al.*[10] for dry plates and the curves calculated by Aron *et al.*[11] for hydrogen and oxygen. Our results are given in Table III from which intermediate curves can be obtained by interpolation. The table gives ranges for various energies and several values of R_w, the volume of water in the emulsion divided by the sum of the water and emulsion volumes. In these calculations it was assumed that $P=1$ and that the stopping power and hence the density of the plates used would be the same as for those used by Lattes *et al.*

If the factor P is found to be different from unity or our measurement of the emulsion density differs from the 3.9 g/cm³ of the Ilford plates used by Lattes *et al.*, we correct the volume of emulsion used to calculate R_w and the ranges of the table assuming that the range-energy curves for the dry emulsions differ only by a constant factor multiplying the ranges. The measured densities of the plates used were in the range 3.5 ± 0.15 g/cm³.

Our reason for dealing with the relative volume R_w is that the results are sufficiently accurate for use with either heavy or ordinary water.

TABLE II. Results of this paper. In combining the results over energy intervals the isotropy coefficient obtained from 11 to 13 Mev has been combined separately, as it differs from the results at lower energies by more than the experimental error.

Energy (Mev)	Isotropy coefficient a/b	Forward asymmetry coefficient c/b
5–6	0.10 ± 0.06	...
6–7	0.04 ± 0.05	...
7–8	0.06 ± 0.05	...
8–9	-0.03 ± 0.06	0.26 ± 0.15
9–10	0.08 ± 0.07	0.24 ± 0.18
10–11	0.03 ± 0.07	0.31 ± 0.19
11–12	0.20 ± 0.08	0.09 ± 0.22
12–13	0.30 ± 0.11	-0.08 ± 0.29
5–11	0.04 ± 0.03	...
11–13	0.24 ± 0.07	...
8–13	...	0.24 ± 0.09

(B) Corrections

The fraction of the tracks that are retained (i.e., satisfy the criteria for acceptance given in Part II) is a function of θ and the ratio of the track length to the wet emulsion thickness considered, L/t (the layers near the surface of the emulsion not being considered). The values of θ and L/t were determined for each track studied and weights equal to the reciprocal of the probability of the track being retained were assigned to each track. This weighting factor, w, is given below for the three possible cases.

Case I. $t/L\geq\sin\theta\leq\sin45°$

$$\frac{1}{w}=1-\frac{2L}{\pi t}\sin\theta.$$

Case II. $\sin\theta\geq\sin45°\leq t/L$

$$\frac{1}{w}=\frac{2}{\pi}\left\{\sin^{-1}\left(\frac{\sin45°}{\sin\theta}\right)-\frac{L}{t}\left[\sin\theta-(\sin^2\theta-\sin^2 45°)^{\frac{1}{2}}\right]\right\}.$$

Case III. $\sin\theta\geq t/L\leq\sin45°$

$$\frac{1}{w}=\frac{2}{\pi}\left\{\sin^{-1}\left(\frac{t}{L\sin\theta}\right)-\frac{L}{t}\left[\sin\theta-\left(\sin^2\theta-\frac{t^2}{L^2}\right)^{\frac{1}{2}}\right]\right\}.$$

These weightings take account of our rejection of tracks with dip greater than 45° as well as loses from the emulsion.

[10] Lattes, Fowler, and Cuer, Proc. Phys. Soc. (London) **59**, 889 (1947).

[11] Aron, Hoffman, and Williams, University of California Radiation Laboratory—121 (1949).

(C) Determination of Parameters

Gamma-ray energies were obtained for each track from plots in terms of proton energy and space angle, θ, in the laboratory. The laboratory angles, θ, corresponding to 15° intervals of θ_{cm} in the center-of-mass system were calculated for each one-Mev interval in gamma-ray energy. The tracks were then grouped into one-Mev intervals of gamma-ray energy and 15° intervals of θ_{cm}. As values of θ were recorded to the nearest degree tracks with values of θ near one of the boundaries were divided between the intervals. For instance, if for a particular energy interval $\theta_{cm}=15°$ when $\theta=14°\ 15'$, one quarter of the tracks with $\theta=14°$ were assigned to the 15–30° interval in θ_{cm}.

The sum of the weights of the tracks in each interval minus the corresponding background was called the corrected number of tracks, Ni, in the interval. Assuming that the differential cross section is of the form indicated in the introduction the number of tracks to be expected in the ith θ_{cm} interval is $aA_i+bB_i+cC_i$ where

$$A_i=2\pi\int_{\theta_i}^{\theta_{i+1}}\sin\theta d\theta,\quad B_i=2\pi\int_{\theta_i}^{\theta_{i+1}}\sin^3\theta d\theta,$$

$$C_i=2\pi\int_{\theta_i}^{\theta_{i+1}}\sin^3\theta\cos\theta d\theta$$

with $\theta_i=0$, 15, 30°—etc.

The experimental values of a, b, and c were found for each energy interval by the method of least squares which requires that

$$\sum_{i=1}^{n}W_i(N_i-aA_i-bB_i-cC_i)^2$$

be a minimum. W_i is a weighting factor which varies inversely as the square of the probable error.

The foregoing method of correcting the data and obtaining the desired parameters avoids the use of average weighting factors and solid angle corrections for the relatively large intervals in θ. This is an advantage as the correct averages for such quantities depend upon the parameters one is measuring. For instance, in one alternative method of analysis the average solid angle for each interval of θ is divided into the data and an average weighting is used to correct for the loses from the emulsion in each interval. The corrected results are then plotted against an average value of $\sin^2\theta$. A straight line is obtained and the isotropy coefficient a/b appears as the negative intercept on the $\sin^2\theta$ axis. Each of the three quantities for which averages are required is a function of $\sin\theta$ so that the correct averages depend upon the parameters in the distribution function.

TABLE III. Calculated proton ranges for various values of R_w, assuming that $P=1$ and the range-energy curves for dry plates is identical with the determination of Lattes *et al.*

Proton energy (Mev)	R_w 0.5	0.6	0.65	0.7	0.75
1	17μ	18μ	18μ	19μ	19μ
2	50	53	54	56	58
3	97	102	105	109	113
4	157	167	173	180	187
5	231	247	255	265	276
6	316	338	350	364	379
7	412	441	457	476	495
8	520	555	575	600	625
9	635	685	710	740	770
10	765	820	855	890	930
11	905	970	1010	1050	1095
12	1050	1130	1175	1225	1280
13	1210	1300	1350	1410	1470
14	1380	1490	1550	1620	1690

(D) Errors and Correction for Uncertainty in Sense

The effect of various errors on the final results has been obtained by numerical calculations, except for the standard statistical error for which we obtain the formulas

$$\Delta(a/b)=\pm0.9n^{-\frac{1}{2}},\quad \Delta(c/b)=\pm2.2n^{-\frac{1}{2}}$$

for large values of the total uncorrected count, n. The effect of an error in the shrinkage factor on the result for (a/b) has been mentioned. Uncertainty in the sense of the tracks results in too low a result for c/b as a part of the forward asymmetry is not observed. This uncertainty was estimated from the opinions of observers who graded about a third of the tracks on an A, B, C, D basis and from the number of disagreements arising when two observers studied the same tracks. We estimate that for gamma-rays above 8 Mev the sense is correct in about 95 percent of the data. This corresponds to our missing 10 percent of the forward asymmetry including that between the laboratory and center-of-mass systems. Hence, 0.05±0.03 has been added to our results for c/b. The uncertainty in sense rises rapidly as one proceeds to the shorter tracks and reliable values for c/b were not obtained below 8 Mev.

Several effects which might be expected to result in too large a value of a/b have been considered and found to contribute negligibly compared to the errors already mentioned. These were the 2° grazing incidence of the beam, variations in gamma-ray direction from the direction at the center of the plate, and random errors of ±1.5° in measuring θ.

Checks on the results from different observers and plates were found to agree within the statistical error.

(E) Azimuthal Distribution

The azimuthal distribution was calculated and tabulated from the data for 15° intervals of θ_{lab} and four energy intervals. The results were consistent within the statistical errors, and have been combined for presen-

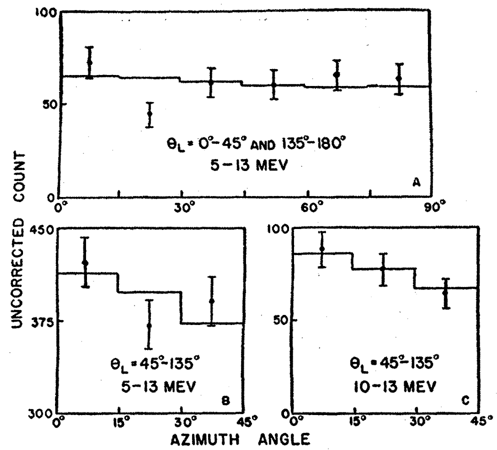

FIG. 1. Observed azimuthal distributions compared to distributions represented by solid histograms calculated for uniform azimuthal distributions altered by losses from the emulsions.

tation in Fig. 1 (A, B). Figure 1 (C) shows the energies and angles for which losses from the emulsion were largest. For $135° > \theta > 45°$ and $\varphi > 45°$ there are a few tracks, but the calculation was not carried out.

IV. RESULTS AND COMPARISON

The values obtained for a/b and c/b after correction of the latter for the uncertainty in track sense are presented in Table II. In Fig. 2 the folded histograms for 5–11 and 11–13 Mev are compared with the histograms calculated from the experimental values of a and b; while Fig. 3 shows the results from 8–13 Mev using the experimental values of a, b, and c (the correction of c/b does not affect this figure).

The results of other workers in the 4–20-Mev range are given in Table I. We have taken the results for the isotropy coefficient from Fuller's graphs rather than his conclusion in which he combined his results at 5 and 7 Mev to obtain $a/b \sim 0.05$ and regarded the results at 9 and 11 Mev as an indication that a/b was near zero.

Theoretical calculations of Marshall and Guth, in which they neglect the possible effects of meson exchange currents and tensor forces, predict no appreciable contribution to the a/b term in the energy region 5 to 20 Mev other than the magnetic dipole contribution. The latter amounts to approximately two percent of the 90° cross section. The same calculations predict that the forward asymmetry coefficient, c/b, should rise from 0.12 at 6 Mev to 0.25 at 17 Mev. Details of the potentials used in the calculations result in differences of the parameters which are small compared to the errors arising in the angular distribution measurements which have been performed in this energy interval.

In the energy region around 7 Mev the experimental observations of the isotropy coefficient are in general agreement with values quoted above with the possible exception of Goldhaber's. Using the D_2O-loaded plate technique the determination of the forward asymmetry is inherently difficult in this energy range because of the large uncertainty in the direction sense of the low energy proton tracks. Fuller[4] has, however, observed a forward asymmetry which agrees with the theory within the experimental error.

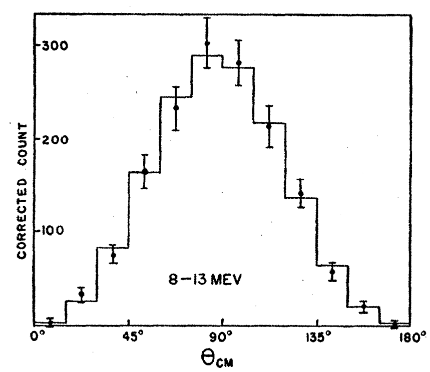

FIG. 3. Angular distribution found in the energy interval where a measure of the forward asymmetry was obtained. The solid histogram was calculated from the distribution function using the experimental values of a, b, and c prior to the correction of c/b for the uncertainty in track sense.

At higher energies the present work indicates a value for a/b of 0.24 ± 0.07 in the energy range 11 to 13 Mev as opposed to Fuller's value at zero in this range. His values, however, start at -0.12 at 9 Mev rising rapidly to 0.22 at 17 Mev. Other values of a/b given for 17.6 Mev are handicapped by large errors due to high background[3] or poor geometry.[6] The background tracks in the present work due to (γ,p) processes or knock-on protons constitute only eleven percent of the usable tracks in this energy region. The forward asymmetry parameter as given by this work is of the right order of magnitude but is subject to a large error. The large experimental error in this parameter, however, has no effect per se in the determination of the value of a/b.

Two papers on similar experiments[10, 11] have appeared

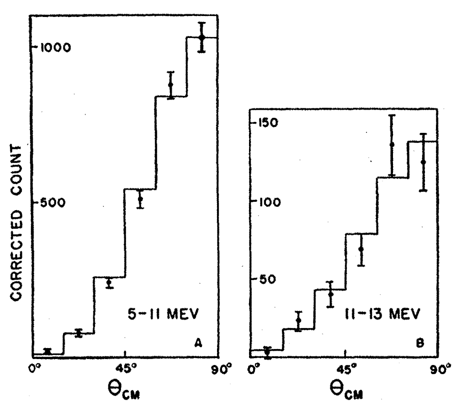

FIG. 2. Experimental angular distributions folded about 90° and compared with the solid histograms calculated from the distribution function using the empirical values of a and b.

[10] K. Phillips, Phil. Mag. **43**, 129 (1952).

[11] H. Wäffler and S. Younis, Helv. Phys. Acta **24**, 483 (1951).

in the literature since the present paper was submitted. The results quoted lead to values of c/b in agreement with the calculations of Marshall and Guth,[9] while the values reported for a/b suggest that this coefficient rises about 0.06 at 6 Mev to 0.14 at 15 Mev. These results are reasonably consistent with those reported here.

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Covariant Theory of Radiation Damping

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Schwinger's expression of the S matrix in the Cayley form, in terms of a Hermitian operator K, is shown to be identical with the previous noncovariant expression used in Heitler's theory of radiation damping. The comparison of the two formalisms leads, furthermore, to a clear understanding of mass renormalization which is necessary for internal consistency, quite independently of the eventual removal of divergences. For the computation of $\bar{K}$ in a covariant way, new formulas generalizing and connecting Gupta's and Fukuda and Miyazima's results are presented. The nth order approximations of $\bar{K}$ and S are closely related, and $\bar{K}_n$ may be expressed in terms of the S_p of order $p\leqq n$ or in terms of their anti-Hermitian parts only.

1. THE TWO FORMS OF THE S MATRIX

THE solution of the Schrödinger equation in the interaction representation

$$i\delta\Psi[\sigma]/\delta\sigma(x)=H(x)\Psi[\sigma] \quad (1)$$

by means of the usual perturbation method leads to the collision S matrix

$$S=1+\sum_{n=1}^{\infty}S_n, \quad (2)$$

with[1]

$$S_n=(-i)^n\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}H(x_1)\theta^+(\sigma_1,\sigma_2)H(x_2)\times\theta^+(\sigma_2,\sigma_3)\cdots H(x_n)dx_1\cdots dx_n, \quad (3)$$

where

$$\theta^+(\sigma_1,\sigma_2)=\begin{cases}1 \text{ if } \sigma_1 \text{ is after } \sigma_2\\ 0 \text{ if } \sigma_1 \text{ is before } \sigma_2.\end{cases} \quad (4)$$

This expression of S has been extensively used because the presence of θ^+ functions alone, which are closely related to the principle of causality, leads very simply to the causal D^c functions of Stueckelberg and Feynman enabling a simple computation of (3) to be made by means of the Feynman rules.

However, even if all the S_n have been made convergent by a suitable regularization, it is not known whether the series (2) is always convergent, although it has a certain similarity with the development of an exponential, as was pointed out by Heisenberg.[2]

In any case, for large values of the coupling constant as in meson theories the convergence is presumably slow, and it is more indicative to write the unitary S matrix in the Cayley form

$$S=(1-\tfrac{1}{2}i\bar{K})/(1+\tfrac{1}{2}i\bar{K}), \quad (5)$$

which is closely connected with the effect of radiation damping. For the computation of transition probabilities one usually makes use of the alternative form

$$S=1-i\bar{R}, \quad (6)$$

which is equivalent to (5), provided $\bar{R}$ is deduced from the Heitler integral equation[3]

$$\bar{R}=\bar{K}-\tfrac{1}{2}i\bar{K}\cdot\bar{R}. \quad (7)$$

The scattering cross sections are then proportional to the square of the modulus of the corresponding matrix elements of $\bar{R}$.

The Hermitian operator $\bar{K}$ can be easily obtained as a series

$$\bar{K}=\sum_{n=1}^{\infty}\bar{K}_n \quad (8)$$

by a suitable perturbation method. According to Schwinger,[4]

$$\bar{K}_n=\left(-\frac{i}{2}\right)^{n-1}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}H(x_1)\epsilon(\sigma_1,\sigma_2)H(x_2)\times\epsilon(\sigma_2,\sigma_3)\cdots H(x_n)dx_1\cdots dx_n, \quad (9)$$

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