higher order reactions in which two or more neutrons are produced. The evidence for this is as follows. We have taken their estimate for the (γ,n) cross section and their measurements of the (γ,pn) cross section and calculated the yield of neutrons to be expected in Jarmie, Jones, and Terwilliger's experiments. The yield comes out to be too low by a factor five. Thus, there seems to be an appreciable cross section from reactions which Katz and Penfold have not taken into account. Of course, this yield might come from neutrons produced by high energy photons in a similar process to the one we have to postulate above to explain the Terwilliger transition curve. In this case the high energy process would have to account for 80 percent of the neutrons observed with 330-Mev bremsstrahlung. This seems rather high. If we assume arbitrarily, that as for heavy elements the high energy process accounts for about 40 percent of the neutron yield, then the (γ,n) cross section alone is too small by a factor three to secure agreement with Terwilliger's results. Thus, the estimate of the (γ,n) cross section is either much too low or else there are higher order reactions with integrated cross sections comparable to that for the (γ,n) process. At any rate, it seems that it would be premature to claim disagreement with the Levinger-Bethe formula until more measurements are made.

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Nuclear Phenomena Deducible from μ-Pair Theory with Pseudoscalar Coupling*

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The case of pseudoscalar coupling between nucleons and μ-field is considered within the framework of μ-pair theory. Besides the usual perturbation treatment, the strong coupling approximation for this case is developed. Both methods are applied to the problems of scattering of μ-mesons by nucleons and the nucleon-nucleon interaction. Interpolation between the extremes of weak and strong coupling suggests that this μ-pair theory may be promising with an intermediate coupling strength, a condition also required by the μ-pair theory of the π-meson.

1. INTRODUCTION

BESIDES the Yukawa theory of nuclear forces, some attention has in the past been devoted to the so-called pair theories according to which the interaction between nucleons may be pictured as being transmitted by a pair of particles instead of a single particle (π-meson). The quanta of a pair may be either bosons[1] or fermions.[2] The latter were first taken to be electron-neutrino or electron-positron pairs and later μ-meson pairs.[3]

One recommending feature of pair theories is the saturation characteristics of the nuclear forces.[2,4-6] A second is the possibility of interpreting the π-meson as a pair of μ-mesons bound together due to a small admixture of a virtual nucleon-pair state,[7] and the possible explanation of the V-particles[8] as excited nuclear states resulting from the binding of μ-mesons, say, by a bare nucleon. The fact that the spin of such excited states may be integral or half-integral, depending on whether an odd or even number of fermions has been bound, may be helpful in understanding the long lifetime of the neutral V-particle (10^{-10} sec), in particular its stability against γ-decay into the neutron ground state.

The most serious objection against pair theories may well be the role played by the momentum cutoff which must be introduced to achieve convergence and which dominantly affects the predictions of the theory in the high energy region. Since it turns out that the cutoff also determines the range of the nuclear forces and the density of nucleons in heavy nuclei,[5] its order of magnitude is roughly that of the meson mass (times c). Therefore, if one takes the cut-off prescription seriously to the extent of applying it, for example, to the scattering of μ-mesons by nucleons, then one would expect at kinetic energies much greater than 100 Mev a very small cross section, while a substantially larger value for energies of the order 100 Mev or somewhat less. According to the measurements of Amaldi and Fidecaro[9] the cross section is at most 4.5×10^{-29} cm² per nucleon for μ-energies between 200 and 320 Mev, and above 320 Mev it is at most 2.3×10^{-30} cm² per nucleon. The cutoff therefore offers an explanation for the presently

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known data, though not a very satisfactory one. Similar is the situation with regard to the μ-pair creation in high energy nucleon-nucleon and photon-nucleon collisions.[7,10] As a result of the cutoff, μ-pairs are much more likely to come out in the bound state, i.e., as a π-meson.

The interaction Hamiltonian of fermion pair theories has the general form

$$\text{const} \int d^3x \Phi^* O \Phi \psi^* O \psi \tag{1.1}$$

where Φ and ψ are the field operators of the nucleons and mesons, respectively. If transitions between neutrons and protons are to be considered, the ψ-field must be of a mixed charged-neutral type. This also appears necessary for any μ-pair theory if it is to give an explanation for the $\pi^\pm$ meson within its framework. For the sake of simplicity, however, we shall always associate ψ with charged (+) μ-mesons, so that any pair produced by (1.1) will have total charge zero. Any more complete treatment with respect to the inclusion of neutral mesons is straightforward and is easily carried out in perturbation theory.[11] The symbol O in (1.1) stands for one of the five types of matrix operators well known from β-decay theory: scalar, vector, tensor, pseudovector, and pseudoscalar coupling. All except the last have been extensively investigated with respect to nuclear forces and μ-scattering. In the "static" case (nucleons infinitely heavy and at rest), there exist exact solutions for scalar and vector coupling,[6,12] whereas in the tensor and pseudovector cases only weak or strong coupling approximations are available.[2,3,13] Very little, however, is known about the pseudoscalar coupling ($O=\beta\gamma_5=i\beta\alpha_1\alpha_2\alpha_3$) because (1.1) vanishes in the strictly static limit, and even in the lowest nonstatic approximation the problems are considerably more involved than in the static tensor and pseudovector cases.

This paper is intended to fill the gap in our knowledge at least partially. Weak and strong coupling approximations will be applied to the pseudoscalar problem, with emphasis on the more difficult strong coupling case. The particular problems to be studied are the scattering of μ-mesons by nucleons (Sec. III), and the two nucleon forces (Sec. IV). The more involved problems (saturation properties, isobar states, magnetic moments) have been left aside. Two other reasons may be mentioned that give relevance to a treatment of the pseudoscalar case. The first is based on our presently held belief of the pseudoscalar nature of the π-meson. The explanation of this particular π-meson type as a bound μ-pair is only possible with pseudoscalar or pseudovector coupling types O, or a mixture of the two,[7] and for pure pseudoscalar coupling, there can be only one bound state, namely, the pseudoscalar one. Secondly, if the coupling constant in (1.1) is determined from the π-meson mass (binding energy), the pseudoscalar coupling leads to a much smaller μ-scattering cross section than all other coupling types. This follows from the fact that the matrix elements of $\Phi^*\beta\gamma_5\Phi$ for small nucleon velocities are proportional to the recoil velocity, which is small as the meson-nucleon mass ratio μ/M. Therefore, it turns out that in the μ-scattering problem, a weak coupling approximation is permissible (even though the π-binding energy requires an intermediate coupling strength), and the cross section becomes comparatively small, due to the factor $(\mu/M)^2$. The pseudoscalar coupling is unique in this respect. Furthermore, for very small meson velocities the cross section will practically vanish. This version of the theory is therefore more likely to be compatible with the fact that no strong nuclear μ-scattering has been observed so far.

2. APPROXIMATIONS

Inserting $O=\beta\gamma_5$ in (1.1), in the limit of large nucleon mass, $\Phi^*O\Phi$ may be replaced by

$$\textstyle\sum_n \delta(\mathbf{x}-\mathbf{x}_n)(\boldsymbol{\sigma}_n\cdot\mathbf{p})/2M,$$

where $\mathbf{p}=-i\nabla$ and n enumerates the various nucleons (Mass$=M$) present. The situation is well known from pseudoscalar Yukawa theory. Indeed, if the nonrelativistic Pauli approximation for the nucleon eigenfunctions is used, the matrix elements of $\int d^3x\Phi^*O\Phi F(\mathbf{x})$ are correctly calculated with this substitution. $F(\mathbf{x})$ is in Yukawa theory the pseudoscalar field operator, and in pair theory it is the bilinear product $\psi^*\beta\gamma_5\psi$. Therefore

$$H_i = \textstyle\sum_n f_n[(\boldsymbol{\sigma}_n\cdot\mathbf{p}_n)/2M]\psi^*(\mathbf{x}_n)\beta\gamma_5\psi(\mathbf{x}_n).$$

(The coupling parameter f_n may be chosen different for neutrons and protons.)

The momentum cutoff which will now be introduced is equivalent to replacing $\psi(\mathbf{x}_n)$ by a spatially averaged operator

$$\bar{\psi}(\mathbf{x}_n) = \int d^3x u(\mathbf{x}_n-\mathbf{x})\psi(\mathbf{x}). \tag{2.1}$$

The source function u is given the property of being different from zero only for $|\mathbf{x}-\mathbf{x}_n| \gtrsim A^{-1}$ and being a real, spherically symmetric function. With this modification

$$H_i = (1/2M)\textstyle\sum_n f_n(\boldsymbol{\sigma}_n\cdot\mathbf{p}_n)\bar{\psi}^*(\mathbf{x}_n)\beta\gamma_5\bar{\psi}(\mathbf{x}_n). \tag{2.2}$$

This is the cut-off procedure customary in pair theories and we shall adopt it here.

The total Hamiltonian is then

$$H = H_\mu + H_i, \tag{2.3}$$

where

$$H_\mu = \int d^3x \psi^*(\mathbf{x})[(\boldsymbol{\alpha}\cdot\mathbf{p})+\mu\beta]\psi(\mathbf{x}). \tag{2.3}$$

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The spinor components of the field operators obey the usual anti-commutation relations

$$\psi_\rho^*(\mathbf{x})\psi_\sigma(\mathbf{x}')+\psi_\sigma(\mathbf{x}')\psi_\rho^*(\mathbf{x})=\delta_{\rho\sigma}\delta(\mathbf{x}-\mathbf{x}'),\ \text{etc.} \tag{2.4}$$

Next we shall develop the strong coupling approximation. In doing so, we may closely follow a paper of Blatt[13] in which the tensor coupling case is treated along similar lines.

The first and common aim of all such theories is the isolation of the modes on which the dominant term H_i depends most. This is achieved by splitting the field operator $\psi(\mathbf{x})$ into parts having the $\mathbf{x}$ dependence of the various source functions $u(\mathbf{x}-\mathbf{x}_n)$ and of their derivatives, and another part $\psi'(\mathbf{x})$ orthogonal to every source function and its derivatives.

$$\psi(\mathbf{x})=\sum_n N^{-\frac{1}{2}}Q_n u(\mathbf{x}-\mathbf{x}_n)+\sum_n \mathfrak{N}^{-\frac{1}{2}}(\mathfrak{Q}_n\cdot\nabla)u(\mathbf{x}-\mathbf{x}_n)+\psi'(\mathbf{x}) \tag{2.5}$$

with

$$\int d^3x\psi'(\mathbf{x})u(\mathbf{x}-\mathbf{x}_n)=0 \tag{2.6}$$

$$\int d^3x\psi'(\mathbf{x})\nabla u(\mathbf{x}-\mathbf{x}_n)=0. \tag{2.7}$$

Note that due to (2.6, 7) the ψ'-part in $\bar{\psi}(\mathbf{x}_n)$ and in $p_n\bar{\psi}(\mathbf{x}_n)$ vanishes, and therefore H_i (2.2) causes no coupling of the ψ'-field with the bare nucleons.

This kind of separation is characteristic of all strong coupling theories. The term n in (2.5) will describe the mesons bound to the nth nucleon, whereas $\psi'(\mathbf{x})$ describes the free mesons whose interaction with "real" or "physical" nucleons turns out to be weak. The normalization factors N and $\mathfrak{N}$ in (2.5) are conveniently defined by

$$N=\int d^3x|u(\mathbf{x})|^2,\quad \mathfrak{N}=\tfrac{1}{3}\int d^3x|\nabla u(\mathbf{x})|^2. \tag{2.8, 9}$$

Assuming for the present that the distance between nucleons is larger than A^{-1}, no overlap between the various sources will occur.

$$\int d^3xu(\mathbf{x}-\mathbf{x}_n)u(\mathbf{x}-\mathbf{x}_m)=0\quad (m\neq n). \tag{2.10}$$

Then it follows from (2.5, 6, 7, 8, 9) that

$$\begin{aligned} Q_{m\rho}&=N^{-\frac{1}{2}}\int d^3x\psi_\rho(\mathbf{x})u(\mathbf{x}-\mathbf{x}_m)\\ \mathfrak{Q}_{m\rho}{}^{(j)}&=\mathfrak{N}^{-\frac{1}{2}}\int d^3x\psi_\rho(\mathbf{x})(\partial/\partial x_j)u(\mathbf{x}-\mathbf{x}_m).\end{aligned} \tag{2.11}$$

The spinor operators $Q_{m\rho}$ and $\mathfrak{Q}_{m\rho}{}^{(j)}$ are the variables of the "physical" nucleons; the nonvanishing anti-commutators containing these are

$$\begin{aligned}[Q_{m\rho}^*, Q_{m'\rho'}]_+&=\delta_{\rho\rho'}\delta_{mm'},\\ [\mathfrak{Q}_{m\rho}{}^{(j)}, \mathfrak{Q}_{m'\rho'}{}^{(j')}]_+&=\delta_{\rho\rho'}\delta_{mm'}\delta_{jj'},\end{aligned} \tag{2.12}$$

as is easily seen from (2.4) and (2.11). Similarly,

$$\begin{aligned}[Q_{m\rho}, \psi_\sigma^*(\mathbf{x})]_+&=N^{-\frac{1}{2}}\delta_{\rho\sigma}u(\mathbf{x}-\mathbf{x}_m),\\ [\mathfrak{Q}_{m\rho}{}^{(j)}, \psi_\sigma^*(\mathbf{x})]_+&=\mathfrak{N}^{-\frac{1}{2}}\delta_{\rho\sigma}(\partial/\partial x_j)u(\mathbf{x}-\mathbf{x}_m).\end{aligned} \tag{2.12'}$$

From (2.4, 12, and 12′) it then follows that

$$\begin{aligned}[\psi_\rho'^*(\mathbf{x}), \psi_\sigma'(\mathbf{x}')]_+ &=\delta_{\rho\sigma}[\delta(\mathbf{x}-\mathbf{x}')-\textstyle\sum_n N^{-1}u(\mathbf{x}-\mathbf{x}_n)u(\mathbf{x}'-\mathbf{x}_n)\\ &\quad -\mathfrak{N}^{-1}\nabla u(\mathbf{x}-\mathbf{x}_n)\nabla' u(\mathbf{x}'-\mathbf{x}_n)].\end{aligned} \tag{2.13}$$

The Hamiltonian (2.2, 3) may now be split into three parts,

$$\begin{aligned} H&=H^0+H'+\Omega,\\ H^0&=\textstyle\sum_n H^0(n),\end{aligned} \tag{2.14}$$

$$\begin{aligned} H^0(n)=\ &i(f/2M)(N\mathfrak{N})^{\frac{1}{2}}\boldsymbol{\sigma}_n\cdot\{\mathfrak{Q}_n^*\beta\gamma_5Q_n+Q_n^*\beta\gamma_5\mathfrak{Q}_n\}\\ &+i(\mathfrak{N}/N)^{\frac{1}{2}}\{Q_n^*(\boldsymbol{\alpha}\cdot\mathfrak{Q}_n)-(\mathfrak{Q}_n^*\cdot\boldsymbol{\alpha})Q_n\}\\ &+\mu\{Q^*\beta Q+\mathfrak{Q}^*\beta\mathfrak{Q}\}.\end{aligned} \tag{2.15⁰}$$

$$H'=\int d^3x\psi'^*(\mathbf{x})[(\boldsymbol{\alpha}\cdot\mathbf{p})+\mu\beta]\psi'(\mathbf{x}) \tag{2.15'}$$

$$\Omega=\textstyle\sum_n \Omega(n)$$

$$\Omega(n)=\mathfrak{N}^{-\frac{1}{2}}\int d^3x[\psi'^*(\mathbf{x})(\boldsymbol{\alpha}\cdot\mathbf{p})(\mathfrak{Q}\cdot\boldsymbol{\nabla})u(\mathbf{x}-\mathbf{x}_n)+\text{c.c.}]. \tag{2.15$^\Omega$}$$

Similarly, the expressions for the charge and the angular momentum of the system may be split up (with no cross terms such as Ω appearing), but they will not be needed here.

The term Ω, in H, which is bilinear in the bound and free meson variables, describes the emission or absorption of free mesons by the physical nucleons. Ω must be a weak perturbation if the separation (2.14) is to be useful. This leads to the "strong coupling condition" which, following the argument of Pauli and Hu, turns out to be

$$(f/2M)A^3\gg 1, \tag{2.16}$$

provided that $A>\mu$. In any strong coupling calculation, condition (2.16) will be assumed to hold, that is to say, Ω will be disregarded. In this approximation there is a complete separation of the variables describing the various physical nucleons $Q_{m\rho}$, $\mathfrak{Q}_{m\rho}{}^{(j)}$ and the free meson variables $\psi'(x)$. The Hamiltonians concerning the various nucleons (H^0) and the free meson field (H') may be studied separately.

In the following applications, namely, the μ-scattering and two-nucleon forces, we will be concerned with the H' problem only. H^0 can be considered an additive constant. A general scheme for solving the H' problem rigorously is the following: Introduce a complete set of orthonormal functions (spinors) ϕ_m subject to the orthogonality restrictions

$$\int d^3x\phi_{m,\rho}^*(\mathbf{x})u(\mathbf{x}-\mathbf{x}_n)=0, \tag{2.17}$$

$$\int d^3x\phi_{m,\rho}^*(\mathbf{x})(\partial/\partial x_i)u(\mathbf{x}-\mathbf{x}_n)=0, \tag{2.18}$$

where $\rho=1, 2, 3, 4$; $i=1, 2, 3$; $m=1, 2, \cdots$; and $n=1, 2, \cdots$. In addition, of course,

$$\int d^3x\phi_m^*\phi_m-1=0. \tag{2.19}$$

These functions, together with

$$N^{-\frac{1}{2}}u(\mathbf{x}-\mathbf{x}_n)C(\sigma) \quad \text{and} \quad \mathfrak{N}^{-\frac{1}{2}}(\partial/\partial x_i)u(\mathbf{x}-\mathbf{x}_n)C(\sigma)$$

(where $C(\sigma)\{\sigma=1, \cdot\cdot 4\}$ designate four spinors obeying $C^*(\sigma)C(\sigma')=\delta_{\sigma\sigma'}$, e.g. $C_\rho(\sigma)=\delta_{\rho\sigma}$) form a complete set suitable for the expansion of the complete field ψ [see (2.5)]. For the part ψ', subject to the orthogonality conditions (2.6, 7) and to the commutation rules (2.13), the most general expansion is

$$\psi_\rho'(\mathbf{x})=\textstyle\sum_m a_m\phi_{m,\rho}(\mathbf{x}), \tag{2.20}$$

where the a's obey the well-known anticommutation relations

$$[a_m^*, a_{m'}]_+=\delta_{mm'}, \quad [a_m, a_{m'}]_+=[a_m^*, a_{m'}^*]_+=0.$$

The ϕ_m's are conveniently constructed as eigenfunctions of the variational problem

$$\delta\int d^3x\phi_m^*(\mathbf{x})[(\boldsymbol{\alpha}\cdot\mathbf{p})+\mu\beta]\phi_m(\mathbf{x})=0, \tag{2.21}$$

with the constraints (2.17, 18, 19). Introducing for these constraints, respectively, Lagrangian multipliers $\lambda_{n\rho}$, $\Lambda_{n\rho}^{(i)}$ and E, one obtains the set of linear equations

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\mu\beta)\phi=E\phi+\textstyle\sum_n(\lambda_n+\mathbf{\Lambda}_n\cdot\mathbf{p})u(\mathbf{x}-\mathbf{x}_n). \tag{2.22}$$

Then, with (2.20) inserted into (2.15′), H' assumes the diagonal form

$$H'=\textstyle\sum_m a_m^*a_mE_m. \tag{2.23}$$

Note that the coupling parameter f does not appear in the H' problem. Finally it may be noted that the introduction of an additional neutral meson field (doubling of all ϕ-components, with O involving isotopic spin operators) causes no computational changes in the H' problem. Of course, the number of eigenvalues E, per interval ΔE, is doubled.

3. ONE-NUCLEON PROBLEM

In the one-nucleon case (2.22) reduces to

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\mu\beta-E)\phi=(\lambda+\mathbf{\Lambda}\cdot\mathbf{p})u(\mathbf{x}). \tag{3.1}$$

To study the μ-scattering, we insert for $\phi(\mathbf{x})$ a plane, incoming wave plus an outgoing spherical wave

$$\phi(\mathbf{x})=\phi_0e^{i(\mathbf{p}_0\cdot\mathbf{x})}+\int_C d^3ka(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x})}. \tag{3.2}$$

ϕ_0 obeys the "source free" equation

$$(\boldsymbol{\alpha}\cdot\mathbf{p}_0+\mu\beta-E)\phi_0=0, \quad E=+(p_0^2+\mu^2)^{\frac{1}{2}} \tag{3.3}$$

and C denotes the countour to be taken in the complex $|\mathbf{k}|$ plane which gives rise only to outgoing waves.

Multiplying (3.1) with $(\boldsymbol{\alpha}\cdot\mathbf{p}+\mu\beta+E)$, one finds

$$(p^2-p_0^2)\phi(\mathbf{x})=(\boldsymbol{\alpha}\cdot\mathbf{p}+\mu\beta+E)(\lambda+\mathbf{\Lambda}\cdot\mathbf{p})u(\mathbf{x}). \tag{3.4}$$

Let

$$u(\mathbf{x})=\int d^3kg(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x})}. \tag{3.5}$$

Inserting (3.2, 5) into (3.4) yields at once

$$a(\mathbf{k})=g(\mathbf{k})(\boldsymbol{\alpha}\cdot\mathbf{k}+\mu\beta+E)(\lambda+\mathbf{\Lambda}\cdot\mathbf{k})/(k^2-p_0^2); \tag{3.6}$$

λ and $\mathbf{\Lambda}$ are still to be determined from (2.19, 20)

$$\int d^3xu(\mathbf{x})\phi(\mathbf{x})=\int d^3xu(\mathbf{x})\mathbf{p}\phi(\mathbf{x})=0.$$

Choosing the "3" axis in the p_0 direction one finds

$$\lambda=\frac{g(p_0)}{p_0^2I(p_0)-3J(p_0)}[p_0\alpha_3-(E-\mu\beta)]\phi_0,$$

$$\Lambda_i=\frac{p_0^{-1}g(p_0)}{p_0^2I(p_0)-3J(p_0)}[\alpha_i\alpha_3(E-\mu\beta)-p_0\alpha_i]\phi_0; \quad (i=1, 2), \tag{3.7}$$

$$\Lambda_3=-p_0^{-1}g(p_0)\left[(E-\mu\beta)\left(\frac{1}{J(p_0)}+\frac{1}{p_0^2I(p_0)-3J(p_0)}\right)-\frac{p_0\alpha_3}{p_0^2I(p_0)-3J(p_0)}\right]\phi_0,$$

where

$$I(p_0)=\int_C dk\frac{k^2g^2(k)}{k^2-p_0^2}, \quad J(p_0)=\tfrac{1}{3}\int_C dk\frac{k^4g^2(k)}{k^2-p_0^2}. \tag{3.8}$$

The contour C is of course the same as in (3.2). From (3.6, 7)

$$a(\mathbf{k})=\frac{b(\mathbf{k})}{k^2-p_0^2}\phi_0, \tag{3.9}$$

where

$$b(\mathbf{k})=\frac{g(k)g(p_0)}{p_0^2I(p_0)-3J(p_0)}(\boldsymbol{\alpha}\cdot\mathbf{k}+\mu\beta+E)$$
$$\times\left[(\boldsymbol{\alpha}\cdot\mathbf{p}_0)-(E-\mu\beta)-\frac{(\boldsymbol{\alpha}\cdot\mathbf{k})(\boldsymbol{\alpha}\cdot\mathbf{p}_0)}{p_0^2}(E-\mu\beta)+(\boldsymbol{\alpha}\cdot\mathbf{k})-\frac{p_0^2I(p_0)-3J(p_0)}{J(p_0)}\left(\frac{\mathbf{k}\cdot\mathbf{p}_0}{p_0^2}\right)(E-\mu\beta)\right].$$

Finally one may in the usual way evaluate the k integral in (3.2) for the wave zone ($|\mathbf{k}||\mathbf{x}|\gg 1$)

$$\phi(\mathbf{x})=[e^{i\mathbf{p}_0\cdot\mathbf{x}}+2\pi^2 b(\boldsymbol{\kappa})\exp(i|\boldsymbol{\kappa}||\mathbf{x}|)/|\mathbf{x}|]\phi_0. \quad (3.10)$$

Here, $\boldsymbol{\kappa}$ stands for the momentum of the meson scattered in the x direction $\boldsymbol{\kappa}=|\mathbf{p}_0|\mathbf{x}/|\mathbf{x}|$. Note that if ϕ_0 refers to a positive energy state ($E>0$), then also the scattered wave contains only positive energy states, as is indicated by the factor $(E+\boldsymbol{\alpha}\cdot\boldsymbol{\kappa}+\mu\beta)$ in (3.9). Therefore, the scattering cross section averaged over the initial polarizations, is simply

$$\begin{aligned} d\sigma/d\Omega&=2\pi^4\,\mathrm{Sp}[b^*(\boldsymbol{\kappa})b(\boldsymbol{\kappa})(E+\boldsymbol{\alpha}\cdot\mathbf{p}_0+\mu\beta)/2E] \\ &=8\pi^4\left|\frac{1}{J(p_0)}\right|^2\frac{(\boldsymbol{\kappa}\cdot\mathbf{p}_0)^2}{p_0^2}(p_0^2+\boldsymbol{\kappa}\cdot\mathbf{p}_0)g^4(p_0). \end{aligned} \quad (3.11)$$

We have evaluated the cross section for the simple cut-off law

$$g(k)=(1+k^2/A^2)^{-1} \quad (3.12)$$

which corresponds to the source function

$$u(x)=2\pi^2A^3(e^{-Ax}/Ax) \quad (3.13)$$

as defined by (3.5). In this special case

$$\left.\begin{aligned} I(p_0)&=i\pi^2[2p_0-iA(1-p_0^2/A^2)]g^2(p_0) \\ 3J(p_0)&=i\pi^2[2p_0^3-iA^3(1+3p^2/A^2)]g^2(p_0) \\ \frac{d\sigma}{d\Omega}&=72p_0^4\frac{\cos^2\vartheta(1+\cos\vartheta)}{[4p_0^6+A^6(1+3p_0^2/A^2)^2]} \end{aligned}\right\} \quad (3.14)$$

where $\vartheta=\mathbf{p}_0\cdot\mathbf{x}/|\mathbf{p}_0||\mathbf{x}|$. Integrating over all directions gives

$$\sigma=96\pi p_0^4\frac{1}{[4p_0^6+A^6(1+3p_0^2/A^2)^2]}. \quad (3.15)$$

It is evident that this strong coupling value of the scattering cross section is much larger than the observed values. As was mentioned earlier, the cut-off momentum A cannot be taken much larger than μ on account of the actual range of the nuclear forces. It is true that σ, with p_0 increasing above A, falls off as p_0^{-2}, but this decrease is not fast enough and sets in too late to make the numerical values compatible with the measurements of Amaldi and Fidecaro at 200 Mev and above. A sharper cutoff would improve the situation for high energies, but for $p\sim A\sim\mu$ Eq. (3.15) would still give an unbelievably large cross section. A computational check of this point was made for $g(k)=(1+k^2/A^2)^{-2}$ which bears out the correctness of this conclusion.

Everything mentioned above about (3.14, 15) applies also to a more general interaction involving the isotopic spin operators. The scattering of either charged or neutral mesons without charge exchange would again be given by (3.14, 15) in the strong coupling approximation.

Equations (3.14, 15) may also be compared with the weak coupling value, obtained by ordinary perturbation theory (first-order approximation)

$$\frac{d\sigma}{d\Omega}=\frac{f}{16\pi^2}[p'g(p')]^4\frac{(1-\cos\vartheta')^2}{[(p'^2+\mu^2)^{\frac{1}{2}}+(p'^2+M^2)^{\frac{1}{2}}]^2} \quad (3.16)$$

$$\sigma=\frac{f}{3\pi}[p'g(p')]^4\frac{1}{[(p'^2+\mu^2)^{\frac{1}{2}}+(p'^2+M^2)^{\frac{1}{2}}]^2}. \quad (3.17)$$

All primed quantities are meant to refer to the center-of-mass coordinate system. (ϑ' = scattering angle.) Equations (3.16, 17) were already obtained by Wentzel[7] (his value in Eq. (22) of reference 7 should be multiplied by 4; $\eta=f/2\pi)^3$). As mentioned in the Introduction, this approximation is even valid for intermediate coupling strengths, and the numerical values are not incompatible with the observed cross sections.

Before concluding the discussion of the one-nucleon problem, a few remarks may be added regarding the H_0 problem, i.e., the "binding" of mesons by a single nucleon leading to excited nucleon states. The main term in H^0 Eq. (2.15) is the one arising from the original interaction term ($\sim f$); neglecting the other terms in a first approximation, one is confronted with the eigenvalue problem

$$\begin{aligned} &H_0{}^0F^0=E^0F^0 \\ &\quad H_0{}^0=i(N\mathfrak{N})^{\frac{1}{2}}(f/2M)\boldsymbol{\sigma}\cdot\{\mathfrak{Q}^*\beta\gamma_5Q+Q^*\beta\gamma_5\mathfrak{Q}\}; \end{aligned}$$

$i\beta\gamma_5$ may of course be chosen diagonal. For the sixteen operators Q_ρ and $\mathfrak{Q}_\rho{}^{(i)}$ which obey the usual Jordan-Wigner anticommutation relations (2.12) it seems simplest to take $Q_\rho{}^*Q_\rho$ and $\mathfrak{Q}_\rho{}^{(i)*}\mathfrak{Q}_\rho{}^{(i)}$ diagonal. So the problem which must be solved is the diagonalization of a matrix of 2^{16+1} rank. The various constants of motion reduce this $H_0{}^0$ to submatrices which are more manageable. The great number of these, however, makes the computation an almost impossible task.

4. THE TWO-NUCLEON INTERACTION

We consider two nucleons at the positions $\mathbf{x}_1$ and $\mathbf{x}_2$, with nonoverlapping sources, i.e., $|\mathbf{x}_1-\mathbf{x}_2|\gg A^{-1}$. Then the H^0 part of the Hamiltonian (2.14, 15^0) can be disregarded in the interaction problem, since it has no dependence on $|\mathbf{x}_1-\mathbf{x}_2|$. H', however, gives rise to an interaction, because the vacuum ψ'-field is changed by the presence of nucleons and this modification depends on the distances of the nucleons. Referring back to (2.17–23), let $E_m(r)$ be the energy of a stationary state m, as a function of $r=|\mathbf{x}_1-\mathbf{x}_2|$; then the interaction energy $J(r)$ is given by

$$J(r)=\sum\nolimits_{m^{(-)}}[E_m(r)-E_m(\infty)]. \quad (4.1)$$

The sum is extended over all occupied negative energy states.[4]

For calculating the eigenvalues, E_m, it is initially necessary to introduce a periodicity condition giving rise to Fourier series. Therefore, let

$$u(\mathbf{x})=8\pi^3V^{-1}\sum g(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x})} \quad \text{with} \quad g(\mathbf{k})=g^*(\mathbf{k})=g(-\mathbf{k}) \tag{4.2}$$

$$\phi(\mathbf{x})=8\pi^3V^{-1}\sum \varphi(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x})}. \tag{4.3}$$

V is the usual periodicity volume. The variational problem (2.21) then leads in the Fourier series representation of (2.22) to

$$(\boldsymbol{\alpha}\cdot\mathbf{k}+\mu\beta-E)\varphi(\mathbf{k})=g(k)\sum_{n=1}^{n=2}(\lambda_n+\mathbf{\Lambda}_n\cdot\mathbf{k})e^{-i(\mathbf{k}\cdot\mathbf{x}_n)}$$

$$\varphi(\mathbf{k})=g(k)\frac{(\boldsymbol{\alpha}\cdot\mathbf{k}+\mu\beta+E)}{k^2+\mu^2-E^2}\sum_{n=1}^{n=2}(\lambda_n+\mathbf{\Lambda}_n\cdot\mathbf{k})e^{-i(\mathbf{k}\cdot\mathbf{x}_n)}. \tag{4.4}$$

Note that no solution of the homogeneous equation, like the plane wave in (3.2), need be added since we are interested in determining stationary states m having a vanishing current density at infinity. Now, as before in the scattering problem, we resubstitute (4.4) into the Fourier transform of the constraints (2.17) and (2.18). These are

$$\int d^3x\phi(\mathbf{x})u(\mathbf{x}-\mathbf{x}_n)\sim\sum g(\mathbf{k})\varphi(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x}_n)}=0$$

$$\int d^3x\partial\phi/\partial x_ju(\mathbf{x}-\mathbf{x}_n)\sim\sum g(\mathbf{k})\varphi(\mathbf{k})k_je^{i(\mathbf{k}\cdot\mathbf{x}_n)}=0.$$

One thus gets from them, respectively,

$$\sum_n\sum_k|g(k)|^2\frac{(\boldsymbol{\alpha}\cdot\mathbf{k}+\mu\beta+E)}{k^2+\mu^2-E^2}\times(\lambda_n+\mathbf{\Lambda}_n\cdot\mathbf{k})e^{-i\mathbf{k}\cdot(\mathbf{x}_n-\mathbf{x}_{n'})}=0 \tag{4.5}$$

$$\sum_n\sum_k|g(k)|^2k_j\frac{(\boldsymbol{\alpha}\cdot\mathbf{k}+\mu\beta+E)}{k^2+\mu^2-E^2}\times(\lambda_n+\mathbf{\Lambda}_n\cdot\mathbf{k})e^{-i\mathbf{k}\cdot(\mathbf{x}_n-\mathbf{x}_{n'})}=0. \tag{4.6}$$

In the two-nucleon case (4.5) and (4.6) form a system of 32 linear equations for the 32 components of $\lambda_{n\rho}$ and $\Lambda_{n\rho}^{(i)}$ whose nontrivial solution is determined by the vanishing of the determinant of their coefficients. The negative values E for which this determinant, denoted by $\Delta(E,r)$, vanishes are the eigenvalues $E_m(r)$ in (4.1). The calculation of $\Delta(E,r)$ is lengthy but straightforward and leads to the following simple result:

$$\Delta(E,r)=\Delta(E,\infty)[\chi(E,r)]^4. \tag{4.7}$$

The function χ will be described below.

When χ is known, one is able to find $J(r)$ by making use of the method of Wentzel.[5] Pauli and Hu[1] used this procedure to obtain for $J(r)$ the expression

$$J(r)=-\frac{1}{2\pi i}\int_C dE\ln\frac{\Delta(E,r)}{\Delta(E,\infty)}=-\frac{2}{\pi i}\int_C dE\ln\chi(E,r). \tag{4.8}$$

The counterclockwise path of this integral in the complex E plane is a loop chosen so as to enclose all the zeros of $\Delta(E,r)$ and $\Delta(E,\infty)$ on the negative (real) axis. χ may now be evaluated in the limit $V\to\infty$. This process causes χ to become discontinuous along the real E axis due to the continuous spread of the zeros. Explicit computation of χ shows that the limiting values, as the real E axis is approached from either side, are complex conjugates to each other. So if ϵ and κ are defined by $E=-[(\kappa^2+\mu^2)^{\frac{1}{2}}+i\epsilon]$, and introducing the notation

$$\chi^{\pm}(\kappa^2,r)=\lim_{\epsilon\to\pm0}\chi(E,r),$$

it turns out that $(\chi^+)^*=\chi^-$. Therefore, we may define a function $\vartheta(\kappa^2,r)$ by

$$\chi^+=\chi^-\exp[2i\vartheta(\kappa^2,r)]$$

and (4.8) may then be written as

$$J(r)=-\frac{4}{\pi}\int_0^\infty d\kappa\frac{\kappa}{(\kappa^2+\mu^2)^{\frac{1}{2}}}\vartheta(\kappa^2,r). \tag{4.9}$$

The phase $\vartheta(\kappa^2,r)$ is uniquely determined by the requirement

$$\lim_{r\to\infty}\vartheta(\kappa^2,r)=0 \quad (J(\infty)=0).$$

In (4.9), strictly speaking, a contour integral over a small circle in the E plane around the point $E=-\mu$ should be added. But it can be shown, with the formulas given later, that no contribution to $J(r)$ arises as the radius of the circle tends to zero. The k-space integrals‡ contained in $\chi^{\pm}$ and ϑ are expressible in terms of the following two types of integrals after angular integration:

$$I_\alpha^{\pm}(\kappa^2,r)=\lim_{\eta\to0}\int_{-\infty}^{\infty}\frac{k^\alpha e^{ikr}g^2(k)}{k^2-(\kappa^2\pm i\eta)}dk \quad (\alpha=1,2,3,4)$$

$$(2n+1)J_n^{\pm}(\kappa^2)=\lim_{\eta\to0}\int_0^\infty\frac{k^{2(1+n)}g^2(k)}{k^2-(\kappa^2\pm i\eta)}dk \quad (n=0,1). \tag{4.10}$$

[Note that for $\kappa^2<0$, or $E>-\mu$ one obtains $I_\alpha^+=I_\alpha^-$, $J_n^+=J_n^-$; this is the reason why only the region $\kappa^2>0$, or $E<-\mu$ contributes to the integrals (4.8) or (4.9).]

For further evaluation we choose again the simple source function (3.12), which allows one to carry out the k integration in (4.10) as contour integrations. The poles $k=iA$ in $I^{\pm}(\kappa^2,r)$ give rise to terms proportional to e^{-Ar} which will be neglected under the assumption $r\gg A^{-1}$ [see (2.10)]. Then the integrals of (4.10)

‡ $\left(\lim_{V\to\infty}V^{-1}\sum\to\frac{1}{8\pi^3}\int\right)$

become

$$(I_\alpha^+ + I_\alpha^-) = i\pi\kappa^{\alpha-1}g^2(\kappa)[e^{i\kappa r} + (-1)^{\alpha-1}e^{-i\kappa r}]$$
$$(I_\alpha^+ - I_\alpha^-) = i\pi\kappa^{\alpha-1}g^2(\kappa)[e^{i\kappa r} - (-1)^{\alpha-1}e^{-i\kappa r}]$$
$$(2n+1)(J_n^+ + J_n^-) = \pi A^{2n+1}(-1)^{n+1}g^2(\kappa) \times[(n-\tfrac{1}{2})+(n+\tfrac{1}{2})\kappa^2/A^2] \tag{4.11}$$
$$(2n+1)(J_n^+ - J_n^-) = i\pi\kappa^{2n+1}g^2(\kappa).$$

We can now summarize the results for the function χ^+ in (4.8) with the choice (3.12) for the source function.

$$\chi^+ = (\chi^-)^* = \chi_1^+\chi_2^+ = (\chi_1^-\chi_2^-)^*$$
$$\chi_1^+ = 1 + \frac{36}{y^2}\frac{(-4iy^3+14y^2+18iy-9)}{(2iy^3+3z^2y+z^3)^2}e^{2iy}$$
$$\chi_2^+ = 1 + \frac{36}{y^2}\frac{(2iy^5-17y^4-68iy^3+142y^2+162iy-81)}{(2iy^3+3z^2y+z^3)^2}e^{2iy}$$

where $y=\kappa r$ and $z=Ar$. The complete expression for χ_2^+ also contains a term with an e^{4iy} factor, which gives, however, a completely negligible contribution to $J(r)$.

In (4.9), χ_1 and χ_2 contribute in an additive manner ($\vartheta=\vartheta_1+\vartheta_2$). The qualitative behavior of χ_1^+ and χ_2^+ is similar, and it will be sufficient to discuss χ_1^+. Three separate intervals on the κ or y axis may be distinguished.

(a) The neighborhood of the point $y=y_1$ defined such that the real part of χ_1^+ vanishes: $\Re(\chi_1^+(y_1))=0$; $y_1\cong 18/z^3\ll 1$. In this region the imaginary part, $\Im(\chi_1^+(y))$, is positive so that $\vartheta_1(y_1)=\pi/z$. The region (a) occupies a y domain, Δy_1, which includes all points where $|\Re(\chi_1^+(y))|\lesssim|\Im(\chi_1^+(y))|$.

(b) If y, starting from y_1, increases toward infinity, χ_1^+ very rapidly approaches 1, i.e., ϑ_1 approaches 0.

(c) To the other side of y_1, y approaching 0, ϑ_1 tends rapidly toward the value π.

The region (a) is very narrow ($\Delta y_1 \sim y_1^4 \ll y_1 \ll 1$) and therefore a good approximation for the integral (4.9) is obtained by setting $\vartheta_1\cong(\chi_1^+-\chi_1^-)/2i$ in the interval b_1: $y_1\leqslant y<\infty$, and $\vartheta_1\cong\pi$ for c_1: $0\leqslant y\leqslant y_1$.

Similarly one finds the contribution of χ_2^+ to be $\vartheta_2\cong(\chi_2^+-\chi_2^-)/2i$ in the interval b_2: $y_2\leqslant y<\infty$, and $\vartheta_2\cong\pi$ for c_2: $0\leqslant y\leqslant y_2$ (there $y_2\cong 54/z^3$). Then, according to (4.9)

$$J(r)=J_b(r)+J_c(r)$$
$$J_c(r)=J_{c1}(r)+J_{c2}(r)=-4/r[-2\mu r+(\mu^2r^2+y_1^2)^{\frac{1}{2}} +(\mu^2r^2+y_2^2)^{\frac{1}{2}}] \tag{4.12}$$
$$J_b(r)=J_{b1}(r)+J_{b2}(r)=-(288/A^6r^7)f(\xi)$$

where

$$\xi=2\mu r,$$
$$f(x)=90I(x)+xK_1(x)[101/2+(19/16)x^2] +x^2K_0(x)[41/4+x^2/16]$$

and

$$I(x)=\int_0^\infty \frac{d\eta}{(\eta^2+1)^{\frac{1}{2}}}\left(\cos\eta x-\frac{\sin\eta x}{\eta x}\right)$$

and K_0, K_1 are Hankel functions of second kind with imaginary argument.[14] These are the same types of integrals as appear in the earlier work on pair theories.[1,6,15]

For two distinct r-intervals simpler expressions for J may be derived by inserting the asymptotic expansions of the Hankel functions: If $y_2\ll 1\ll\xi$; $r\gg 1/2\mu\gg(54)^{\frac{1}{3}}/A$ (supposing, here, $A\gg\mu$),

$$J(r)=-\frac{144}{(\pi\mu r)^{\frac{1}{2}}}\left(\frac{\mu}{A}\right)^4\frac{1}{A^2r^3}e^{-2\mu r}.$$

If, however, $\xi\ll y_2\ll 1$; $(54)^{\frac{1}{3}}/A\ll r\ll 1/2\mu$ (again: $A\gg\mu$),

$$J(r)=-288/(A^3r^4).$$

For all values of r covered by our calculations, i.e., $r\gg A^{-1}$, the potential is attractive.

At smaller values of r the source functions of the nucleons overlap and also H^0 contributes to the potential. This problem is even more complicated than the one-nucleon H^0 problem mentioned at the end of Sec. III.[16]

One inadequacy of a really strong coupling assumption appears rather certain, and this is the absence of long range spin-dependent forces of the proper magnitude. The Ω-interaction (2.15^Ω) between the ψ'-field and the physical nucleons will certainly produce some spin-dependence of the forces, derivable by a second- (or perhaps fourth-) order perturbation treatment,[13] which, however, requires the knowledge of the H_0 eigenvalues and eigenfunctions. As long as the strong coupling condition (2.16) is fulfilled, the resulting spin-dependence will presumably be quite weak.

Since we would actually expect the coupling to be of intermediate strength, we want to present, in addition, the results of a weak coupling calculation, carried out with the usual perturbation methods. Here we adopt a charge-symmetric theory,[11] involving both charged and neutral mesons, because otherwise no resemblance with reality is obtained. The spin and isotopic spin dependence of the potential turns out to be the same as known from the pseudoscalar Yukawa theory, namely:

$$(8\pi^3)^2J(\mathbf{r}) =(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\nabla)(\boldsymbol{\sigma}_2\cdot\nabla)U(r)$$
$$=(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left[\left(\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}-\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right)r\frac{d}{dr}\left(\frac{1}{r}\frac{dU}{dr}\right) +\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU}{dr}\right)\right]. \tag{4.13}$$

[14] See G. N. Watson, *A Treatise on the Theory of Bessel Functions* (The Macmillan Company, New York, 1944).
[15] J. Jauch (see reference 12).
[16] See Appendix II of reference 2.

The spatial dependence of $U(x)$, instead of being given by a simple momentum space integral as in Yukawa theory, is now expressible by the double integral

$$U(x)=\frac{f^2}{M^2}\int d^3p\int d^3q\frac{g^2(p)g^2(q)e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{x}}}{[(p^2+\mu^2)^{\frac{1}{2}}+(q^2+\mu^2)^{\frac{1}{2}}]}\left[1+\frac{(\mathbf{p}\cdot\mathbf{q})+\mu^2}{(p^2+\mu^2)^{\frac{1}{2}}(q^2+\mu^2)^{\frac{1}{2}}}\right]. \tag{4.14}$$

After angular integrations this expression may be written as

$$U(x)=\frac{16\pi^2f^2}{M^2}\int pdp\int qdq\frac{g^2(p)g^2(q)}{(p^2+\mu^2)^{\frac{1}{2}}+(q^2+\mu^2)^{\frac{1}{2}}}\left[\sin px\,\sin qx\left(1+\frac{\mu^2}{(p^2+\mu^2)^{\frac{1}{2}}(q^2+\mu^2)^{\frac{1}{2}}}\right)\frac{1}{x^2}\right.$$
$$\left.+\frac{(px\cos px-\sin px)(qx\cos qx-\sin qx)}{(p^2+\mu^2)^{\frac{1}{2}}(q^2+\mu^2)^{\frac{1}{2}}}\frac{1}{x^4}\right]. \tag{4.15}$$

Again, the cutoff due to $g^2(p)$, $g^2(q)$ is essential for insuring the convergence of the integrals. In previous calculations[3] this was overlooked, and the results are therefore doubtful. Even for simple functions g, as (3.12), the integrals (4.15) are not expressible in terms of known functions, for arbitrary values of the parameters x, μ^{-1}, and A^{-1}. A first approximation for $x<\mu^{-1}$ is obtained by taking $\mu/A=0$. This appears sensible since, for $A>\mu$, the contribution that the integrand makes to $U(x)$ for p, $q<\mu$ is small. A convenient choice for g then is $g^2(p)=e^{-p/A'}$ (with $A'\sim A$). After the introduction of the coordinates $p=\rho\cos^2\vartheta$ and $q=\rho\sin^2\vartheta$ (4.15) may be evaluated in a straightforward computation with the result

$$U(x)=\frac{16\pi^2f^2}{M^2}A'^5\left[\frac{3}{2}\left\{\arc\tan A'x-\frac{A'x}{1+(A'x)^2}\right\}\frac{1}{(A'x)^5}\right.$$
$$\left.-\frac{1}{\{A'x[1+(A'x)^2]\}^2}\right]. \tag{4.16}$$

The substitution of this expression into $J(\mathbf{r})$ yields for the central-force term a strong short-range repulsion and attraction at larger distances. The tensor-force term, on the other hand, has always the same sign for all r values and it vanishes at $r=0$. Some characteristics of a charge-independent, "hard-core" type[17] of theory are therefore found in a weak coupling approximation of pseudoscalar μ-pair theory, especially the possibility of explaining nuclear saturation in terms of a short-range repulsion. It is easily checked that, for larger r values ($r\sim\mu^{-1}$), $J(\mathbf{r})$ exhibits qualitatively the same spin dependence as the pseudoscalar Yukawa theory. Therefore, as far as the weak coupling approximation goes, the spin and charge dependence of the forces agree, at least qualitatively, with the deuteron data, as, e.g., with the sign of the electric quadrupole moment. The range of the forces, for $A>\mu$, is, of course, given by A^{-1}, and so for $r>\mu^{-1}$ the potential is already negligibly small. For $(16\pi^2)(8\pi^3)^{-2}f^2A^5M^{-1}\sim1$, $J(\mathbf{r})$ has roughly the right order of magnitude. It is, however, doubtful and hard to ascertain whether a perturbation treatment is legitimate for such f values.

Since for intermediate coupling strengths no computational method is known, the best one can do is to resort to a plain interpolation between the two extreme cases. Within this broad margin, and as far as our results go, there appears to be no serious discrepancy with experience.

In conclusion, I would like to express my gratitude to Professor G. Wentzel, who suggested this problem and gave me invaluable guidance and encouragement during the course of this work.

[17] R. Jastrow, Phys. Rev. **81**, 165 (1951).