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The Dynamic Characteristics of a Low Pressure Discharge*

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Square-wave modulation of current at a frequency of 167–400 cps was imposed on a dc discharge in a tube containing mercury vapor at a pressure of 6μ and argon at 3.5 mm (Hg). The intensity of λ2537 and other data were recorded on oscillograms. Each sudden change of the arc current i was accompanied by an instantaneous change in the same direction, of the longitudinal voltage gradient E, the electron temperature T_e, the 2537 output, and the ion current to the wall. These instantaneous changes were followed by much slower adjustments toward the steady-state values corresponding to the arc current. The time required for a rough adjustment was 0.2–0.5 msec when the current was increased, and 1–3 msec when it was decreased, for ratios of maximum to minimum current ranging from 1.6 to 3.9. When the current was decreased, there was an excess of ions that had to diffuse to the tube wall.

When the maximum arc current was 2.75 times the minimum value, the ratio of the electron concentration n_0 measured at the tube axis to that at the inner edge of the wall sheath (n_w) varied by a factor of 3 during each cycle. Some of this variation can be attributed to greater ion production per electron near the axis; some of it may be an artifact, since the wall probe may alter the radial distribution of electrons.

The number of electrons N_e per cm of arc was computed from the measured values of electron concentration by assuming that the radial variation follows the Bessel function $J_0(\mu r/a)$, where $J_0(\mu)=n_w/n_0$. The results for the major part of the cycle do not agree with those computed from the mobility equation. Part of the discrepancy can be ascribed to changes of the radial distribution of electrons during a short period when ion production is very high. Two-stage ionization predominates throughout the cycle.

Granovsky's approximate solutions of the basic equations for transient states of a low pressure discharge are not applicable to a Hg discharge with a rare-gas pressure of the order of 1 mm (Hg) or more. Since the concentrations of excited atoms are not single-valued functions of T_e, these equations are insoluble. However, step-by-step graphical or tabular solutions might be made for particular cases by use of experimental values of mean free paths and of the probabilities of exciting, ionizing, and radiative transitions. Useful relationships between i, E, T_e, and N_e can be obtained by combining the mobility and the energy-balance equations for the electrons. The variation in T_e during a transient depends mainly on the range of i/N_e. Instantaneous values of E are roughly proportional to the square root of the power expenditure w_e per electron. Since T_e is the principal factor fixing w_e, E and T_e are closely interdependent.

The modulation of radiant energy must decrease continuously with increasing frequency if a dc discharge is operated with a fixed amplitude of current modulation.

INTRODUCTION

IN the summer of 1943, the author was using an oscillograph to study the effects of a sudden change in the current through a fluorescent lamp operated on direct current. The purpose was merely to make measurements within a period over which the pressure of the mercury vapor was practically constant. However, the oscillograms showed an unexpected effect: an instantaneous change in lamp voltage that was much larger than the net long-term change, and in the opposite direction. This initial change was reversed in less than one millisecond when the change of current was an increase, and in one to five milliseconds when it was a decrease. It seemed certain that the transient was an indication of the time required for the ion concentrations in the discharge to increase or decrease to the equilibrium values corresponding to the new value of arc current.

Galkena and Granovsky[1] made similar observations on a low pressure discharge in mercury vapor without any rare gas present. In this case the mobility of the ions is many times greater than it is in a fluorescent lamp, hence the duration of the transient is only a few microseconds. Recently McClure and Holt[2] have reported results obtained by the use of current pulses with a duration of only 30 microseconds.

If the initial change of current is made by adding to the resistance in a lamp circuit or by shorting out part of it, the lamp current changes during the transient because the lamp impedance is changing. The results are more striking and more easily interpreted if the current is controlled by use of pentodes in series with the discharge, so that square-wave changes of current can be made, practically independent of subsequent changes in the impedance of the discharge.

EXPERIMENTAL TECHNIQUE

Circuits for square-wave control of current and for measuring the characteristics of the discharge during a transient have been described elsewhere.[3] The data in the present article have been taken on discharge tubes with an internal diameter of about 36 millimeters. The tubes had a well-distributed supply of liquid mercury and also contained argon (filling pressure 3.5 mm (Hg) at 27°C). They were operated fully immersed in a water bath which was held at 40°C. To prevent electrical leakage, the leads to the electrodes and to the probes were brought above the water level in glass tubing which was sealed to the discharge tube. The wall probe used to measure positive ion current was a disk 4.8 mm in diameter, surrounded by a guard ring with internal and external diameters of 6.2 and 9.5 mm. A mica disk behind the disk probe and its

* A preliminary report on this subject was made at the M.I.T. Conference on Physical Electronics, on April 2, 1948. Further results were given at the Conference on Gaseous Electronics at Pittsburgh, on November 5, 1949.

[1] Z. P. Galkena and V. L. Granovsky, J. Tech. Phys. (U.S.S.R.) 18, 585 (1948).

[2] B. T. McClure and R. B. Holt, Phys. Rev. 83, 878 (1951).

[3] B. T. Barnes and S. Eros, J. Appl. Phys. 21, 1275 (1950).

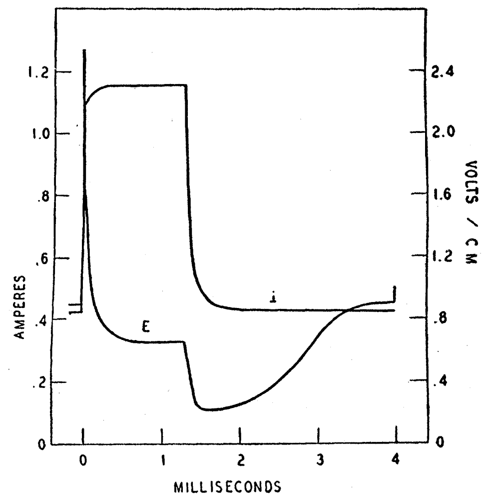


FIG. 1. Arc current i and longitudinal voltage gradient E during a cycle of modulation.

guard ring shielded their rear surfaces and the leads. The other probes were 3-mil wires with an exposed length of 5 mm, and with a short section adjacent to the collecting area surrounded by a thin-walled glass tube not quite in contact with the probe. This arrangement was intended to prevent extension of the collecting area by sputtered material from the probe, yet to provide the minimum interference with diffusion of electrons to the probe. There was a 5-mm gap between the wire probes used to measure electron temperature and a 12.3-cm spacing between the pair used to measure the longitudinal voltage gradient. The exposed areas of the probes were made small in order to reduce their effect on the discharge. The current drawn at space potential by a wire probe or by the wall probe, with the guard ring floating,[4] was 3–4 percent of the arc current.

At the middle of the discharge tube for which the intensity of λ2537 was determined, there was a side tube long enough to rise above water level, with a re-entrant section of the same length ending in a fused-quartz window. A heating coil prevented condensation of mercury between the inner and outer walls of this side tube.

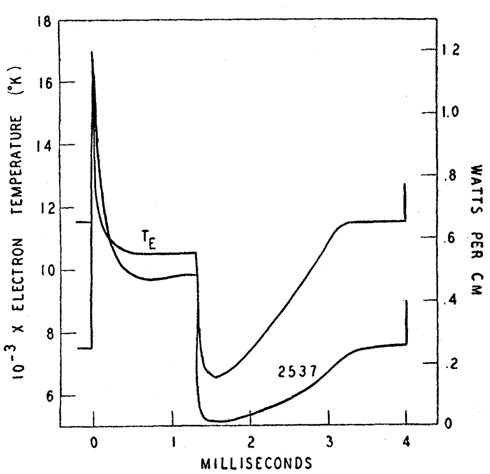


FIG. 2. Electron temperature T_e and output, per cm of arc, in 2537 line. Current modulation same as in Fig. 1.

[4] The guard ring was connected only for measurements of ion current.

RESULTS

The arc current i and the longitudinal voltage gradient E during a cycle of modulation are shown in Fig. 1. Each sudden change in arc current causes the voltage gradient to change by a slightly larger factor. When the current has been increased, the initial rise in voltage gradient is followed immediately by a very rapid decrease; when the current is reduced, a correspondingly rapid decrease in voltage gradient is followed by a relatively slow increase. In the latter case, the excess ions must flow to the wall of the discharge tube. The cycle is made long enough to permit the

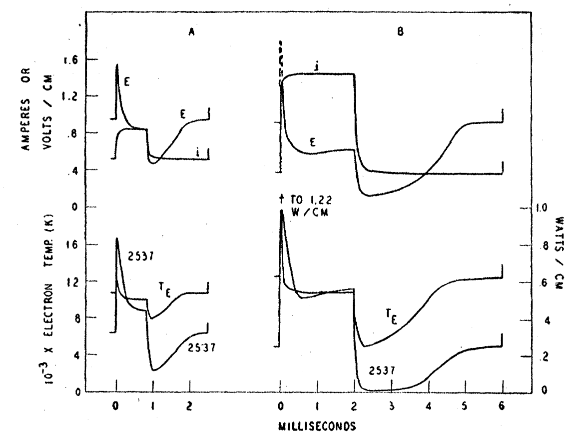


FIG. 3. Upper half of graph corresponds to Fig. 1, lower to Fig. 2, but amplitude of current modulation is less in case A, greater in case B.

discharge to nearly reach its steady-state condition in each constant-current period.

The curves in Fig. 2 show the electron temperature T_e and the power output in the 2537 line. The curves for E, T_e, and λ2537 are quite similar, but T_e changes less and the 2537 output changes more than the voltage gradient does.

Figure 3 shows the effect of changing the amount of modulation while keeping the average arc current roughly constant. The adjustment toward equilibrium after a decrease in current becomes slower when one increases the factor by which the current is changed. This decrease in the speed of adjustment is caused by the greater reduction in T_e in the early part of the low current period, since the reduction in T_e produces a corresponding decrease in the rate of ambipolar diffusion. To offset this effect, one must reduce the frequency of modulation when the amplitude is increased, if each period at constant current is to be roughly of the same duration as the transient.

The voltage between the arc axis and the inner edge of the wall sheath is shown in Fig. 4. (The results given in Figs. 4–11 inclusive correspond to the amount of current modulation shown in Fig. 1.) The percentage variation in the radial voltage drop within the plasma is somewhat less than that of T_e (Fig. 2). If the radial distribution of electron concentration remained constant, one would expect the voltage drop to be proportional to T_e.

The rate at which ions arrive at the wall of the tube is plotted in Fig. 5. The sudden rise in ion current when the arc current increases abruptly is due to the increase in voltage gradient at every point along a radius of the tube, since ion velocities change proportionately. The subsequent gradual rise in ion current, in spite of falling T_e, reflects the increase in ion density during the period of high arc current. At the beginning of the low current period, the sudden fall in T_e causes a corresponding drop in ion current. The latter continues to fall throughout the period of low arc current because the ion concentrations decrease faster than the radial voltage gradients increase.

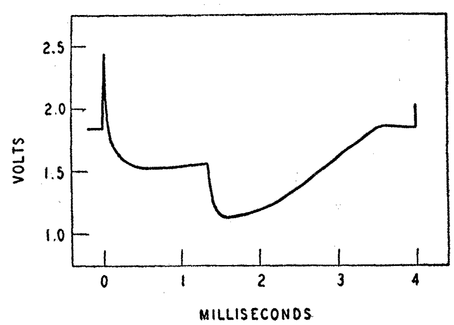


FIG. 4. Voltage between arc axis and inner edge of wall sheath. Current modulation same as in Fig. 1, for Figs. 4–11.

Figure 6(a) is an oscillogram of the current to a wire probe centered about the tube axis; 6(b) is the corresponding one for a wall probe. In each case the series of traces above the zero line was obtained by making successive changes of 1.5 volts in the bias voltage between an auxiliary probe and a special cathode-follower circuit[5] attached to the probe whose current was being recorded. By correcting for changes in the grid-cathode bias and for the varying voltage drop in the measuring resistor, one obtains data for a current-voltage characteristic for any instant during a cycle of modulation. Plotting these data, one finds the probe current at space potential and can then calculate the ion concentration in the region surrounding the probe.

Comparison of Figs. 6(a) and (b) shows that at the beginning of the high current period the current to a probe near the axis of the tube rises more rapidly than that to a wall probe. This indicates that the rate of ionization per electron is greater near the axis than near the wall—doubtless because the concentration of atoms in the 6^3P states is maximum at the axis.

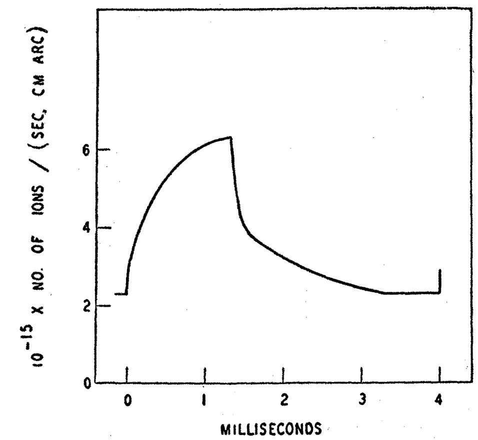
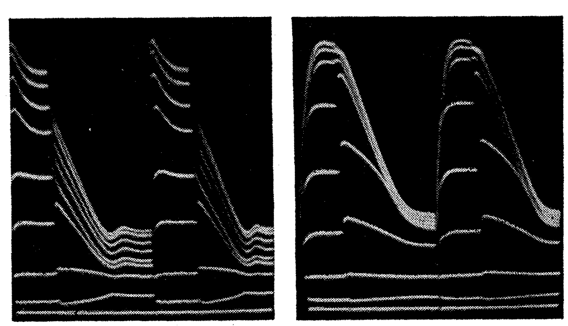


FIG. 5. Positive ion current to wall during a cycle.

The electron concentrations n_0 and n_w at the axis and at the inner edge of the wall sheath are shown in Fig. 7. The measured values of n_w for the latter part of the high current period are too low because only one 6L6 tube was used in the cathode-follower[5] when two were needed to prevent saturation. The dashed part of the curve in Fig. 7 probably represents the approximate course of n_w in this part of the cycle.

The values of n_w shown in Fig. 7 are based on the assumption that the concentration of electrons at the inner edge of the space charge sheath is not affected appreciably by the potential of a wall probe. With a mercury vapor pressure of a few microns and with no rare gas present, Langmuir and Mott-Smith obtained a ratio of electron current at space potential to positive ion current that was in reasonable agreement with this assumption.[6] With argon present at a pressure of 3.5 mm this might no longer be true. This point needs more detailed theoretical treatment. The wide variation of



a b

FIG. 6. Probe currents at voltages near space potential for probe centered about axis (a) and for wall probe (b). Bias voltage (between auxiliary probe and cathode follower connected to measurement probe) changed in 1.5-volt steps. Zero line at bottom.

[5] See reference 3, Fig. 4.

[6] See L. Tonks and I. Langmuir, Phys. Rev. 34, 917 (1929).

n_w/n_0 shown by Fig. 7 also deserves further investigation.

To compute the total number of electrons N_e per centimeter of arc from the data in Fig. 7, one must make an assumption as to the radial distribution of electron concentration. I have assumed that this distribution corresponded to the equation

$$n = n_0 J_0(\mu r/a), \quad (1)$$

where r = distance from the tube axis, a = tube radius, and J_0 is the zero-order Bessel function of the first kind. Values of μ for use in Eq. (1) were obtained from the curves shown in Fig. 7, using the relationship $J_0(\mu) = n_w/n_0$.

Equation (1) corresponds to the case where ionization is either single-stage or two-stage with no radial variation of the concentrations of atoms in the 6^3P states. Actually, the radial variation of n must be closer to a linear one than is indicated by Eq. (1), and the corresponding integration factors would be a few percent less than those with which N_e was computed. Since the radial variation of the concentration of metastable atoms is not known, the true integration factor cannot be computed.

A second method of computing N_e is available. The mobility equation for the electrons in a low pressure discharge may be written in the form[7]

$$i = 3.84 \times 10^{-10} N_e L_e E / T_e^{\frac{1}{2}}, \quad (2)$$

where i, E, and T_e are in amperes, volts/cm, and °K, respectively, and L_e is the electron mean free path (cm). Values of L_e for 3.5 mm (Hg) of argon plus 6μ of mercury vapor (both pressures reduced to terms of 0°C), have been published elsewhere.[8] These values were computed for Maxwellian distributions of electron velocity, taking into account the variation of the probability of collision with electron velocity.

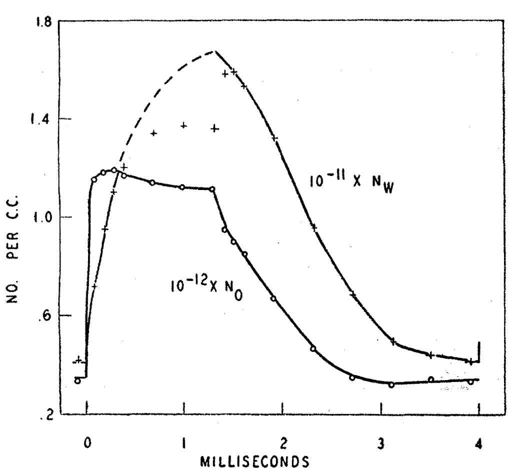


FIG. 7. Electron concentration at axis (n_0) and at inner edge of wall sheath (n_w) during a cycle.

The results obtained with these two methods of computing N_e are given in Fig. 8. The circles show the values calculated from n_0 and n_w using Eq. (1); the squares, those obtained from Eq. (2). In the early part of the high current period the discrepancy between the two curves is quite large. Here one would expect the values shown by the circles to be too high because most of the ions present have just been formed and are relatively near their points of origin. Under these conditions, the deviation of the radial distribution of electrons from that indicated by Eq. (1) is at its maximum, and the values of N_e obtained by use of this equation must be too high. However, it seems doubtful that this factor alone accounts for the disagreement between the curves in Fig. 8.

During the first part of the low current period, T_e is sufficiently low that ion production probably is almost negligible. If this is true, then Fig. 5 shows that N_e should be decreasing at the rate indicated by the curve drawn through the circled points. Later in the low current period, the rate of production of ions is not known and one cannot predict accurately the course of the curve for N_e. However, since the curve with the circled points seems to have the correct shape, it has been used in all subsequent calculations.

The rate of production of ions obviously is equal to the rate at which they flow to the wall plus the rate at which the number present in the discharge increases (Figs. 5 and 8). Figure 9 shows the ionization per electron, plotted against the reciprocal of T_e. Because of possible errors in dN_e/dt and in the data on ion flow to the walls, the computed values of ion production are not reliable when dN_e/dt is negative and nearly equal to the ion flow to the walls, per centimeter of arc. The curves through the circled points in Fig. 8 and the squares on Fig. 9 were drawn in such a way that these curves and the wall-current data (Fig. 5) are mutually consistent. This consistency, then, does not prove that the upper curve on Fig. 9 has the right slope, since the square that is farthest to the left on this graph could have been shifted up or down a relatively large distance by a comparatively small change in the course of the curve though the circled points on Fig. 8.

The dashed curve on Fig. 9 shows the computed rate of ionization for a single-stage process,[9] assuming a Maxwellian distribution of electron velocities and using the cross sections given by Nottingham.[10] In the discharge under investigation, one would expect a predominance of two-stage ionization. This is just what Fig. 9 shows.

The slope of the experimental curve on this graph has no special significance. At any value of T_e, the probability of two-stage ionization depends on the concentration of excited atoms. During a transient at roughly constant current, these concentrations may

[7] K. T. Compton and I. Langmuir, Revs. Modern Phys. **2**, 218 (1930), Eq. (107).
[8] Kenty, Easley, and Barnes, J. Appl. Phys. **22**, 1010 (1951).
[9] I am indebted to Miss M. A. Easley, who gave me these results.
[10] W. B. Nottingham, Phys. Rev. **55**, 214 (1939).

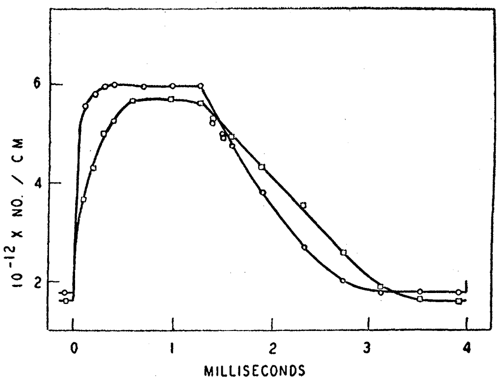


FIG. 8. Electrons per cm of arc computed from probe measurements of electron concentration (○) and from mobility equation for electrons (□).

differ considerably from those that would exist if the same values of T_e were obtained in the steady state by changing the arc current, the mercury vapor pressure, or both. Thus the variation with T_e, of the ion production per electron may be quite different from that found in steady-state operation of a similar discharge.

BASIC EQUATIONS

Granovsky has made an extensive theoretical analysis[11] of nonstationary states of a low pressure discharge. Since the fundamental equations form such a complex system that they defy any attempt to find a general solution for them, Granovsky has made simplifying assumptions in order to derive solutions for special cases. Unfortunately several of his assumptions are not applicable to a discharge in mercury vapor plus a rare gas. The principal reasons why his analysis is not applicable are the following:

(1) In his derivation of the equation for the rate of change of the concentration of excited atoms, Granovsky ignores the effect of radiative transitions. In a typical low pressure discharge in mercury vapor, the loss of excitation by radiation of the 2537 line is much more important[12] than transitions from 6^3P levels to the ground level during collisions of the second kind.

(2) Granovsky assumes that the elastic loss per electron per second is proportional to $T_e - T_g$. (T_g = gas temperature in °K). In argon, because of the Ramsauer effect, the elastic loss per electron increases almost exponentially with T_e. In a discharge of the sort considered here (6μ Hg, 3.5 mm A), the elastic loss is almost entirely due to collisions of electrons with argon atoms.

(3) For determining the variation of inelastic losses with T_e, Granovsky suggests that measurements be made on steady-state discharges. This is not practical if either multiple excitation or collisions of the second kind play an important role, since the concentrations in the excited levels during a transient ordinarily will not have the equilibrium values, at a particular T_e, attained in steady-state operation.

(4) For the case where the concentrations in excited states approach a Boltzmann equilibrium with those in the normal level, Granovsky concludes that there is practically no variation in the concentrations in excited levels during a transient. However, T_e varies during every transient, and Boltzmann populations are exponential functions of T_e. Thus, if the transient is of sufficient duration, changes in the concentrations of excited atoms are an important factor. In the discharge described in the present article, the populations of the 6^3P levels are far from a Boltzmann equilibrium,[12] but even if this were not the case, the duration of transients is sufficient to allow large variations in the concentrations of excited atoms to take place when the amplitude of modulation is large.

A complete solution of the fundamental equations for transient states of a discharge seems unattainable, but there are several very useful relationships between the variables. The mobility equation for the electrons is applicable at every instant during a transient. It may be written in the following form:

$$E/i = 2.60 \times 10^9 T_e^{\frac{1}{2}}/(L_c N_e). \qquad (3)$$

The instantaneous power balance in the positive column is expressed by the equation

$$Ei = N_e(w_e + dW_e/dt), \qquad (4)$$

where W_e is the average energy of an electron and w_e is its average expenditure of power. This equation ignores the power spent by the longitudinal field in the acceleration of ions, but for a mercury discharge, this power is less than 0.1 percent of the input.

Rough calculations show that dW_e/dt is negligible compared to w_e except for a period of a few microseconds after a sudden large change in i. Since this short period of initial adjustment of T_e was not observable on the oscillograms on which Figs. 1–9 are based, one may use the following equation for the analysis of the

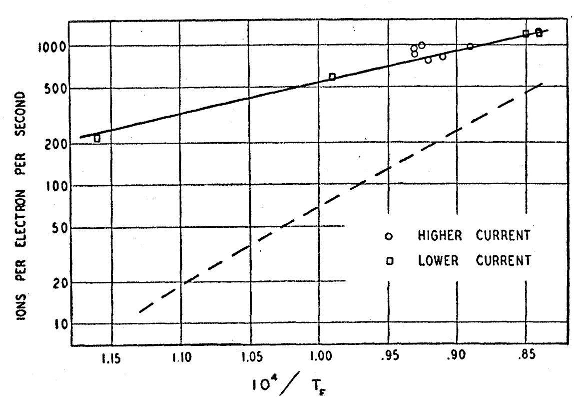


FIG. 9. Ion production per electron *vs* $10^4/T_e$ during a cycle of modulation. Dashed line shows values computed for a single-stage process, using Nottingham's data.

[11] B. Granovsky, J. Phys. U.S.S.R. **3**, 195 (1940).
[12] C. Kenty, J. Appl. Phys. **21**, 1309 (1950).

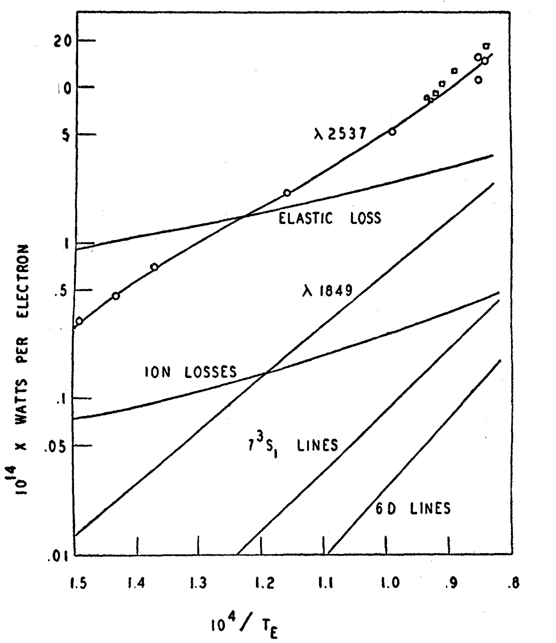


FIG. 10. Power per electron spent in elastic collisions, in flow of electrons and ions to wall, and in radiation of principal lines transmitted by or absorbed in tube wall.

data plotted on these graphs:

$$Ei = N_e w_e, \tag{5}$$

(3)÷(5) gives

$$w_e = 2.60\times 10^9 \frac{T_e^{\frac{1}{2}}}{L_e}\left(\frac{i}{N_e}\right)^2 \tag{6}$$

and (3)×(5),

$$E = 5.1\times 10^4 (T_e)^{\frac{1}{4}}(w_e/L_e)^{\frac{1}{2}}. \tag{7}$$

Since recombination of ions and electrons in the plasma of a low pressure discharge ordinarily is negligible, the variation of N_e during a transient obeys the following equation:

$$dN_e/dt = \lambda N_e - i_w/e, \tag{8}$$

where λ = number of ions formed per electron per sec, i_w = positive ion current to the wall per cm of arc, and e = electronic charge.

To what extent do the equations listed above define the behavior of a discharge during a transient? Since Eqs. (6) and (7) are derived from (3) and (5), these four equations fix the values of only two unknowns. If one assumes that i is determined solely by the external circuit, that N_e is known from the previous history of the discharge, and that the relationships of w_e and of L_e to T_e are known, then Eq. (5) can be used to compute instantaneous values of T_e and Eq. (3) shows the value of E at any instant.

To test some of the equations derived above, w_e will be determined by summing its component parts, each of which is known or can be computed as a function of T_e. This set of values will then be compared with those computed by substituting the values shown in Figs. 1, 2, and 8 into Eqs. (5) and (6).

One of the principle components in w_e is the λ2537 output per electron, which can be computed from Figs. 2 and 8. The other major component is the elastic loss per electron; its variation with T_e for Hg and A is shown in another article.[8]

The output in the 1849 line was not measured, but its order of magnitude is known from experiments in which the contribution of this line to the excitation of the phosphor in a fluorescent lamp was determined. The variation with T_e of the 1849 output is not known either, and cannot be calculated accurately because there is a possibility of appreciable two-stage excitation. As a rough approximation, the intensity has been assumed proportional to $E^{-6.7e/kT_e}$, where k is the Boltzmann constant. Similar exponentials have been used for the lines originating in the 7^3S_1 and in the $6D$ levels. Absolute values were computed from data obtained on ac and dc tubes similar to those operated on modulated direct current.

The power spent, per electron present, in accelerating ions toward the wall and in providing both ionization energy (10.4 ev) and the kinetic energy of the electrons ($3kT/2$) has been computed and labeled as ion losses. The result is plotted in Fig. 10, together with the other types of power expenditure discussed above. The values of w_e obtained from Fig. 10 are shown by the dashed line in Fig. 11; those computed from Eq. (6) are indicated by squares, and those from Eq. (5) by circles. The two sets of plotted points are in reasonable agreement, but the dashed line lies far below them at low values of T_e. Since elastic losses account for two-thirds of the computed expenditure of power at the lowest value of T_e, it seems likely that the curve given by Brode[13] for the probability of collisions of electrons with argon atoms is too low in the region where $\sqrt{V} < 2$.

Equation (3) has been tested already, since it was used to compute one of the sets of N_e values plotted in Fig. 8. The agreement between the two curves on this figure is relatively poor, but the differences may be chiefly due to sources of error already mentioned: uncertainty as to L_e at low values of T_e, and difficulty in determining the correct integration factor for computing N_e from n_0.

In testing Eq. (7) one might use the values of w_e obtained from Eq. (6), but this would merely duplicate the results discussed in the preceding paragraph, since one obtains Eq. (3) when Eqs. (6) and (7) are combined. It seems more logical to test Eq. (7) by using the values of w_e obtained from Fig. 10. The dashed curve in Fig. 12 shows the result of this substitution, and the plotted points are the experimental values for two amplitudes of modulation. The curve through these points is in reasonable agreement with the computed curve except

[13] R. B. Brode, Revs. Modern Phys. 5, 263 (1933).

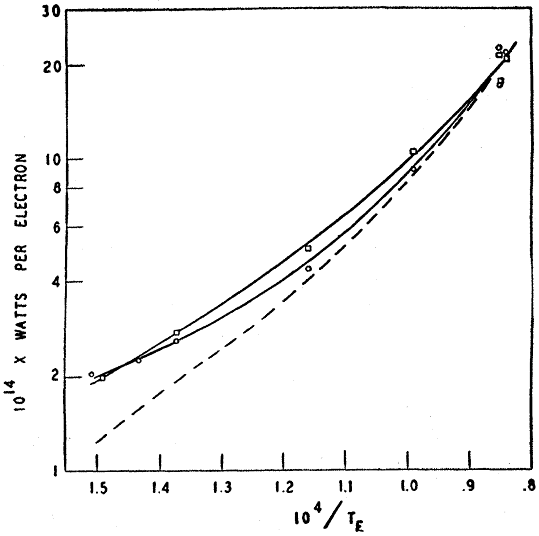


FIG. 11. Total power spent per electron, computed from input per cm arc and N_e values ○, from Eq. (6) □, and from sum of losses shown in Fig. 10 — — —.

at low values of T_e where the values of w_e obtained from Fig. 10 seem to be too low.

For a typical discharge in mercury vapor, computation of N_e by use of Eq. (8) is not possible because there are no detailed data on the probability of two-stage ionization. Klarfeld has estimated[14] that, for an equal excess of energy over the minimum required, the probability of ionization of an atom in a 6^3P level is an order of magnitude greater than that of an unexcited atom. Data accumulated here, but not analyzed in detail, indicate that for electron energies within 1–2 ev of the minimum for ionization, and for the same excess of energy above the minimum required value, the cross sections for ionization from the 6^3P_2 state are several times Nottingham's values[10] for the single-stage process. This tentative conclusion is based on measurements of ion production under steady-state conditions, which can be made fairly accurate, but it also involves computations of metastable atom concentrations and measurements of electron velocity distributions, both of which are much less reliable. Nevertheless, it should be possible to deduce curves for the cross sections for ionization from the 6^3P levels of mercury which would be useful over a considerable range of experimental conditions, even though one has insufficient data for confirmation of the details of the curves chosen to represent the ionization from the individual levels. If these cross sections were known, one could determine values of λ for use in Eq. (8), since fairly reliable computations of the concentrations in the 6^3P_1 and 6^3P_2 levels can be made,[12] and a rough estimate suffices for the 6^3P_0 one. With λ known, Eq. (8) would be usable and one could predict the behavior of a low pressure Hg discharge during any sort of transient.

[14] B. Klarfeld, J. Tech. Phys. (U.S.S.R.) 5, 921 (1938).

DISCUSSION

Actually to use the equations in the preceding section, one would make a step-by-step computation of the changes in the variables over successive small intervals of time. This laborious procedure is necessary whenever two-stage processes are of importance because w_e and λ are not single-valued functions of T_e and of the pressure of mercury vapor. These two variables also depend on the concentrations of atoms in the 6^3P levels, and changes in these concentrations lag far behind changes in T_e. Thus, there are no simple expressions for w_e and λ that one can substitute in the equations describing the dynamic behavior of a discharge, in order to obtain a general solution for these equations.

The variation in T_e during a transient can be explained qualitatively by analyzing Eq. (6). This equation shows that $w_eL_eT_e^{-\frac{1}{2}}$ is proportional to $(i/N_e)^2$. Since L_e changes relatively slowly with changes in T_e, while w_e increases with T_e much faster than the first power, $w_eL_eT_e^{-\frac{1}{2}}$ always changes in the same direction as T_e. Thus an increase in i/N_e is accompanied by a rise in T_e; and a decrease, by a fall in T_e. Therefore, if a sudden change in i is made, T_e must change in the same direction initially, before N_e has adjusted to its equilibrium value for the new current.

During transients caused by sudden changes in current, the value of T_e at any particular instant will depend mainly on the value of i/N_e. This is illustrated by Fig. 13, which gives data for two of the three amplitudes of modulation shown in Figs. 1 and 3. The fact that, with the maximum modulation, the points for the high current period lie above the smooth curve through the other points may not be due to errors in the data. When the arc current is high, the concentration of atoms excited to the 6^3P levels is correspondingly high,

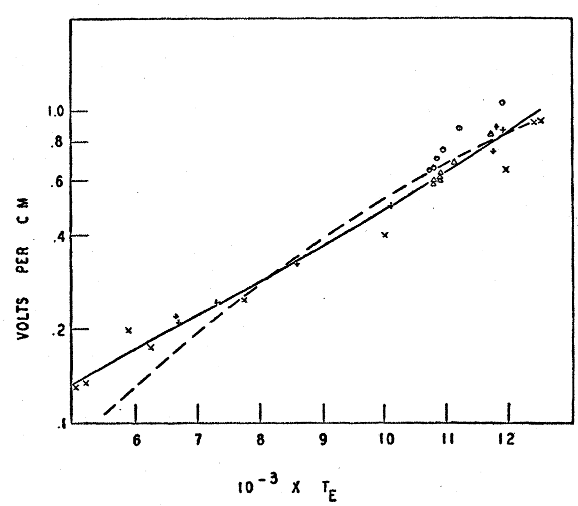


FIG. 12. Longitudinal voltage gradient *vs* $10^4/T_e$. Plotted points and full-line curve show experimental values; dashed curve was computed using Eq. (7). Values obtained at higher and lower currents with maximum modulation (Fig. 3) shown by △ and ×, respectively; those for medium modulation (Fig. 1) by ○ and +.

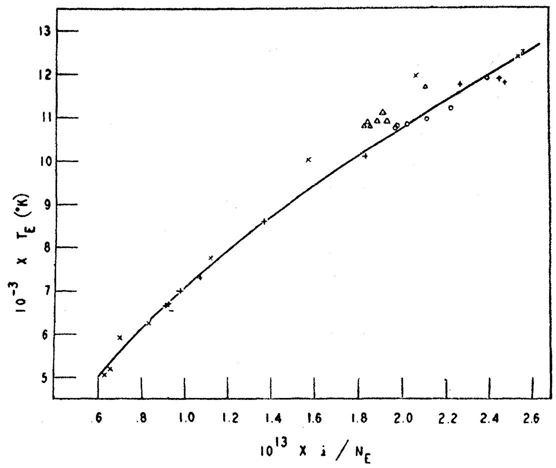


FIG. 13. Electron temperature *vs* i/N_e. Values at higher and lower currents at maximum modulation shown by △ and ×, respectively; those for medium modulation by ○ and +.

and a considerable proportion of the excitations to these levels are offset by collisions of the second kind which return the excitation energy to the electrons. As a result, the value of w_e is less for a particular T_e, and conversely [from Eq. (6)] the value of T_e corresponding to a particular value of i/N_e is greater than at lower currents.

Inspection of Eq. (7) shows that for a particular discharge E is nearly proportional to $w_e^{1/2}$, since $T_e^{1/4}$ and $L_e^{1/2}$ change relatively little during a transient. Since T_e is the principal factor determining the value of w_e, for a particular discharge tube, Eq. (7) shows that there is a close correlation between E and T_e. This was evident from the experimental data discussed earlier (Figs. 1 and 2).

Since the computation of λ [Eq. (8)] is the most difficult step in the use of Eqs. (3) to (8), these equations are particularly useful when $\lambda \doteqdot 0$. This is true in the early stages of a transient following a major reduction in current, since T_e is too low for appreciable ionization (see Fig. 2).

When the frequency of modulation or of ac operation is sufficiently high, instantaneous values of λ are of no importance because the number of ions formed during a cycle is small compared to the number present. Thus N_e varies relatively little during a cycle, and one needs to know only its average value in order to compute the dynamic properties of the discharge. Such a computation should shed some light on the factors which decrease the modulation of the radiant energy from a discharge as the frequency is increased, with the modulation of arc current held constant.[15] From Eq. (6), one sees that the modulation of T_e would decrease with increasing frequency because i/N_e would have a smaller range of values as N_e approached constancy. At still higher frequencies, one would have to use $w_e + dW_e/dt$ on the left side of Eq. (6). Since the ratio of the maximum to the minimum value of i/N_e would have become practically constant, w_e and therefore T_e would vary less when the frequency became high enough that dW_e/dt was of the same order of magnitude as w_e during a major part of the cycle.

Another factor which must be considered in the use of Eq. (6) for calculating the modulation of T_e is the storage of energy in metastable and resonance levels and its subsequent expenditure. These processes make w_e higher than one would otherwise expect when T_e is increasing, and lower when it is decreasing. With the modulation of w_e fixed by Eq. (6), any process which increases w_e at high values of T_e and decreases it when T_e is low reduces the variation of T_e during a cycle. Rough calculations indicate that the maximum rates of storage and expenditure of excitation energy will increase with frequency, even though the ratio of the maximum to the minimum stored energy decreases. If this is the case, this storage process will decrease the modulation of T_e with increasing frequency.

When the gas temperature is of the same order of magnitude as the electron temperature, storage of energy in the thermal motion of the gas atoms is another factor that reduces the modulation of T_e with increasing frequency. This effect was mentioned in the second article listed under reference 15.

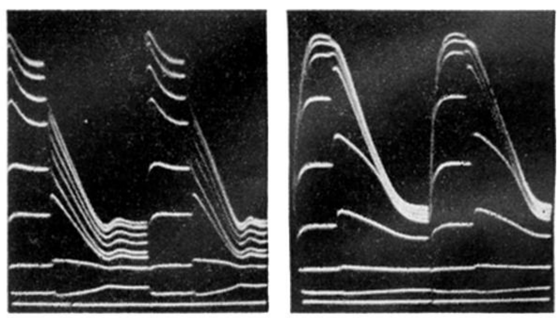
All of the processes discussed in the preceding paragraphs make the radiant energy per electron vary less as the frequency of modulation is increased, with constant amplitude of current modulation. To find how the modulation of radiant energy changes, one must take account of the reduction in the fluctuation of N_e as the frequency is increased. Since N_e is higher at the maximum T_e and lower at the minimum, as the frequency is increased, the percentage modulation of the radiant intensity decreases less rapidly with increasing frequency than does the percentage variation of the radiated power per electron.

The detailed discussion of modulation given above illustrates the many factors to be considered when one uses Eqs. (3) to (8) to predict or interpret the dynamic behavior of a discharge. These equations obviously can also be used to analyze the wave shape of the voltage across an ac discharge, or the corresponding curve for the spectral intensity of some line from the discharge. Although the graphs refer to a particular discharge in mercury vapor with argon present, the equations and the method of analysis are applicable to any low pressure discharge.

ACKNOWLEDGMENT

I am indebted to Miss M. A. Easley, to Carl Kenty, and to C. G. Found, who furnished data on various dc and ac discharges in mercury vapor and discussed many phases of their behavior, thus contributing much of the background for this investigation.

[15] N. C. Beese, J. Opt. Soc. Am. **36**, 555 (1946). Frank, Huxford, and Wilson, J. Opt. Soc. Am. **37**, 718 (1947).



a b

FIG. 6. Probe currents at voltages near space potential for probe centered about axis (a) and for wall probe (b). Bias voltage (between auxiliary probe and cathode follower connected to measurement probe) changed in 1.5-volt steps. Zero line at bottom.