energies (above 3 Mev) in an attempt to observe the upper limit which should be about 2.20 Mev for the bombarding energies used. The observations showed a general decline in background in this region of the spectrum. Unfortunately a sharp limit in the weak background could not be established in the presence of strong groups from carbon and oxygen which, if they do not actually interfere, contribute to the instrumental background.

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Spectroscopic Isotope Shift and Nuclear Shell Structure*

G. BREIT
Yale University, New Haven, Connecticut
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FROM a theoretical standpoint views of nuclear shell structure, NSS, present the difficulty of reconciliation of a one-body, individual particle approach with the apparent existence of many particles within each other's range of force. An analogy with the theory of solids brought out by Weisskopf[1] is helpful in removing some of the contradiction. In the similar atomic electron problem the exclusion principle plays an important role, as in Weisskopf's explanation; in addition the geometrical relations, which make orbits that are added with higher Z have larger radii, also matter, producing an ordering according to values of the principal quantum number n. In the present note it is speculatively considered whether the centrifugal barrier associated with high values of the azimuthal quantum number L contributes to the possibility of assigning the usual shell structure quantum numbers as reasonably "good" quantum numbers. The considerations are based on empirical evidence from the spectroscopic isotope shift, SIS, which shows marked irregularities at the closure of nuclear neutron shells.[2] The recent finding of an anomaly in the addition of two neutrons after the completion of the neutron number 126 substantiates the view[3] that magic numbers and anomalies in the SIS are intimately connected, as mentioned by Kopfermann and Brix themselves.[2] A way of correlating the facts which appears adequate for explaining the apparent goodness of the NSS quantum numbers is as follows: (A) It is supposed that there exists a nuclear core[4] and that for even Z the protons form a part of it. (B) The radius of the core is smaller than the radius of the maximum of $|r\psi|^2$ for the $1i_{13/2}$ neutrons when $Z\sim 80$ and the $1h_{11/2}$ neutrons when $Z\sim 50$. (C) Within the core the neutrons with higher L move somewhat as hypothesized by Weisskopf; outside the core neutrons move essentially as free particles. (D) The density of the core is approximately constant.

The second part of (C) is hardly a hypothesis, since on a sphere of radius $1.5\times 200^{1/3}\times 10^{-13}$ cm one can arrange 14 neutrons at distances of $\sim 0.83\times 10^{-12}$ cm from each other. The jump in the SIS at $N=126$ is explicable as the result of adding two $1i_{13/2}$ neutrons to Pb^{206} to form Pb^{208} and two $2g_{9/2}$ neutrons to Pb^{208} to form Pb^{210}. Taking the radius to be $\hbar/(Mmc^2)^{1/2}$ the centrifugal potential barrier, B, for i neutrons is 21 mc^2 and the neutron density of an i neutron falls off by a factor 1/2.7 in a fractional radius change $\Delta r/r\sim 1/13$. For g neutrons $B=10$ mc^2 and $\Delta r/r\sim 1/9$. The ratio of surface thicknesses affected is $\sim(21/10)^{1/2}=1.45$ on the JWKB approximation. The agreement of this number with the observed ratio 1.5 in the SIS of the $(210-208)/(208-206)$ differences is probably fortuitous but there is seen to be no difficulty in accounting for the observed magnitude of the effect. For $N=82$, $B\sim 20$ mc^2 for an h neutron and $\Delta r/r\sim 1/11$. The ratio of core thicknesses affected by h and by f neutrons is $\sim(20/12)^{1/2}=1.58$. It should be remarked that if the $f_{7/2}$ neutrons were to have a maximum of their radial density distribution slightly inside the core, a much larger ratio would result; the neutron density at the core surface and the core thickness affected would both be larger. The observed jump in the SIS on closing $N=82$ corresponds to a factor ~ 4. The existence of further anomalies close to $N=90$ suggests that the above explanation must be combined with other considerations, and there is a strong suggestion[5] of the participation of the effect of deviations from spherical symmetry for such nuclei. It is not intended, therefore, to explain all of the anomalies as an effect of a change in the penetrability of the core to the neutron wave function. The fact that the calculated isotopic shift is usually larger than the observed fits with the proposed outlying character of the $1i_{13/2}$ shell for $N=126$; the SIS in most of the isotopes Pb, Hg, Tl has presumably to do with the addition of i neutrons and the usual calculations assume a uniform density model of the nucleus, overestimating the expected effect. The general possibility of such a connection has already been noted by Kopfermann. The considerations on the effect of the centrifugal barrier presented above increase the probability of this view. For stable elements with $Z\cong 50$, the above picture when combined with the Bethe-Levinger-Courant view[6] of $\gamma-n$ and $\gamma-p$ reactions gives an increase in $\sigma(\gamma-n)/\sigma(\gamma-p)$ for these elements in comparison with the value of this quantity for lighter elements.[7] A proton in the nuclear core has to move through an envelope of neutrons before emergence, so that there results an increased probability of neutron emission as a result of collisions.

An obvious objection to the explanation of irregularities in the SIS is found in the fact that the spin of Pb^{207} is 1/2 and that this value cannot be explained as the spin of the $i_{13/2}$ shell with one missing neutron. On the other hand this objection disappears if one supposes that in Pb^{207} a neutron from an inner shell, perhaps the $3s_{1/2}$ shell, has been promoted to the $1i_{13/2}$ shell. The observed direction of staggering of the SIS could then be thought of as being partially caused by a decrease in nuclear volume produced by promoting the neutron from a penetrating to an external orbit. Since there is another way[5] of explaining odd-even SIS staggering, a quantitative consideration appears to be too difficult at this time.

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jj-Coupling in Nuclei

B. H. FLOWERS*
Department of Mathematical Physics, The University, Edgbaston, Birmingham, England
(Received February 21, 1952)

ACCORDING to the Mayer-Jensen spin-orbit coupling shell model of nuclear structure,[1,2] the states of a single nucleon moving in the average field of the rest of the nucleus are defined by the quantum numbers $nljm$, the degeneracy in j being destroyed by strong spin-orbit forces. One is, therefore, led to investigate the splitting of the states of the nuclear configuration $(l_j)^N$ under the influence of central forces considered as a perturbation.