the multiplication with a γ_i of the order of 1×10^3. Thus the rate of increase in luminosity is accounted for. Assuming a relatively sudden cessation of ionization, the decline in light intensity is that caused by emission from excited states, as recombination is negligible. If the choking starts gradually and builds up rapidly, the plateau of luminosity with decreasing excitation and radiation is accounted for. With average electron energies of the order of 3 ev in the fields of $X/p\sim30$ to 70, near the end of the negative glow region, the cross section for dissociative attachment in air is 6×10^{-20} cm^2.[14] The random electron velocity is of the order of 10^8 cm/sec. At 760 mm the rate of negative ion forming encounters is 1.6×10^8 per second and the average lifetime τ of electrons is 6×10^{-9} sec. The electrons will then be reduced to $1/e$ of their number in 0.06 cm if the drift velocity in this region is 1×10^7 cm/sec at an X/p of about 26. At worst this reduction will occur at 0.12 cm with an electron velocity of 2×10^7 cm/sec. The negative ion density, with center of charge at 0.12 cm from the cathode surface, is of the order of 5×10^{13} ions/cm^3.[5] This creates a field distortion superposed on that inferred at the height of the luminosity[5] sufficient to terminate all ionization. The ionization becomes negligible within 6×10^{-9} sec after the maximum ionization is reached. This is the rate of choking required, and the observations of English are now understood. The time duration of the quenching process is given by

$$\tau=1/(\sigma\bar{c}N_1p),$$

where σ is the attachment cross section, $\bar{c}$ the average random velocity, N_1 the number of attaching molecules per cm^3 at 1-mm pressure and p the pressure in mm. No data on the change of τ with these variables exists to date except the general observations of Weissler,[15] Miller and Loeb,[6] and Bandel[8] that pulses appear to last longer at high potentials, low pressures, or low percentages of O_2. It may be added that the distortion of the field[16] by the negative ion space charge is such that after the pulse, when excitation and ionization cease near the point, there is still excitation and some ionization on the anode side of the negative ion space charge. This pushes out into the gap as the space charge clears. The resulting potential distribution accounts for the intense and extensive fan-like positive column extending to 0.15 cm which characterizes Trichel pulses in air but is absent in coronas in N_2 and in air where no pulses occur.

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Electrodynamic Corrections to the Fine Structure of Positronium

ROBERT KARPLUS AND ABRAHAM KLEIN
Harvard University, Cambridge, Massachusetts
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THE accuracy of experiments that are currently being carried out to measure the separation of the ground-state doublet of positronium[1,2] has called attention to the need for a more precise theoretical treatment of the problem. Investigations reported in the literature[3–5] have considered only the effects to order α^2Ry of the exchange of one virtual quantum between the electron and the positron. Electrodynamic corrections due to the exchange of two virtual quanta and relativistic corrections to the first-order calculation, each of relative order α, may be expected to amount to an appreciable fraction of a percent.[5]

The starting point of this investigation was an equation for the two-particle electron-positron wave function that was obtained by the obvious adaptation of Schwinger's derivation of the two-electron wave function.[6] The interaction operator appearing in this equation is formally the same as the one obtained by Bethe and Salpeter,[7] except for the occurrence of a term that describes the virtual annihilation force[3,4] between the electron and the positron.

The smallness of the fine structure effects makes it possible to treat them as perturbations on the two-particle system that is bound by the Coulomb potential.[8] Our problem is simplified by the fact that only the spin-dependent part of the perturbation energy in an S-state is needed for an evaluation of the experimentally accessible 1^3S-1^1S separation. We have, therefore, considered only the effect of the corrections on this separation.

It is possible to divide the result meaningfully into two parts: effects in which Dirac particles are always present (direct interaction) and effects that involve at least one intermediate state in which only photons are present (exchange interaction or virtual annihilation force). The former contribute an amount

$$\Delta E_1=\tfrac{2}{3}\alpha^2\mathrm{Ry}_\infty[1-\tfrac{2}{3}\alpha/\pi+\alpha/\pi]=\tfrac{2}{3}\alpha^2\mathrm{Ry}_\infty[1-\tfrac{1}{3}\alpha/\pi],$$

where the large term is the first-order effect of the Breit interaction,[9] the negative term includes relativistic and retardation corrections and the effects of the exchange of two virtual photons, and the last term represents the increase in the Breit interaction due to the anomalous magnetic moments[10] of the particles. The exchange interaction contributes

$$\Delta E_2=\tfrac{1}{2}\alpha^2\mathrm{Ry}_\infty[1-8\alpha/9\pi-4\alpha/\pi+2(1-\ln 2)\alpha/\pi];$$

the large term is the first-order effect of the virtual annihilation force, the two negative terms arise, respectively, from the polarization of the vacuum by the virtual photon and from the interactions of the pair before and after the virtual annihilation, and the last term represents the contribution of two-quantum virtual annihilation. This last term, which is effective only in the singlet state, is really complex, the imaginary part corresponding to the width of the state due to the real decay rate[11]

$$(2\pi\tau)^{-1}=\alpha^3\mathrm{Ry}_\infty/h=1.28\times10^9\ \mathrm{sec}^{-1}.$$

Numerically the predicted separation is

$$\Delta E_1+\Delta E_2=(1.1666+0.8671)\times10^5\ \mathrm{Mc/sec} =2.0337\times10^5\ \mathrm{Mc/sec}.$$

When the experiments of Deutsch are interpreted on the basis of a Zeeman effect that depends on the total magnetic moment $(e\hbar/2mc)(1+\alpha/2\pi)$ of each particle, the value of the separation obtained by him is

$$(2.037\pm0.003)\times10^5\ \mathrm{Mc/sec}.$$

Theory and experiment are thus in satisfactory agreement.

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