Interpretation of Experiments on the Photonuclear Effect in Heavy Elements*

L. EYGES

Radiation Laboratory, Department of Physics, University of California, Berkeley, California†

(Received December 26, 1951)

An analysis of the experiments to date on photonuclear reactions in heavy nuclei leads to the following results. There is a correlation between the energy at which the (γ, n) cross section is a maximum and the $(\gamma, 2n)$ threshold. The shape of the total photon absorption cross sections in Sb and Ta can be estimated up to 22 Mev, and there is an indication that the cross section drops off strongly above that energy. Using the estimate of the shape of the total cross section in Ta, one can calculate the neutron yield to be expected in experiments with 330-Mev bremsstrahlung. The calculated value is only about 60 percent of the experimental value. Evidence is presented that the discrepancy is due to neutrons produced by high energy photons, presumably due to mesonic effects. The integrated cross sections for Zn⁶⁴, Sb, and Ta¹⁸¹ can be evaluated and by comparison with the Levinger-Bethe formula lead to values of 0.56, >0.44, and 0.50, respectively, for the fraction of exchange force in the neutron-proton interaction, assuming that all of the photoeffect is due to electric dipole transition.

HERE have been many experiments on the photonuclear effect.¹⁻³⁹ Some of these have been interpreted as agreeing with Bohr's model of the compound nucleus, whereas others have seemed to disagree. Thus, Hirzel and Wäffler² measured the ratio of emitted protons to neutrons from nuclei irradiated by 17.5-Mev

- ¹ G. C. Baldwin and G. S. Kaiber, Phys. Rev. 70, 259 (1946).
- C. Hirzel and H. Wäffler, Helv. Phys. Acta. **19**, 373 (1947).
 J. L. Lawson and M. L. Perlman, Phys. Rev. **74**, 1190 (1948).
- ⁴ M. L. Perlman and G. Friedlander, Phys. Rev. 74, 445 (1948); M. L. Perlman, Phys. Rev. 75, 988 (1949).

 - ⁵ H. Wäffler and O. Hirzel, Helv. Phys. Acta. **21**, 200 (1948).
 - ⁶ N. W. Curtis et al., Phys. Rev. 77, 290 (1950).
- ⁷ E. R. Gaerttner and M. L. Yeater, Phys. Rev. 77, 570, 714 (1950).
 - G. A. Price and D. W. Kerst, Phys. Rev. 77, 806 (1950).
 - ⁹ H. L. Poss, Phys. Rev. 79, 539 (1950).
 - ¹⁰ D. W. Kerst and G. A. Price, Phys. Rev. 79, 725 (1950).
 - ¹¹ B. C. Diven and G. M. Almy, Phys. Rev. 80, 407 (1950).
- ¹² A. K. Mann and J. Halpern, Phys. Rev. 80, 470 (1950).
 ¹³ Seishi Kikuchi, Phys. Rev. 80, 492 (1950).
- ¹⁴ McDaniel, Walker, and Stearns, Phys. Rev. 80, 807 (1950).
 ¹⁵ H. E. Johns, *et al.*, Phys. Rev. 80, 1062 (1950).
- ¹⁶ P. R. Byerly, Jr., and W. E. Stephens, Phys. Rev. 81, 473
- (1951). ¹⁷ R. N. H. Haslam and H. M. Skarsgard, Phys. Rev. 81, 479
- (1951). ¹⁸ L. Katz and A. S. Penfold, Phys. Rev. 81, 815 (1951).
- ¹⁹ L. Katz and A. S. Penfold, Phys. Rev. 83, 169 (1951)
- ²⁰ N. Sugarman and R. Peters, Phys. Rev. 81, 951 (1951).
 ²¹ Karl Strauch, Phys. Rev. 81, 973 (1951).
- ²² Seishi Kickuchi, Phys. Rev. 81, 1060 (1951).
- ²³ Richard D. Miller, Phys. Rev. 82, 260 (1951)

- ²⁴ Haslam, Johns, and Horsely, Phys. Rev. 82, 270 (1951).
 ²⁵ L. Katz et al., Phys. Rev. 82, 271 (1951).
 ²⁶ E. R. Gaerttner and M. L. Yeater, Phys. Rev. 82, 461 (1951).
- Stephens, Halpern, and Sher, Phys. Rev. 82, 511 (1951).
 M. E. Toms and W. E. Stephens, Phys. Rev. 82, 709 (1951).

- ²⁹ A. K. Mann and J. Halpern, Phys. Rev. 82, 733 (1951).
 ³⁰ Terwilliger, Jones, and Jarmie, Phys. Rev. 82, 820 (1951).
 ³¹ C. Levinthal and A. Silverman, Phys. Rev. 82, 822 (1951).
 ³² P. R. Byerly, Jr., and W. E. Stephens, Phys. Rev. 83, 54
- (1951) ³³ E. R. Gaerttner and M. L. Yeater, Phys. Rev. 83, 146 (1951).
 ³⁴ R. Sagane, Phys. Rev. 83, 174 (1951).
 ³⁵ Marshall, Rosenfeld, and Wright, Phys. Rev. 83, 305 (1951).

 - L. Lundby and L. Marshall, Phys. Rev. 83, 323 (1951).
 L. Marshall, Phys. Rev. 83, 345 (1951).

 - ³⁸ J. Halpern and A. K. Mann, Phys. Rev. 83, 370 (1951).
 ³⁹ D. H. Wilkinson and J. H. Carver, Phys. Rev. 83, 466 (1951).

 γ -rays and found that this ratio was much higher than that predicted by compound nucleus theory. Another apparent discrepancy arose when it was established that the (γ, n) cross section in medium weight elements had a maximum around 20 Mev and dropped off sharply for higher energies. According to compound nucleus theory, this drop-off should be due to the competition to the (γ,n) reaction afforded by the $(\gamma,2n)$. But experimentally, it appeared that the $(\gamma, 2n)$ cross section was much too small to furnish appreciable competition. It was this discrepancy, among others, that led Goldhaber and Teller⁴⁰ to propose their special model of nuclear dipole vibrations. Finally, various observers have found that the angular distribution of high energy protons from (γ, p) reactions is not spherically symmetrical, in contradiction to the predictions of the compound nucleus.

On the other hand, there were experiments which were in good agreement with compound nucleus theory. Thus, the angular distribution of the low energy neutrons and protons produced in the nuclear photoeffect was measured by various workers and found to be spherically symmetrical.^{10,11,30} Also, the energy distribution of the neutrons and protons was in good agreement with the compound nucleus model.¹¹ Moreover, Byerly and Stephens¹⁶ measured the ratio of neutrons to protons emitted from Cu when irradiated with 24-Mev bremsstrahlung and found excellent agreement with the predictions of Weisskopf and Ewing,41 which were based on the compound nucleus model.

These apparently contradictory results are, we think, resolved in part by the suggestion of Courant⁴² that there may be a direct photoeffect on protons, in which they are ejected from the nucleus without a compound nucleus being formed. The cross section for this process can be quite small and still explain the anomalously large number of protons in Hirzel and Wäffler's experiment. Also, this hypothesis explains naturally the

- ⁴¹ V. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1940). ⁴² E. Courant, Phys. Rev. **82**, 703 (1951).

^{*} This work was performed under the auspices of the AEC.

[†] Present address: American Embassy, 2 Ave. Gabriel, Paris 8, France.

⁴⁰ M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948)

TABLE I. Summary of data on photonuclear reactions.

Isotope	Thresh- old	Wmax	Half- width	$\int \sigma dW$ (Mev-barns)	Refer- ence
		(γ, n) read	tions		
(C ¹²	18.7	22.4	•••	0.047	24
C12	• • •		•••	0.086	35
C12	•••	•••	• • •	0.107	21
Ål ²⁷	14	19.6		•••	• • •
$\mathbf{P^{31}}$	12.4	19.0	7.5	0.120	18
Fe ⁵⁴	18.3		5.7	0.42	25
Ni ⁵⁸	18.5	•••	4.6	0.33	25
(Cu ⁶³	10.9	17.5	6.0	0.70	15
{Cu ⁶³	10.9	• • •	• • •	0.77 ± 0.15	35
Cu ⁶³	10.9	17.5	•••	0.60	11
Ċu ⁶⁵	10.2	19.0	6.0	1.40	15
Cu ⁶⁵	• • •	19.0		0.89	21ª
(Zn ⁶⁴	19.0	•••	•••	0.61	21ª
{Zn ⁶⁴	18.5	• • •	7.1	0.83	25
Zn ⁶⁴	• • •	• • •	•••	0.77	35
Àg ¹⁰⁹	• • •	16.5	• • •	1.65	11
Sb121	9.3	14.5	5.5	>1.2	15
Sb ¹²³	9.3	14.5	5.5	2.0	15
Ta ¹⁸¹	8.0	13.5	4.5	>0.39	15
		(γ, p) rea	ctions		
C ¹²		21.5 ± 0.5	1.7	0.063 ± 0.016	38
Mg^{25}		21		0.056 ± 0.03	28
A129		21.2 ± 0.5	5.4	0.12 ± 0.03	38
Ni 67% Ni58		18.7	5.4	0.32 ± 0.08	38
27Co ⁵²		21.5	5.7	0.14 ± 0.04	38
30Zn 68				0.068	21
41Cb93		21.3	6.6	0.12 ± 0.03	38
		Higher order	reaction	IS	
Reaction					
$S^{32}(\gamma, np+d)$				0.004	18
$Zn^{64}(\gamma, pn)Cu^{62}$	•			0.26	21
$Zn^{66}(\gamma, pn)Cu^{64}$	£			0.13	21
$Zn^{64}(\gamma,2n)Zn^{62}$				0.047	214
$\operatorname{Cu}^{63}(\gamma,2n)\operatorname{Cu}^{61}$	t			0.035	34

^a In this reference the measurements of integrated cross sections are relative to that of Cu⁵⁵. To convert these relative values to the absolute ones given in this table we have taken as the integrated cross-section for Cu⁶⁵ the mean of the three values given above, i.e., 0.69-Mev barn.

angular asymmetry of high energy protons mentioned above. Moreover, it does not destroy the agreement with the compound nucleus theory that Byerly and Stephens found. The directly-ejected protons are important only when the proton binding energy is so high that the number of evaporated protons is small. This is the case in the experiments of Hirzel and Wäffler but is not so for Cu⁶³, the element measured by Byerly and Stephens.

There is also no contradiction between compound nucleus theory and the fact mentioned above, that the $(\gamma, 2n)$ cross section in Cu⁶³ is much too small to provide competition for the (γ, n) reaction. It appears experimentally that the (γ, np) cross section is much larger than the $(\gamma, 2n)$ cross section and, indeed, is sufficiently large to afford competition.

There is some other evidence that has not been presented before for the validity of the statistical model. For heavy nuclei, the emission of a proton is strongly inhibited by the Coulomb barrier. The only important processes appear to be the emission of neutrons. If the statistical theory and the idea of competition is correct, we would expect to find a correlation between the energy at which the (γ, n) cross section is maximum and the threshold of the $(\gamma, 2n)$ reaction. Evidence for such a correlation is presented in Sec. III.

The statistical model predicts what happens after the nucleus absorbs a photon. There are also theories which say something about the absorption of the photon in the first place. In particular, we shall consider the work of Levinger and Bethe.⁴³ Using so-called "sum rules" these authors calculate the integrated cross section for dipole absorption of a photon. They get the formula

$$\int_{0}^{\infty} \sigma_{\text{total}}(W) dW = 0.060 (NZ/A) (1+0.8x).$$
(1)

Here x is the fraction of exchange force in the neutronproton potential (assumed to be due to central forces only). $\sigma_{\text{total}}(W)$ is the sum of all processes in which a photon is absorbed, i.e., it is the sum of the cross sections for all such processes as (γ, n) , (γ, p) , $(\gamma, 2n)$, etc. If we have an experimental value for this sum, we have a means of deducing the fraction of exchange force in the neutron-proton potential, always remembering the assumption of dipole transition. The integral in Eq. (1) can be evaluated from the available data in some cases. This is discussed in Sec. VI.

The experiments that we have analyzed to obtain the results stated above have been of two major kinds. The first kind is that in which the integrated cross section and sometimes the detailed shape of a particular reaction is measured, detecting the reaction by the method of induced radioactivity. The second kind is that in which the total neutron yield is measured from a target irradiated by bremsstrahlung or some other source of photons. The latter method has the advantage that one gets information about all reactions in which a neutron is emitted, but it has the corresponding disadvantage that one cannot immediately isolate the contribution to the total cross section from particular reactions. This disadvantage can be overcome to some extent by comparing the results of experiments done at different maximum bremsstrahlung energies. Among the experiments which we shall use, are those of McDaniel et al.¹⁴ on neutron yields using photons from the $Li(p,\gamma)$ reaction, that of Price and Kerst⁸ using bremsstrahlung at 18- and 22-Mev maximum energy and those of Terwilliger et al.30 and of Kerst10 using bremsstrahlung at about 330-Mev maximum energy. By comparing the difference between the 18- and 22-Mev yields, one gets information about the cross section for neutron emission between these energies; this is mainly information about the $(\gamma, 2n)$ cross section for heavy elements. By further comparing the 22-Mev results with the 330-Mev results, we can find something about the contribution of the cross section between 22 and 330 Mev.

In the next section, we present a compilation of the

⁴³ J. S. Levinger and H. A. Bethe, Phys. Rev. 78, 115 (1950).

data on individual reactions and in the following section, show that for heavy elements there is a strong correlation between the $(\gamma, 2n)$ threshold and the peak of the (γ,n) cross section. In Sec. IV, we try to estimate the magnitude of the $(\gamma, 2n)$ cross section in some elements by comparing the 18- and 22-Mev data of Kerst and Price and by using the results of McDaniel mentioned above. We can then use this estimate and the 330-Mev bremsstrahlung data to try to find something about the cross section between 22 and 330 Mev. This work, plus other evidence presented in Sec. V, indicates that the total cross section drops off quite sharply above 22 Mev and begins to rise again at energies of the order of 200 Mev. This rise may be due to mesonic effects. In Sec. VI, we use our deductions from the experimental data to calculate the integrated cross section and hence the fraction of exchange force in the n-p interaction.

II. SUMMARY OF DATA

In Table I we present a summary of the data available on photonuclear reactions. There are some interesting features to note in this table. The first is the rather regular rise of the integrated cross section for (γ, n) reactions. Also, there is a slight indication that for elements heavier than Cu, the (γ, n) half-width decreases slowly with Z. As opposed to the (γ, n) reactions, the integrated cross section for (γ, p) reactions rises with Z up to about Z=28 and then begins to drop off. This trend is confirmed by the work of Mann and Halpern,²⁹ who measure the yields of (γ, p) reactions as a function of Z. These yields are roughly proportional to the integrated cross section. Mann and Halpern also find a maximum yield at around Z=30. At this point the yields are of the same order of magnitude as for the (γ, n) reaction. Beyond Z=30, however, the proton yield drops off rapidly and at Z=50 is about one-hundredth the neutron yield. This is easily understandable as due to the effect of the Coulomb barrier. It suggests that in heavy elements one can confine attention to processes in which a neutron is emitted. The last point to be noted in Table I is that the integrated cross section for the (γ, pn) reaction in a medium weight element can be much larger than that for the $(\gamma, 2n)$ reaction. For $_{30}$ Zn⁶⁴, e.g., the integrated (γ, pn) cross section is about 0.4 of the integrated (γ, n) cross section. This suggests that, if the statistical model holds, competition in medium weight elements may be provided by (γ, pn) rather than $(\gamma, 2n)$ reactions.

III. CORRELATION BETWEEN ENERGY AT WHICH (γ, n) CROSS SECTION IS MAXIMUM AND THRESHOLD OF THE $(\gamma, 2n)$ REACTION

According to the Bohr concept of the compound nucleus, the cross section $\sigma_{\gamma,a}(W)$ for a reaction in which a particle a^{44} is emitted when a photon of energy W is incident on a nucleus, is $\sigma_{\gamma,a}(W) = \sigma_{abs}(W) \times P_a(W)$. Here $\sigma_{abs}(W)$ is the cross section for absorption of a photon of energy W, and $P_a(W)$ is the probability that at excitation energy W, the nucleus emits a particle type a. Now, as is well known, the cross section of the (γ,n) reaction, $\sigma_{\gamma,n}(W)$, has a maximum around 19 Mev for medium weight elements and at somewhat smaller values for heavier elements. In all measured cases, the cross section is small or zero near threshold and rises sharply to the maximum. We will say nothing about this initial rise here. A maximum in the (γ,n) absorption cross section could be caused by a maximum in $\sigma_{abs}(W)$, or, if we assume that $\sigma_{abs}(W)$ is more or less flat near W_m , the maximum could be caused by a dropping off of $P_a(W)$. The last assumption seems to us to be much less restrictive.

If we make it, we would expect to find a correlation in heavy elements between the energy W_m at which the (γ,n) cross section is maximum and the $(\gamma,2n)$ threshold. We have restricted this statement to heavy elements because, as we have discussed in the introduction, it is only for these that one can be reasonably certain *a priori*, that essentially only neutrons are emitted. For medium weight elements, we would expect a correlation if the $(\gamma,2n)$ process dominated the (γ,pn) . We are reasonably sure that this is the case for Cu⁶⁵, for which Byerly and Stephens have found that proton emission is small compared with neutron emission. Thus, we include Cu⁶⁵ along with four other heavy elements in Table II, which seems to show that there is a real correlation between W_m and the $(\gamma,2n)$ threshold.

A word about the method of calculating the $(\gamma, 2n)$ thresholds given in the third column of Table II may be useful here. That for Ag¹⁰⁹ is obtained by adding the value given by Sher *et al.*⁴⁵ for the photoneutron threshold in Ag¹⁰⁹ to Harvey's⁴⁶ value for the photoneutron threshold in Ag¹⁰⁸. Similarly, for Sb¹²³ we have used Sher's value for the threshold in Sb¹²³ and Harvey's for the threshold in Sb¹²¹. For the other elements, we have had to add the measured values of the (γ, n) threshold (from Harvey and/or Sher) in the original nucleus and the value calculated from the semi-empirical mass formula for the final nucleus, or vice versa. The measured values have probable errors of about 0.2 Mev. A comparison of Harvey's data on some neighboring

TABLE II. Comparison of the $(\gamma, 2n)$ threshold and energy W_m at which the (γ, n) cross section is maximum.

Element	W_m	$(\gamma, 2n)$ threshold
enC1165	19.0	18.1
47 A p109	16.5	16.05 ± 0.28
1Sh121	14.5	15.6 ± 0.3
5155 51Sh123	14.5	15.4
73 Ta ¹⁸¹	14.0ª	14.1

^a This value is a later one than that presented in Table I and is taken from a preprint of a paper by R. N. H. Haslam, L. A. Smith, and J. G. V. Taylor.

⁴⁵ Sher, Halpern, and Mann, Phys. Rev. 84, 387 (1951).
 ⁴⁶ J. A. Harvey, Phys. Rev. 81, 353 (1951).

⁴⁴ We shall let the subscript a signify several particles if the excitation energy W is high enough that they can be emitted.

elements with the calculated values for those elements, indicates that the calculated values that enter here should be good to within 0.3 or 0.4 Mev. Therefore, we expect the $(\gamma, 2n)$ thresholds given in Table II for Cu, Sb¹²³, and Ta to be accurate to perhaps 0.5 Mev. Probably, this is the magnitude of error in the position in the maximum of the cross section. Thus, our values seem to be sufficiently accurate as not to destroy the correlation, if there is one.

IV. ESTIMATE OF (γ, n) AND $(\gamma, 2n)$ CROSS SECTION FOR Sb AND Ta

In this section, we attempt to estimate the (γ, n) and $(\gamma, 2n)$ cross sections as a function of photon energy and hence, the total cross section below the $(\gamma, 3n)$ threshold, for Sb and Ta. The reason for limiting ourselves to these two elements will appear shortly.

The shape of the (γ, n) cross section for Ta¹⁸¹ (the only isotope of Ta) is known from the work of Johns et al.,15 but the absolute magnitude is not. Naturally occurring Sb consists of two isotopes, Sb¹²¹ (56 percent) and Sb¹²³ (44 percent). The (γ, n) cross section for Sb¹²³ has been measured by Johns et al.14 The shape of the (γ, n) cross section in Sb¹²¹, for those reactions in which the nucleus is left in the 16.4-minute isomeric state, has also been measured. The shape of this cross section curve is, within experimental error, the same as that for Sb¹²³. We will assume here that the shape of the (γ, n) reaction in which Sb is left in the ground state is the same as that measured for the reaction in which Sb is left in the isomeric state.⁴⁷ Thus, both isotopes of Sb will be assumed to have the same total cross section shape. It would be very natural, both a priori and in view of the assumed similarity in shape between the cross sections in Sb¹²¹ and Sb¹²³, to assume further that the magnitudes are the same. We shall not do this here. but instead shall estimate independently the magnitude of the cross section in Sb¹²¹; we shall find that it does come out to be very close to that for Sb¹²³.

We already know something about the total cross section in Sb and Ta, since we know the shape of the (γ,n) cross section and up to the $(\gamma,2n)$ threshold this is the total cross section. In the last section, we found that the (γ,n) cross section had a maximum, i.e., was flat just at and below the $(\gamma,2n)$ threshold. In view of the correlation we have found between the energy at which the (γ,n) reaction is maximum and the $(\gamma,2n)$ threshold, it seems very reasonable to assume that the total cross section is fairly flat just beyond the maximum. Starting from this, we can estimate the total cross section above the $(\gamma, 2n)$ threshold in more detail using the data of Price and Kerst⁸ on neutron yields with 18- and 22-Mev bremsstrahlung and the data of McDaniel *et al.*¹⁴ on neutron yields with γ -rays from the Li⁷ (p, γ) reaction.

Price and Kerst have made measurements of the neutron yields from a large number of elements, using 18- and 22-Mev bremsstrahlung. The ratio of these yields is a smoothly varying function of Z for Z greater than about 30. The difference in yields between the 18- and 22-Mev experiments is due to the difference in the bremsstrahlung spectra at these energies, weighted by the cross section [mainly $(\gamma, 2n)$]. We know what the difference between the 18- and 22-Mev spectra looks like from the work of Schiff.⁴⁸ Thus, we can estimate roughly what the $(\gamma, 2n)$ cross section must be between these energies to agree with Kerst and Price's experiment. This first estimate is rather ambiguous. since there is, of course, more than one shape which will give agreement, but it is a starting point, and we shall see later how it can be made more accurate.

Now, using our guess of the shape of the $(\gamma, 2n)$ cross section we can estimate the absolute cross sections in a way which is insensitive to errors in the $(\gamma, 2n)$ cross section. Price and Kerst have also measured the relative neutron yields of various elements at 18 Mev, among them Cu. Now, the (γ, n) cross sections for Cu⁶³ and Cu⁶⁵ are quite well known. The integrated cross section for Cu⁶³ has been measured by three different observers11,14,32 with fairly good agreement among the three measurements. The measurements of the Cu63 cross section by Diven and Almy¹¹ and by Johns et al.¹⁵ agree quite well with each other. The cross section curve for Cu⁶⁵ has been measured only by Johns et al., but it agrees very well with the Cu⁶³: Cu⁶⁵ cross-section ratio measurements at 17.5 Mev of Hirzel and Wäffler. Thus, there is reason to have considerable confidence in an average of the different sets of measurements on Cu⁶³ and of Johns' measurements of Cu⁶⁵. Then, using the 18-Mev bremsstrahlung spectrum due to Schiff, we can calculate the absolute neutron yield to be expected from natural copper. Using the relative cross-section shapes as estimated above for the element considered, we can also calculate what the absolute magnitude must be to agree with Kerst's 18-Mev data on the yield relative to Cu.

To successfully carry out the analysis outlined above, we must know the shape of the (γ, n) cross section. This is known for all the isotopes in Table II. But now we see why the only elements we can use are Sb and Ta and not Ag and Cu. This is because Kerst and Price use natural isotopic mixtures of the elements, and Ag¹⁰⁹ and Cu⁶⁵ represent only 48.1 percent and 29.9 percent of the natural isotopic mixture, respectively. On the other hand Ta¹⁸¹, and Sb¹²¹ and Sb¹²³ together, account

⁴⁷ Evidence for the correctness of this assumption is had from the work of R. Montalbetti and L. Katz, Phys. Rev. **83**, 892 (1951). These authors have measured the cross-section curves for $Mo^{92}(\gamma,n)Mo^{91}$ when Mo^{91} is left both in the ground state and in a 15.5-min isomeric state. They find a peak for both reactions at 18.7 Mev and also find that above 15.5 Mev the shape of the cross-section curves are the same. The cross sections begin to differ below 15.5 Mev, but they are small compared to their peak values in that region. Similar results have been found by R. Sagane (private communication).

⁴⁸ L. I. Schiff, Phys. Rev. 83, 252 (1951).

for approximately 100 percent of the isotopic abundances of Ta and Sb.

Our estimate of the absolute magnitude of the (γ,n) cross section is insensitive to the estimated shape of the $(\gamma,2n)$ cross sections, since most of the neutron yield with 18-Mev bremsstrahlung from Sb and Ta comes from the (γ,n) reaction.

We can check and refine the above rough estimates using other experimental data. McDaniel *et al.*⁴ have measured the neutron yield relative to Cu from various elements irradiated with the γ -rays from the Li⁷(p, γ) reaction. These γ -rays comprise a sharp line at 17.6 Mev and another component of about half the intensity with a half-width of about 2 Mev centered at 14.8 Mev. Since we know the cross sections for Cu and have an estimate of the cross sections for other elements, we can calculate the relative yields to be expected in the experiment of McDaniel and if necessary, correct our previous estimates. The relative yields in McDaniel's experiment are particularly sensitive to the absolute value of the (γ ,2n) cross section at 17.6 Mev. Our results are given in Fig. 1.

The curves in this figure are such as to agree within a few percent with the experimental data we have used to check them, namely the ratio of neutrons yields to that of Cu in the experiments of Kerst and Price, and of McDaniel and the ratio of the 18- and 22-Mev yields in the Kerst and Price experiment. We estimate that they are not in error by more than 15 percent. Although we have drawn in the total cross section up to 22 Mev, the data does not really tell much about the cross sections above 20 Mev. Thus, as far as this data goes, it is quite possible that above 20 Mev the cross-section curve for Sb does not drop off as sharply as indicated, i.e., that there may be a long flat tail which contributes appreciably to the integrated cross sections. For Ta, the situation is quite different. Here the cross section has dropped to less than one-fifth of its maximum value by 20 Mev; hence it seems very unlikely that there is appreciable contribution to the integrated cross section from energies above 22 Mev.

The integrated cross section for the (γ, n) reaction up to 22 Mev in Sb is given by our estimates as 1.82-Mev barns, which is very close to the value of 2.0 Mev-barns found by Katz for Sb¹²³. This is very reasonable, as we have discussed earlier, in the light of the similarity in cross-section shape between Sb¹²¹ and Sb¹²³.

V. YIELDS WITH 330-MEV BREMSSTRAHLUNG. EVIDENCE FOR AN INCREASE IN THE TOTAL CROSS SECTION AT HIGH ENERGIES

Terwilliger *et al.*³⁰ and Kerst⁹ have measured the absolute yields of neutrons using bremsstrahlung with a 330-Mev maximum energy. We can calculate the yields to be expected in Sb and Ta from the (γ,n) and $(\gamma,2n)$ reactions by photons of energy up to 22 Mev by using the results given in Fig. 1 for the shape and ab-



Fig. 1. Estimate of the (γ, n) and $(\gamma, 2n)$ cross sections in Sb: (56 percent Sb¹²¹+44 percent Sb¹²³) and τ_3 Ta¹⁸¹. The shape of the (γ, n) cross section is assumed known from the work of Johns *et al.* The shape of the total cross section (and hence of the $(\gamma, 2n)$ cross section) is estimated roughly from Price and Kerst's data on the ratio of neutron yields with 18- and 22-Mev bremsstrahlung. The absolute magnitude of the cross sections can then be found by using the known cross sections in Cu and comparing Price and Kerst's yield at 18 Mev for the element concerned with that for Cu. This estimate is insensitive to errors in the shape of the $(\gamma, 2n)$ cross section. Finally, the procedure is checked and refined by using the data of McDaniel *et al.* on the yields relative to Cu using γ -rays from the Li (β, γ) reaction. This data is particularly sensitive to the absolute value of the $(\gamma, 2n)$ cross section at 17.6 Mev.

solute magnitude of the cross-section curves. The formula for the yields with 330-Mev bremsstrahlung is

$$\begin{aligned} \text{Yield} & \left(\frac{\text{neutrons}}{\text{erg} \times \text{mole}/\text{cm}^2} \right) \\ &= 1.77 \times 10^3 \int \left[(\sigma_{\gamma, n}(W) + 2\sigma_{\gamma, 2n}(W) \cdots) \right] \frac{dW}{W}, \end{aligned}$$

for cross sections in barns. Computing from this formula, one gets for the yields up to 22 Mev of Ta¹⁸¹ and the natural isotopic mixture of Sb, 526 and 320 neutrons/(erg \times mole/cm²), respectively. The measured



FIG. 2. Decomposition of the transition curve for neutron yield due to 330-Mev bremsstrahlung in Pb. Curve I is the original transition curve as measured by Terwilliger *et al.* Curve II is the transition curve for a spectrum of photons with peak energy at 14 Mev. Curve III is the difference between curves I and IIand is presumably the transition curve due to neutrons produced by mesons. It corresponds to an energy of the order 200 Mev.

yields⁴⁹ are 940 and 552 neutrons/(erg \times mole/cm²), and are thus higher than the calculated yields by almost a factor two. Thus, there is evidence that a considerable fraction of the neutron yield in experiments with 330-Mev bremsstrahlung comes from photons with energy greater than 22 Mev. *A priori*, there is no way to decide whether the photon absorption cross section drops off fairly quickly beyond the maximum or whether there is an appreciable tail.

For Sb, as we have seen, one cannot say with any certainty whether such a tail exists or not. For Ta the situation is different. Here, our reconstruction of the cross-section curves indicates that the total cross section is small at 22 Mev and that there is no evidence for an appreciable tail. Moreover, for Pb there is other evidence that shows much more strongly that the extra neutrons which are needed above 22 Mev to give agreement with the 330-Mev data come not from a long tail to the photon absorption cross section, but rather from an increase in the neutron yield for photon energies of the order of 200 Mev.

This evidence is as follows: Terwilliger *et al.* have measured the transition curve for the neutron yield in Pb due to 330-Mev photons, i.e., the curve of yield per unit thickness *vs* thickness in Pb. This curve is reproduced in Fig. 2. Now, we would expect the photon absorption cross section for Pb to be quite similar to that for Ta, except that the maximum should be shifted to a somewhat lower energy, say 14 Mev. But we have a good idea of what a transition curve resulting from a spectrum of photons with maximum energy around 14 Mev should look like. Such a transition curve is essentially determined by the energy in the photon absorption cross section for which the cross section is a maximum. Its main features do not depend on whether the cross section has a tail or not, provided it is not very large. Strauch²¹ has measured transition curves for photon spectra with peaks at 16 and 19 Mev (the (γ, n) cross sections in Ag¹⁰⁹ and Cu⁶³, respectively). These curves, normalized to unity at the origin, do not differ by more than 30 percent at most and usually by much less than that. One can then quite reliably extrapolate them to find the transition curve due to a photon spectrum with 14-Mev maximum energy. This extrapolated curve is shown in Fig. 2, as curve II, normalized to Terwilliger's curve at radiative units. We see that for thicknesses greater than a radiation length, this curve has the same shape within the experimental error as the transition curve in Pb measured by Terwilliger et al. If we subtract the two, we get the transition curve for neutron yield from those photons which are not contained in the spectrum around 14 Mev. The subtracted curve is given by curve III of Fig. 2. We see that it falls off very sharply and thus corresponds to a very high

TABLE III. Evaluation of $\int \sigma_{\text{total}} dW$ and x, the fraction of exchange force in the neutron-proton interaction, for ${}_{30}\text{Zn}^{64}$.

Reaction	$\int \sigma dW$ (Mev-barns)	Source
(γ ,n)	0.730	Arithmetic mean of the values in references 21, 25, and 36.
(γ, p)	0.14	Estimated from evaporation model.
$(\gamma,2n)$	0.047	Reference 21.
$(\gamma, 2p)$	~ 0	Reference 4.
(γ, pn)	0.260	Reference 21.
$(\gamma, b2n)$	~ 0.12	(From values in reference 21 for
$(\gamma, b3n)$	~ 0.066	³⁰ Zn ⁶⁶ , multiplied by two to take
$(\gamma, p4n)$	~0.028	account of the fact that $\int \sigma_{\gamma, pn} dW$ for ${}_{30}\text{Zn}^{64}$ is twice that for ${}_{30}\text{Zn}^{66}$.
	$\int \sigma_{ m total} dW$	=1.39; x=0.56.

energy. We estimate this energy to be of the order of 200 Mev. The obvious explanation of the source of this high energy yield of neutrons is from the stars produced by mesons, either real or virtual.

The relative contribution to the neutron yield due to the high energy photons as compared with those around 14 Mev in the Terwilliger *et al.* experiments is given approximately by the relative height of the two transition curves at t=0, since in these experiments the thickness of the targets was small compared with a radiation length. From the relative heights at t=0, we conclude that for Pb the high energy photon component contributes about 40 percent of the neutron yield and the low energy component about 60 percent. This is in excellent agreement with our results for Ta, where we concluded that the high energy component contributed 44 percent of the neutron yield.

If we make the guess indicated by the dotted line in Fig. 1 for the cross section of Sb above 22 Mev, we find that this cross section gives an absolute yield of 328 neutrons/(erg \times mole/cm²). This is 59 percent of the yield found by Terwilliger *et al.* Thus for this estimate

⁴⁹ The values of Kerst are 27 percent lower than those of Terwilliger. We have taken as the measured value the arithmetic mean of Terwilliger's and Kerst's values.

of the Sb cross section, 41 percent of the yield comes from high energy photons.

Finally, our hypothesis that the photon absorption cross section for heavy elements drops off quite sharply beyond the maximum is supported by direct experimental evidence. Anderson and Duffield⁵⁰ have measured the total absorption cross section of U²³⁸ and find that at 23 Mev, the cross section is a small fraction of its value at 15 Mev, the energy at which the maximum cross section occurs.

VI. EVALUATION OF $\int \sigma_{\text{total}} dW$. COMPARISON WITH LEVINGER-BETHE FORMULA

In this section, we try to evaluate the total integrated cross section for photon absorption from experimental data. When this is done we can use this value in Eq. (1) to try to find the fraction of exchange force x in the n-p interaction. The total cross section for photon absorption is the sum of the cross sections for all processes in which a photon is absorbed and a particle (or photon) is emitted. There is sufficient data for the elements ${}_{30}$ Zn⁶⁴ and ${}_{73}$ Ta¹⁸¹ to evaluate the total cross section with reasonable accuracy. Also, one can get a lower limit from the data on Sb. First, we present our data in

TABLE IV. Evaluation of $\int_{0}^{22} \sigma_{\text{total}} dW$ and x, the fraction of exchange force in the neutron-proton interaction, for Sb.

Reaction	$\int \sigma dW$ (Mev-barns)	Source
(γ,n) $(\gamma,2n)$ $(\gamma,n>2)$	1.82 0.59 ?	Secs. II and III Secs. II and III
	$\int_{0}^{22} \sigma_{\text{total}} dW > 2.41;$	<i>x</i> >0.44.

Tables III, IV, and V, and then we discuss the data briefly.

We consider first, the data for ${}_{30}Zn^{64}$ given in Table III. There are several reactions listed, the source of which is the work of Strauch.²¹ Strauch has measured only values of the integrated cross sections relative to that of Cu⁶³. To obtain absolute values from his data, we have combined his relative measurements with three measurements of the integrated cross sections for the (γ, n) reaction on Cu⁶³ by Johns *et al.*, Marshall, and Diven and Almy. These authors give this integrated cross section as 0.70, 0.77 ± 0.15 and 0.60 Mev-barns, respectively. We have used the mean of these three values, 0.69 Mev-barns, to get absolute values from Strauch's relative measurements.

The integrated cross section for the (γ, p) reaction is obtained from the known (γ, n) cross section combined with the calculations of Weisskopf and Ewing on the relative probability of evaporation of a proton or neutron. This should give a fairly reliable estimate, since the proton is bound much less tighly than the neutron. As we have discussed in the introduction, in such a case

TABLE V. Evaluation of $\int \sigma_{\text{total}} dW$ and x, the fraction of exchange force in the neutron-proton interaction, for $_{73}\text{Ta}^{181}$.

Reaction	$\int \sigma dW$ (Mev-barns)	Source	
(γ,n)	2.96	Secs. II and III	
$(\gamma, 2n)$	0.81	Secs. II and III	
(n n > 2)	~ 0	Secs. II and III	

the evaporation theory probably predicts the relative numbers of neutrons and protons fairly accurately. Our calculation neglects the protons which are emitted below the threshold for neutron production, but it is very likely that the number of such protons is small. We would expect the integrated $(\gamma, 2p)$ cross section to be very small, since the integrated (γ, p) cross section is small. There is also evidence for this from the data of Perlman⁴ on the neighboring element Cu⁶³. Perlman finds that the relative yields for the $Cu^{63}(\gamma,n)$ and $Cu^{63}(\gamma, 2p)$ reactions from 50-Mev bremsstrahlung are 35 and 0.16, respectively. Assuming that the energy at which the $(\gamma, 2p)$ cross section is a maximum is about twice that for the (γ, n) gives the result that the integrated $(\gamma, 2p)$ cross section is about 1 percent of the integrated (γ, n) cross section. We would expect a similar result from Zn. Also, we would expect that the cross sections for higher order reactions involving two protons as $(\gamma, 2pn)$, $(\gamma, 2p2n)$, etc., are small, as well as the (γ, γ) and (γ, α) reaction. Thus, it is likely that the data listed in Table II includes all reactions whose integrated cross sections are appreciable and that the value for $\int \sigma_{\text{total}} dW$ is fairly accurate. Putting this value into the formula (1) gives a value x=0.56 for the fraction of the exchange force in the n-p interaction. This is in good agreement with the ratio postulated by Serber to explain the high energy n-p scattering. In Tables IV and V, we present data for the evaluation of $\int \sigma_{\text{total}} dW$ for Sb and 73Ta¹⁸¹. As we have noted before, the cross section for Sb is not known above 22 Mev. Hence, the integrated cross section given must be considered to be a lower limit. For Ta, on the other hand, we expect the integrated cross to be, to a good approximation, the sum of the (γ,n) and $(\gamma,2n)$ integrated cross sections. Comparing the total integrated cross sections with formula (1), one gets the values of x shown. Considering the errors involved in their calculation, the three values of x deduced here must be considered to be in good agreement with one another. In these calculations we have had to neglect contributions to the electric-dipole photoeffect from photons with energy greater than 330 Mev, but we expect this to be a very small contribution.

Katz and Penfold¹⁸ have made similar calculations to that of this section for S³² and conclude that the integrated cross section is much less, about $\frac{1}{8}$ of that expected from the Levinger-Bethe formula. We think the reason for this disagreement is that among other things, they neglect to include the cross sections for

⁵⁰ R. E. Anderson and R. B. Duffield, Bull. Am. Phys. Soc. 26, No. 6, 32 (1951).

higher order reactions in which two or more neutrons are produced. The evidence for this is as follows. We have taken their estimate for the (γ, n) cross section and their measurements of the (γ, pn) cross section and calculated the yield of neutrons to be expected in Jarmie, Jones, and Terwilliger's experiments. The yield comes out to be too low by a factor five. Thus, there seems to be an appreciable cross section from reactions which Katz and Penfold have not taken into account. Of course, this yield might come from neutrons produced by high energy photons in a similar process to the one we have to postulate above to explain the Terwilliger transition curve. In this case the high energy process

would have to account for 80 percent of the neutrons observed with 330-Mev bremsstrahlung. This seems rather high. If we assume arbitrarily, that as for heavy elements the high energy process accounts for about 40 percent of the neutron yield, then the (γ, n) cross section alone is too small by a factor three to secure agreement with Terwilliger's results. Thus, the estimate of the (γ, n) cross section is either much too low or else there are higher order reactions with integrated cross sections comparable to that for the (γ, n) process. At any rate, it seems that it would be premature to claim disagreement with the Levinger-Bethe formula until more measurements are made.

PHYSICAL REVIEW

VOLUME 86, NUMBER 3

MAY 1, 1952

Nuclear Phenomena Deducible from y-Pair Theory with Pseudoscalar Coupling*

L. M. TANNENWALD[†] Department of Physics, University of Chicago, Chicago, Illinois

(Received November 21, 1951)

The case of pseudoscalar coupling between nucleons and μ -field is considered within the framework of μ -pair theory. Besides the usual perturbation treatment, the strong coupling approximation for this case is developed. Both methods are applied to the problems of scattering of μ -mesons by nucleons and the nucleon-nucleon interaction. Interpolation between the extremes of weak and strong coupling suggests that this µ-pair theory may be promising with an intermediate coupling strength, a condition also required by the μ -pair theory of the π -meson.

1. INTRODUCTION

B^{ESIDES} the Yukawa theory of nuclear forces, some attention has in the past been devoted to the so-called pair theories according to which the interaction between nucleons may be pictured as being transmitted by a pair of particles instead of a single particle (π -meson). The quanta of a pair may be either bosons¹ or fermions.² The latter were first taken to be electron-neutrino or electron-positron pairs and later μ -meson pairs.³

One recommending feature of pair theories is the saturation characteristics of the nuclear forces.^{2,4-6} A second is the possibility of interpreting the π -meson as a pair of μ -mesons bound together due to a small admixture of a virtual nucleon-pair state,7 and the possible explanation of the V-particles⁸ as excited

- ⁸ R. Marshak, Phys. Rev. 57, 1101 (1940).
 ⁴ Wigner, Critchfield, and Teller, Phys. Rev. 56, 530 (1939).
 ⁵ G. Wentzel, Helv. Phys. Acta 15, 111 (1942).

 ⁶ G. Wentzel, Helv. Phys. Acta 16, 111 (1942).
 ⁶ A. Houriet, Helv. Phys. Acta 16, 529 (1943).
 ⁷ G. Wentzel, Phys. Rev. 79, 710 (1950).
 ⁸ G. D. Rochester and C. C. Butler, Nature 160, 855 (1947); Seriff, Leighton, Hsiao, Cowan, and Anderson, Phys. Rev. 79, 001 (1970). 204 (1950).

nuclear states resulting from the binding of μ -mesons, say, by a bare nucleon. The fact that the spin of such excited states may be integral or half-integral, depending on whether an odd or even number of fermions has been bound, may be helpful in understanding the long lifetime of the neutral V-particle (10⁻¹⁰ sec), in particular its stability against γ -decay into the neutron ground state.

The most serious objection against pair theories may well be the role played by the momentum cutoff which must be introduced to achieve convergence and which dominantly affects the predictions of the theory in the high energy region. Since it turns out that the cutoff also determines the range of the nuclear forces and the density of nucleons in heavy nuclei,5 its order of magnitude is roughly that of the meson mass (times c). Therefore, if one takes the cut-off prescription seriously to the extent of applying it, for example, to the scattering of μ -mesons by nucleons, then one would expect at kinetic energies much greater than 100 Mev a very small cross section, while a substantially larger value for energies of the order 100 Mev or somewhat less. According to the measurements of Amaldi and Fidecaro⁹ the cross section is at most 4.5×10^{-29} cm² per nucleon for μ -energies between 200 and 320 Mev. and above 320 Mev it is at most 2.3×10^{-30} cm² per nucleon. The cutoff therefore offers an explanation for the presently

^{*} This paper was written as a Ph.D. thesis in the Department of Physics, University of Chicago, Chicago, Illinois. † Present address: U. S. Naval Ordnance Test Station, Inyo-

kern, China Lake, California. ¹ W. Pauli and N. Hu, Revs. Modern Phys. 17, 267 (1945), for

literature until 1945.

² G. Wentzel, Prog. Theoret. Phys. 5, 584 (1950), for literature until 1950.

⁹ E. Amaldi and G. Fidecaro, Phys. Rev. 81, 339 (1951).