The Photodisintegration of the Deuteron at Intermediate Energies. II.

D. H. WILKINSON
Cavendish Laboratory, University of Cambridge, Cambridge, England
(Received December 14, 1951)

The measurements of Barnes, Carver, Stafford, and Wilkinson on the absolute cross section for photodisintegration of the deuteron in the range of gamma-ray energy from 4.45 to 17.6 Mev are analyzed. The cross sections for the electric dipole disintegration from the ^3S-state are obtained by subtracting small computed magnetic dipole and ^3D-state contributions. The electric dipole cross sections are analyzed from the point of view of both square well and Yukawa-type interactions and yield the following effective ranges (in units of 10^{-13} cm):

Effective range	Square well	Yukawa well
$\rho_t(-\epsilon,-\epsilon)$	1.73	1.79
$\rho_t(0,-\epsilon)$	1.75	1.70
$\rho_t(0,0)$	1.77	1.62

The probable error in $\rho_t(-\epsilon,-\epsilon)$ is $\pm 0.09\times 10^{-13}$ cm. ^3P-state forces have been neglected. The meson mass associated with the Yukawa well is 291 ± 24 electron masses.

INTRODUCTION

THE preceding paper[1] described measurements of the absolute cross section for photodisintegration of the deuteron at six gamma-ray energies from 4.45 to 17.6 Mev. This paper discusses these measurements and analyzes them to yield the effective range of triplet neutron-proton interaction appropriate to the deuteron ground state.

THE ISOLATION OF THE ELECTRIC DIPOLE CROSS SECTION

The deuteron may be disintegrated by absorption of any order of multipole. We need here consider only dipole disintegration as, even at 17.6 Mev, the combined electric and magnetic quadrupole cross sections are less than $\frac{1}{2}$ percent of the dipole cross section.[2,3] The electric dipole cross section σ_e and the magnetic dipole cross section σ_m combine to give the total cross section $\sigma_t=\sigma_e+\sigma_m$. We wish to find σ_e and so have chosen a range of gamma-ray energy where σ_m is never more than 4.3 percent of σ_m; although the theory of σ_m[4-7] is not yet free from objection, it is good enough to make this correction with adequate accuracy. We have tied σ_m to the neutron-proton capture cross section, and have assumed that the percentage contribution to the matrix element from meson interaction effects is independent of gamma-ray energy. We have used the neutron-proton capture cross section derived from the results of Whitehouse and Graham[8] and the cross section of boron for slow neutrons.[9]

Table I shows the calculated σ_m together with σ_t taken from I and the resulting σ_e. σ_m/σ_e shows the importance of the correction. As σ_m/σ_e is small and probably correct to 20 percent as a pessimistic estimate, we have applied the experimental error in σ_t to σ_e. The percentage probable error in σ_e is given in the last column. The gamma-ray energy is in Mev and the cross sections are in units of 10^{-28} cm^2.

Confidence that this calculation of σ_m is good enough derives from the experimental work of Hough[10] who finds, at 6.14 Mev, $\sigma_m/\sigma_e=0.03(+0.06,-0.03)$.

DISCUSSION

The calculation of the electric dipole cross section for forces of zero range, σ_{e0}, was made by Bethe and Peierls[11] who find

$$\sigma_{e0}=\frac{8\pi}{3}\frac{e^2}{\hbar c}\frac{\hbar^2}{m}\frac{\epsilon^{\frac{1}{2}}(h\nu-\epsilon)^{\frac{3}{2}}}{(h\nu)^3}, \tag{1}$$

where m is the nucleon mass, ϵ the binding energy of the deuteron, and $h\nu$ the photon energy.[12] We use $\epsilon=(2.227\pm 0.003)$ Mev,[13-16] then

$$\sigma_{e0}=11.39\times w^{3/2}(w+1)^{-3}\times 10^{-27}\text{ cm}^2.$$

$w=(h\nu/\epsilon)-1$. The error (± 0.1 percent)[17] in the constant is determined almost entirely by that in ϵ. This calculation assumes that the outgoing particles are free.

When forces of finite range are introduced Eq. (1)

[1] Barnes, Carver, Stafford, and Wilkinson, preceding paper [Phys. Rev. 85, 359 (1952)], referred to as I.
[2] L. I. Schiff, Phys. Rev. 78, 733 (1950).
[3] J. F. Marshall and E. Guth, Phys. Rev. 78, 738 (1950).
[4] H. A. Bethe and C. Longmire, Phys. Rev. 77, 647 (1950).
[5] E. E. Salpeter, Phys. Rev. 82, 60 (1951).
[6] N. Austern and R. G. Sachs, Phys. Rev. 81, 710 (1951).
[7] N. Austern, unpublished paper.
[8] W. J. Whitehouse and G. A. R. Graham, Can. J. Research A25, 261 (1947).
[9] Summarized by R. K. Adair, Revs. Modern Phys. 22, 249 (1950).
[10] P. V. C. Hough, Phys. Rev. 80, 1069 (1950).
[11] H. A. Bethe and R. E. Peierls, Proc. Roy. Soc. (London) A148, 146 (1935).
[12] We should use the energy of the gamma-ray in center-of-gravity space; this correction is about 10 kev at $E_\gamma=6$ Mev.
[13] R. E. Bell and L. G. Elliott, Phys. Rev. 79, 282 (1950).
[14] R. C. Mobley and R. A. Laubenstein, Phys. Rev. 80, 309 (1950).
[15] Taschek, Argo, Hemmendinger, and Jarvis, Phys. Rev. 76, 325 (1949).
[16] T. R. Roberts and A. O. Nier, Phys. Rev. 77, 746 (1950).
[17] Throughout this paper $\pm$ means probable error.

must be modified in several ways. For the moment we assume that the ground state of the deuteron is entirely a ^3S-state. The radial wave function multiplied by r has the asymptotic form (unnormalized)

$$\psi_g = e^{-\gamma r},$$

where

$$\gamma = \frac{1}{\hbar}(\epsilon m)^{\frac{1}{2}} = 0.2318\times 10^{13}\ \text{cm}^{-1}.$$

The corresponding wave function for the outgoing ^3P-wave behaves as r^2 for small r; this leads to the well-known result that the matrix element for the transition is not strongly dependent on the behavior of the true ground-state wave function near the origin and that the asymptotic form may be used with fair accuracy. The matrix element computed using the asymptotic wave function must be multiplied by a factor F, close to unity, to take this approximation into account.

We must arrange that u_g, the true, unnormalized ground-state wave function, and ψ_g are asymptotic, but we must normalize the ground-state wave function on u_g. This leads immediately[4,5] to the result that σ_{e0} should be increased by a factor

$$\left(1-2\gamma\int_0^\infty(\psi_g{}^2-u_g{}^2)dr\right)^{-1}$$

$$2\int_0^\infty(\psi_g{}^2-u_g{}^2)dr=\rho_t{}'$$

is the effective range of neutron-proton triplet interaction appropriate to the deuteron ground state,[18,19] $\rho_t(-\epsilon, -\epsilon)$ in the notation of Bethe.[19] So

$$\sigma_e=\sigma_{e0}(1-\gamma\rho_t{}')^{-1}F^2. \tag{2}$$

Equation (2) is accurate under three assumptions: (1) the ground state of the deuteron is a pure S-state; (2) the outgoing particles are free; (3) there are no specifically mesonic effects.

These assumptions must be considered separately.

(1) The deuteron ground state contains about 4 percent ^3D-state to account for the quadrupole moment of the deuteron. This diminishes the population of the ^3S-state and lowers the cross section; on the other hand, disintegrations can take place from the ^3D-state in compensation. It may be shown, as has been remarked by Austern[7] and others, that, if we neglect the disintegrations from the ^3D-state, then Eq. (2) remains correct. ρ_t must be re-interpreted, but is the same as that determined from neutron-proton scattering, which is modified in exactly the same way by the presence of the noncentral forces that generate the ^3D-wave. If, then, we are able to calculate the effect of transitions from the ^3D-state and subtract them out, we may still compare the effective range as determined from deuteron photodisintegration with that determined by neutron-proton scattering despite the existence of noncentral forces.[20] The importance of $^3D-{}^3P$-transitions has been estimated by Austern[7] and amounts to a few tenths of a percent. The numerical correction is given later.

(2) The phenomena of high energy neutron-proton scattering imply that the strength of the P-state forces is not more than about 10 percent of that of the S-state forces, so the assumption of free outgoing particles is a good one. Detailed calculations at 6.14 Mev[21] suggest that the 10 percent limit corresponds to an uncertainty of 0.6 percent in the cross section. At 17.6 Mev[7] the uncertainty is about $1\frac{1}{2}$ percent, unless the noncentral forces have a very strong exchange character.[22] We have taken P-state forces to be zero.

(3) It has long been known[23] that there are no specifically mesonic effects in the electric dipole disintegration provided the Hamiltonian of the interaction between the gamma-ray and the deuteron is assumed to contain nucleon co-ordinates only.[24] The validity of this assumption has never been directly investigated, but it is expected to be good for the energies used in this investigation where the chief contribution to the matrix element comes from large nucleon separations.

TABLE I. Total, magnetic dipole, and electric dipole cross sections for photodisintegration of the deuteron, as functions of γ-ray energy.

E_γ	σ_t	σ_m	σ_e	σ_m/σ_e	±%
4.45	24.3	1.01	23.3	0.043	7.3
6.14	21.9	0.59	21.3	0.028	4.7
7.39	18.4	0.44	18.0	0.024	8.3
8.14	18.0	0.39	17.6	0.022	7.5
12.5	10.4	0.22	10.2	0.022	9.8
17.6	7.7	0.14	7.6	0.018	12

THE EFFECTIVE RANGE

Before evaluating the effective range from Eq. (2) we must know F. F depends on the shape of the interaction potential, and we have chosen to analyze the results in terms of extreme short- and long-tailed wells, namely, the square well and Yukawa well. For the square well F remains within 1 percent of unity throughout the range of gamma-ray energy used; we have set it equal to unity. For the Yukawa well F has been estimated through the approximation of Hulthén to the

[18] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).
[19] H. A. Bethe, Phys. Rev. **76**, 38 (1949).
[20] There remains the question of the effect of noncentral forces on the energy dependence of the effective range; in the absence of detailed information we must assume that this dependence is unchanged.
[21] I. F. E. Hansson and L. Hulthén, Phys. Rev. **76**, 1163 (1949).
[22] Both these corrections refer to an interaction potential of Yukawa form.
[23] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).
[24] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).

Yukawa potential.[2,3,25] Then

$$F=1-\left(\frac{\gamma^2+k^2}{\beta^2+k^2}\right)^2, \qquad (3)$$

$$k^2=\frac{1}{\hbar^2}m(h\nu-\epsilon),$$

and β is the second constant of the Hulthén potential, chosen to give the correct effective range; we find $\beta=1.35\times10^{13}$ cm^{-1}. Although the Hulthén potential is a good stand-in for the Yukawa potential at high energies, we must expect some disagreement at the energies of these experiments. The only directly-computed value of F^2 for the Yukawa potential is at 6.14 Mev where its value is 0.984.[26] Equation (3) suggests 0.988. We have used F as given by Eq. (3) in the first instance. Table II shows the various quantities of interest. σ_D is the computed cross section for disintegration from the D-state; $\sigma_S=\sigma_e-\sigma_D$ is the deduced cross section from the S-state only. ρ_{tS}' and ρ_{tY}' are the effective ranges appropriate to square and Yukawa (Hulthén) wells, respectively. $\pm$ is the probable error in ρ_t'.[27]

The units of ρ are 10^{-13} cm, the rest as in Table I. ρ_t' is a good constant[28] over the range of E_γ; this establishes the first objective of this investigation (see I) namely that the form of dependence of σ_e on E_γ is correctly given by quantum mechanics.[29] We cannot distinguish between well shapes on the basis of the constancy of ρ. The probable error in ρ we take to be ±0.09; it is largely governed by uncertainties common to all the cross sections. The weighted means are

$$\rho_{tS}'=1.73\times10^{-13}\text{ cm.}$$
$$\rho_{tY}'=1.78\times10^{-13}\text{ cm.}$$

The direct computation of F^2 at 6.14 Mev suggests that, in the lower energy region of our range where the most accurate experiments lie, the approximation for F^2 based on the Hulthén potential is too high by about 0.4 percent and that ρ should be accordingly increased by about 0.6 percent. We prefer the result deriving from the more physically-plausible Yukawa potential[30] and quote:

$$\rho_t(-\epsilon,-\epsilon)=(1.79\pm0.09)\times10^{-13}\text{ cm.}$$

The determination of this quantity was the second objective of this investigation.

TABLE II. Photodisintegration cross sections and effective ranges (for square well and Yukawa well), as functions of γ-ray energy.

E_γ	σ_e	σ_D	σ_S	σ_{e0}	F^2	ρ_{tS}'	ρ_{tY}'	$\pm$
4.45	23.3	0.10	23.2	14.23	0.994	1.67	1.69	0.18
6.14	21.3	0.13	21.2	12.65	0.988	1.74	1.77	0.12
7.39	18.0	0.14	17.9	11.00	0.982	1.67	1.72	0.21
8.14	17.6	0.14	17.5	10.09	0.980	1.83	1.88	0.18
12.5	10.2	0.14	10.1	6.37	0.958	1.59	1.69	0.24
17.6	7.6	0.14	7.5	4.18	0.925	1.89	2.06	0.35

COMPARISON WITH OTHER RESULTS ON THE EFFECTIVE RANGE

Photodisintegration is the most direct method for the determination of the triplet neutron-proton effective range. The only other measurements available are of $\rho_t(0,-\epsilon)$ and of these the most accurate is due to Hughes[31] who measures the hydrogen coherent scattering amplitude and combines it with the free proton cross section for slow neutrons[32] to find $\rho_t(0,-\epsilon)=(1.70\pm0.03)\times10^{-13}$ cm. Earlier measurements[33,34] gave a rather lower value of ρ_t with a considerably larger probable error.

In order to compare our results for $\rho_t(-\epsilon,-\epsilon)$ with this value of $\rho_t(0,-\epsilon)$ we must correct them for the energy dependence of ρ. This may be done using the data of Blatt and Jackson;[18] the shape-dependent parameter P of these authors is -0.04 and $+0.14$ for square and Yukawa wells, respectively; this leads to $\rho_t(0,-\epsilon)=1.75$ and $1.72_5\times10^{-13}$ cm for the two wells, respectively. It appears, however, that the linear dependence of ρ on energy as prescribed by effective range theory is not an accurate approximation, at least for the Yukawa potential. Hulthén and Nagel[26] have computed the energy dependence of ρ for the Yukawa potential and a better estimate of $\rho_t(0,-\epsilon)$ and $\rho_t(0,0)$ comes from their work. Table III shows the three effective ranges of chief interest. The two $\rho_t(-\epsilon,-\epsilon)$ are our own experimental values for the two wells; the others are derived from them for the square well by effective range theory[18] and for the Yukawa from the computations of Hulthén and Nagel.

The agreement between $\rho_t(0,-\epsilon)$ deduced from photodisintegration and that measured by Hughes and previously quoted is satisfactory. We can make no

[25] J. S. Levinger, Phys. Rev. 76, 699 (1949).

[26] Bengt Nagel, private communication.

[27] The quantity which is determined experimentally is the range factor $R=(1-\gamma\rho)^{-1}$; $d\rho/\rho=(R-1)^{-1}dR/R$. $R\sim1.7$, so the percentage error in ρ is about 1.5 times that displayed in Table I for σ_e.

[28] It might appear that ρ is too good a constant in view of the stated probable errors. These errors are in the absolute values of ρ; constancy of ρ is a matter for the relative cross sections which are considerably better known than the absolute ones, as some principal errors in σ_e are common to all gamma-ray energies.

[29] E. G. Fuller [Phys. Rev. 79, 303 (1950)] has investigated the relative variation of σ with E_γ over a range closely corresponding to ours. He finds a rather slower fall of σ with E_γ than given by quantum mechanics; this may be due to his use of a calculated betatron spectrum.

[30] The results of other long-tailed potentials such as the exponential are very close to this.

[31] Ringo, Burgy, and Hughes, Phys. Rev. 82, 344 (1951).

[32] E. Melkonian, Phys. Rev. 76, 1744 (1949).

[33] Sutton, Hall, Anderson, Bridge, de Wire, Lavatelli, Long, Snyder, and Williams, Phys. Rev. 72, 1147 (1947).

[34] Shull, Wollan, Morton, and Davidson, Phys. Rev. 73, 842 (1948).

TABLE III. The three effective ranges of chief interest.

Effective range	Square well	Yukawa well
$\rho_t(-\epsilon, -\epsilon)$	1.73	1.79
$\rho_t(0, -\epsilon)$	1.75	1.70
$\rho_t(0, 0)$	1.77	1.62

distinction between well shapes on the basis of this comparison. The achieving of this agreement constitutes the third objective of this investigation.

MESON MASS

It is tempting to inquire what meson mass m_π is implied if we accept the Yukawa potential. We may use our value of $\rho_t(0, 0)$ to compute an intrinsic range from which the meson mass is obtained immediately. This gives $m_\pi=(291\pm24)m_e$ where m_e is the electron mass. It is improper to regard the accord between these figures and the experimental π-meson mass[35] of (276.1 $\pm$2.3) as other than fortuitous.

CONCLUSION

The electric dipole component of the photodisintegration of the deuteron seems to be adequately represented by current quantum-mechanical theory up to gamma-ray energies of 10–20 Mev. No information about the shape of the interaction potential is forthcoming from this investigation.

ACKNOWLEDGMENTS

I am very much indebted to Professor L. Hulthén and Mr. Bengt Nagel for acquainting me with the results of their calculations and for much helpful comment.

I would like to thank Dr. Norman Austern, Dr. E. E. Salpeter, and Professor E. Guth for allowing me to see their papers prior to publication.

[35] Birnbaum, Smith, and Barkas, Phys. Rev. 83, 895 (1951).