and hence

$$\int_0^\infty n_2 dt = W - 2R_\pi. \tag{20}$$

The sum of n_1 and n_2 gives the resulting shower curve, and then one proceeds to progressively higher energies by feeding this solution back into the method.

For the case of incident electrons the procedure is exactly the same, except that the primary distribution $n_1(t)$ is due to the incident electron itself and, as shown in reference 6, is given by the integral Gaussian form,

$$n_1(t) = \tfrac{1}{2}\{1 - \mathrm{erf}[(x-R)/(2y)^{\frac{1}{2}}]\}, \tag{21}$$

R and y being given by (6) and (7). The photon spectrum is given by (12) or by its simple approximation dk/k with cutoff at $k = E_0 - R$. The spacial source distribution is given by (15).

Multiple scattering effects are included in much the same manner as in the Monte Carlo work. We calculate the number of electrons below the random energy, given in shower units of energy, approximately by

$$E_r = (10/\beta)^{\frac{1}{2}}, \tag{22}$$

or more accurately by Eq. (14) of reference 6. (Notice that β is in Mev in these formulas.) The fraction of the primary electrons, n_1, below E_r is just r/R for an incident electron or r/R_π for an incident photon, where $r = \ln(E_r + 1)$. The fraction of the secondary electrons n_2 below E_r is just $r/\bar{R}$, where $\bar{R}$ is the pair range given by (7) averaged over the spectrum (12) or (13).

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Comparative Models in Nuclear Scattering*

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The "optical" and "compound" methods for treating nucleon-nucleus encounters are compared on the basis of a very simple model. It is concluded that the optical procedure is valid for incident energies $E > \sim 80$ Mev, the compound for $E < \sim 30$ Mev. The statistical model of the excited nucleus can be applied only when the excitation energies of individual nucleons may all be considered to be within the lower limit. For π-mesons incident on nuclei similar considerations show that the optical method must be used under all circumstances.

1. INTRODUCTION

ELASTIC scattering and absorption of nucleons have been treated by two different methods, which may be designated as the optical[1] and compound[2] procedures. The present note compares these approaches and attempts to define their respective regions of validity, with a view toward determining which approach is suitable for π-meson scattering and absorption by nuclei; it appears that the optical model is preferred under all circumstances.

In the compound procedure the logarithmic derivative f at the nuclear surface is represented by a suitable phenomenological form that reflects the complex situation inside the compound nucleus; namely,

$$f = -K_0 \tan\{\pi/D(E - E_0 + i\Gamma_a/2)\}$$

where $K_0 \approx 10^{13}$ cm^{-1} is a wave number appropriate to the interior of the nucleus and corresponds to a potential well about 30 Mev deep. The energy of the system is E, a resonance energy is E_0, the average spacing of successive resonance levels is D, and Γ_a is the half-width for absorption. This approach yields the Breit-Wigner form for isolated levels at low energy and has been extended[3] to incident nucleon energies as high as 25 Mev, where the resonances are completely smeared out.

The optical method has been used for incident nucleon energies on the order of 90 Mev or more and consists in integrating the phase difference over all paths through the nucleus. The wave number external to the nucleus is k, internally is $k' = k + k_1 + iK/2$, where $|k' - k| \ll k$. The beam may be analyzed into partial waves of angular momentum l, for which the appropriate W.K.B. path length extends radially from R to r_l, the classical turning point radius. A characteristic feature of this treatment is the complete neglect of reflection at the nuclear surface, which implies an infinitely diffuse boundary.

A superficial difference between the models is the question of "sharp" *vs* "diffuse" boundaries: to argue that it is unimportant, consider the error introduced by assuming a sharp boundary when a diffuse boundary is correct. The parameter measuring the sharpness of the boundary is $1/(k\Delta R)$, where the uncertainty in nuclear radius ΔR happens roughly to equal $1/K_0$ in magnitude. In the simplest case of no external potential the phase shift is given by

$$k \cot(kR + \delta) = k \cot K'R + (K' - k) \cot K'R, \tag{1}$$

* This work is supported by the research program of the AEC.
[1] Fernbach, Serber, and Taylor, Phys. Rev. 75, 1352 (1949).
[2] See, for example, Feshbach, Peaslee, and Weisskopf, Phys. Rev. 71, 145 (1947).
[3] H. Feshbach and V. F. Weisskopf, Phys. Rev. 76, 1550 (1949).

TABLE I. Ratio N_1/N_2 for various incident energies E_0.

E_0(Mev)	≈ 0	10	25	50	75	100
Γ(Mev)	0.3×10^{-6}	1.5	4	8	12	16
v/c	0.2	0.3	0.3	0.4	0.4	0.4
$\lambda(10^{-13}$ cm)	1.4	1.4	2	3	4	5
$\bar{\lambda}(10^{-13}$ cm)	1.4	1.4	1.4	2.1	2.8	3.5
f	1	0.85	0.71	0.3	0.1	0.03
N_2/N_1	10^8	26	6	1	0.2	0.07

where $K'=(k^2+K_0^2)^{\frac{1}{2}}$ is the wave number of the interior region. The first term on the right-hand side of (1) would be given by a perfectly diffuse boundary, leading to a phase shift $\delta=(K'-k)R$; the second becomes dominant as the hard sphere condition is approached. The relative importance of reflection at the boundary is thus measured by $(K'-k)/k$; and under the diffuse boundary condition, $K_0/k\ll1$, this becomes $(K'-k)/k\approx\frac{1}{2}(K_0/k)^2\ll1$, so that no appreciable error is made by insertion of the sharp boundary in this case. With long-range potentials $V(r)$ in the internal and external regions, the relation (1) may be approximately preserved by replacing k with $k_{\text{eff}}=[k^2-2MV(R)/\hbar^2]^{\frac{1}{2}}$. Then the sharp boundary holds for $k_{\text{eff}}\ll K_0$ and $k_{\text{eff}}\gg K_0$, being respectively necessary and inconsequential. In any situation, only a minor fraction of the partial waves will satisfy $k_{\text{eff}}\approx K_0$, so that a relatively small total error is expected by assuming sharp boundaries throughout. There is some experimental sanction for a sharp boundary with 90-Mev neutrons,[4] as well as at 25 Mev and 14 Mev.[3]

2. COMPARISON OF METHODS

The underlying physical models constitute the essential difference between the methods: in the optical case the entire incident energy is focused in a single nucleon, and one computes its elastic or inelastic scattering (absorption) in passing through the nucleus. On the other hand, the resonances always implied if not resolved in the compound model involve the sharing and exchange of the incident energy among many nucleons; elastic scattering is achieved only if the energy is relocalized in a single nucleon before being degraded. The models are generically related, since absorption of a nucleon in the optical model is a necessary first step in the establishment of an excited compound state. It is not a sufficient step, however, for the nucleus may lose energy by inelastic emission in an early stage of the sharing and exchange process. Thus if a large number of such systems are being formed by bombardment at a constant rate, the relative numbers in the optical (1) and compound (2) states are given by the formula familiar from a simple radioactive decay chain:

$$N_2/N_1=(\tau_2/\tau_1)f \tag{2}$$

where τ_1, τ_2 are the respective mean lives and f is the fraction of state 1 that upon decay ultimately leads to state 2.

The fraction f is roughly the probability that the incident energy will become uniformly distributed over the nucleus before any of it escapes by nucleon emission (γ- and α-emission are negligible in comparison). The ultimate simplification is one in which all collisions in the nucleus are averaged to produce at each encounter two nucleons of half the original energy. Then the numbers of participating nucleons at any time is $n(t)=e^{0.7t/\tau}$, where $\tau=\lambda/v$ is the mean interval between collisions. The nucleons that escape between successive collisions lie in a surface layer of depth λ, and the fraction of these with the proper direction of motion to excape is about $\frac{1}{2}\int_0^1(1-\mu)d\mu=\frac{1}{4}$. If $P(t)$ is the probability that no nucleons have escaped by time t, and $n(t)$ is assumed uniformly distributed over the volume of the nucleus,

$$dP=-\frac{1}{4}\left\{\frac{4\pi R^2\lambda}{(4/3)\pi R^3}\right\}n(t)\left[\frac{vdt}{\lambda}\right]P$$

$$P(t)/P(0)=P(t)=\exp\left\{-(3/4)v/R\int_0^t n(t)dt\right\}$$
$$\approx\exp\{-(\lambda/R)[n(t)-1]\}. \tag{3}$$

Then $f=P(t_0)$, where t_0 is the earliest time at which the excited nucleons cannot escape. Assuming the average binding of a nucleon in the Fermi well to be about 15 Mev, and including the incident nucleon, one has $n(t_0)\approx E_0/15+1$ where E_0 is the incident kinetic energy, and

$$f\approx\exp\{-\bar{\lambda}E_0/15R\}. \tag{4}$$

The mean free path $\bar{\lambda}$ in (4) is an average over the slowing-down process, while $\tau_1=\lambda/v$ with $\lambda>\bar{\lambda}$ the mean free path of the incident nucleon and v its velocity inside the nucleus. The mean life $\tau_2=\hbar/\Gamma$, where for high excitation energies E_0 the compound model[5] gives the order of magnitude $\Gamma\approx\frac{2}{3}E_0A^{\frac{1}{3}}$. In computing "representative" values of N_2/N_1, an average was taken between the values of Γ and of f for $A^{\frac{1}{3}}=3$ and $A^{\frac{1}{3}}=6$. The indicated assumptions of λ, $\bar{\lambda}$ lead to the estimates given in Table I. The values of N_2/N_1 cannot be taken too literally but suggest that in the region around 30–80 Mev neither model is adequate alone and a mixture must be used with both states at least 20 percent abundant. Measurements in this region would be especially complicated to analyze.

3. APPLICATIONS

An obvious corollary of the foregoing conclusion is that the evaporation model for nuclear reactions[5] cannot be relied upon for excitation energies much above 30 Mev for the individual nucleons. This feature is allowed for in describing the production of stars in nuclear emulsions and the diffusion of the nucleonic

[4] S. Pasternack and H. S. Snyder, Phys. Rev. 80, 921 (1950).

[5] V. F. Weisskopf and D. H. Ewing, Phys. Rev. 57, 472 (1940).

component in cosmic rays. As a first approximation the production of secondary nucleons in high energy nuclear events can be described entirely by methods appropriate to the optical model;[6] the statistical model can be applied as a correction to the nucleons remaining inside the nucleus with excitations of 30 Mev or less.

The present analysis is in harmony with the interpretation of the high energy tail in γ-n excitation curves[7] as largely due to a direct photoelectric effect, especially in light elements. It strongly suggests, however, that this interpretation is not to be applied to the large resonances found in the 17-Mev region among medium and heavy elements, as is already clear from the large peak cross sections.

In case the incident particle to be elastically scattered or absorbed is a π-meson, an important modification is made by the possibility of catastrophic absorption in which the incident particle is destroyed. If this occurs, the nucleus will be excited by more than 140 Mev, and the arguments above indicate that fast nucleon emission will preclude the formation of a compound state. On the other hand, if a compound state is to be formed with the meson as one particle, catastrophic absorption must be avoided for a relatively long time and is the dominant factor determining f. Thus,

$$N_2/N_1 \approx (\tau_2/\tau_1)p^n \approx np^n, \tag{5}$$

where $n \approx \tau_2/\tau_1$ is the number of collisions made by the meson during the establishment of a compound state, and p is the average probability per collision of escaping catastrophic absorption. It appears,[8] at least at moderate energies, that $p \ll 1$; then maximizing (5) shows that $(N_2/N_1)_{\max} \ll 1$, and that this maximum is achieved for $n \ll 2$, which is certainly too few collisions to establish a compound state. Thus one concludes that only the optical model is applicable to calculation of π-mesons on nuclei for all energies. The argument concerning sharp *vs* diffuse boundaries, however, still holds in favor of including nuclear boundary effects in meson calculations.

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[6] M. L. Goldberger, Phys. Rev. **74**, 1269 (1948).
[7] R. Sagane, Phys. Rev. **84**, 587 (1951).
[8] Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951).

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Extinction Effects in Neutron Transmission of Polycrystalline Media*

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The effects of primary and secondary extinction are considered for neutron transmission work in the energy region where diffraction is important. It is shown that in typical studies the grain size is the most important parameter affecting extinction, with the mosaic block size and the angular spread of the mosaic blocks of secondary importance. Experiments were performed to corroborate the theory, and criteria are set up to avoid extinction effects. It is shown how to determine the mosaic block size and the angular spread of the mosaic blocks in substances with large grain size by using fine resolution near the last crystalline cutoff, where the breadth of the Bragg peak becomes large compared to the angular misalignment of the mosaic blocks.

IT is convenient in neutron transmission studies of polycrystalline media to have the apparent cross section per nucleus of the sample, as given by the usual expression $\bar{\sigma} = (\ln I_0/I)/Nx$ (where N is the number of nuclei per cc and x is the thickness of the sample), proportional to the coherent cross section of the nuclei. Previous theoretical treatments[1,2] have assumed this to be the case under the experimental conditions that the microcrystals are randomly oriented and are small enough to give negligible primary extinction. Microcrystals are small coherent domains (commonly called mosaic blocks) and are usually misaligned over a range of several seconds in perfect crystals to several minutes in the macroscopic grains of imperfect crystals like metals. (In large single crystals the gross lineage may cause further misalignment to the extent of several degrees,[3] but we shall confine our attention here to small grains $<10^{-1}$ cm.) It is the purpose of this note to show that the conditions postulated in references 1 and 2 are not sufficient and to point out under what conditions the proportionality between the apparent and coherent nuclear cross sections are assured. We shall also show how it is possible to secure information about the mosaic block size even if primary extinction is negligible.

To begin, we consider the cross section for scattering into Bragg peaks of a perfect microcrystal small enough to make use of the Born approximation,

$$\sigma = \frac{\sigma_{\mathrm{coh}}}{4\pi} \prod_{ijk} \frac{\sin^2(N_i s d\mathbf{q}\cdot\mathbf{i})}{\sin^2(sd\mathbf{q}\cdot\mathbf{i})}, \tag{1}$$

* Research carried out at Brookhaven National Laboratory under contract with AEC.
[1] Halpern, Hamermesh, and Johnson, Phys. Rev. **59**, 981 (1941).
[2] Fermi, Sturm, and Sachs, Phys. Rev. **71**, 589 (1947).
[3] Weiss, Hastings, and Corliss, Phys. Rev. **83**, 863 (1951).