tion. The magnetic 2^4-pole assignment gives the best agreement between the measured half-life and the theoretical half-life calculated from Weisskopf's formula. Table II shows this agreement. Thus, from the agreement between theory and experiment is is reasonable to conclude that the multipolarity of the electromagnetic radiation from the decay of the 0.39-Mev isomeric state of In^{113*} is magnetic 2^4-pole. This assignment predicts an angular momentum change of ± 4 units and a change of parity as shown in the decay scheme of D. A. Thomas *et al.*[4]

It is thought that the comparison method for measuring conversion coefficients, described in this paper, may offer better ultimate accuracy than any other method now used for the determination of the conversion coefficient of an isomeric transition which is not accompanied by a beta-spectrum. The accuracy of the method should be considerably increased by the use of a hollow crystal scintillation spectrometer[11] to reduce the effect of electron scattering out of the crystal.

TABLE II. Half-life of 0.39-Mev radiation from In.

In^{113*}	Half-life Electric 2^5	Electric 2^4	Magnetic 2^4
Weisskopf theory	4.9×10^5 min	0.8 min	$6.6\ \times10^2$ min
Measured			1.04×10^2 min

Note added in proof:—A recent magnetic beta-ray spectrum of the Cs^{137} used in this work indicates the presence of several (upper limit 7 or 8) percent Cs^{134}. Almost if not all of the Cs^{137} discrepancy can be accounted for by the extra Compton electrons from the Cs^{134} gamma-rays.

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The Shapes of Negative Beta-Spectra at the Low Energy End*

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The disposition of the excess atomic binding energy liberated in a negative beta-decay is of interest in connection with the shape of the low energy end of the beta-spectrum. Previous theoretical work points to the carrying away of this energy by the beta-particle in most cases, with a resulting practical cutoff at the low end of the energy spectrum. However, the present approach, in which the nuclear and atomic electron transitions are treated as a unified process, leads to the conclusion that to the first order that shape is practically unaffected. A possibility for experimental study of this question is pointed out.

I. INTRODUCTION

THIS note is concerned with the following question: What experimentally verifiable predictions can be made at present regarding the low energy end shapes of the spectra of negative beta-emitters for which the beta-decay theory of the corresponding bare nucleus can be taken as known? This question should apply, in the light of presently accumulated evidence, at least to the case of beta-transitions of the theoretically allowed type.

The condition of "experimental verifiability" cannot be applied, of course, with any high degree of precision. It does, however, delimit our problem within certain practical bounds. Thus, it appears to be hardly within the scope of present measuring techniques to determine reliably the effect upon spectral shapes of such second-order processes associated with beta-decay as the so-called inner bremsstrahlung,[1] or the excitation and ionization of the system of atomic electrons surrounding the decaying nucleus.[2] It is possible, in fact, that by thus limiting our discussion only to what can be expected to admit of a reliable check by present experimental means, only two effects have to be considered in connection with the above question: the effect of "screening" by the atomic electrons of the nuclear Coulomb field and the effect of the readjustment of the system of atomic electrons to the changed nuclear charge resulting from the beta-decay. These two effects are obviously not completely independent, but for our purpose they can be treated as such.[3]

The effect of atomic electron screening can be incorporated in the Fermi theory, at least in principle, to any accuracy warranted by the experimental potentialities, inasmuch as this is accomplished by employing as wave functions for the beta-particle the continuum

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[1] L. Madansky and F. Rasetti, Phys. Rev. **83**, 187 (1951). This is the latest published report on this effect and deals with its experimental determination; it contains references to the theoretical and earlier experimental work on this problem.

[2] L. Goldstein, J. phys. et radium **8**, 235 and 316 (1937); E. L. Feinberg, J. Phys. (U.S.S.R.) **4**, 423 (1941); A. Migdal, J. Phys. (U.S.S.R.) **4**, 449 (1941). These are theoretical papers. No experimental work on this effect has as yet been announced.

[3] Provided the formulation proposed in this note is essentially correct.

eigenfunctions for the Coulomb potential due to the combined nuclear and atomic electron charge systems of the daughter atom. It is known that this effect is very small in the case of negative beta-decay.[4]

The other effect, that connected with the transition between the ground states of the atomic electron system corresponding to the change in nuclear charge, will be discussed in Sec. II. The conclusion is reached that to the first order the shape of the beta-spectrum is hardly affected. This conclusion is at variance with that to be gathered from previous work related to this problem.[5] Although presumably the approach in each of these papers is different,[6] the results can be seen to imply the following common conclusion: to the first order, the energy

$$D_Z = E_{Z+1} - E_Z \tag{1}$$

(E_Z= numerical binding energy of an atom of atomic number Z), which is liberated in the atomic electron system transition in question,[7] is carried away by the departing beta-particle. From this conclusion it follows that, practically, there should appear a cutoff at the low energy end of negative beta-spectra corresponding to this energy D_Z.

The above conclusion can be readily shown to be a consequence of the assumption, which is tacitly made (except for the first reference 5) in the quoted papers, to the effect that the atomic electron readjustment accompanying beta-decay is a process essentially independent of the nuclear transition and can be considered as taking place subsequent to that transition. Inasmuch as the neutrino has practically no direct coupling with charged particles, one has only to show that the probabilities for electromagnetic radiation or for direct nonradiative absorption of the energy D_Z by the nuclear charges are relatively small, at least in the case of Z large enough for D_Z to be measurably appreciable.

The two alternative conclusions regarding the first-order disposition of the energy D_Z, namely, its being shared by the neutrino and the beta-particle (as is implied by the treatment given in Sec. II), or solely by the beta-particle, are strikingly illustrated by the corresponding consequences regarding the shape of the beta-spectrum of Pu^{241}. On the second view, since D_Z is in this case very nearly of the same order of magnitude as its reported maximum energy, the shape should be practically that of a line spectrum about the end point; whereas on the first view, this shape should be practically that of an allowed, high Z, and small end-point spectrum and thus having an ordinate which increases continuously towards the zero energy point. Recent measurements of the spectrum of Pu^{241} are consistent with this latter shape.[8] There are, of course, other nuclides besides Pu^{241} which are suitable for a test of the question under discussion, and beta-spectrum measuring techniques developed recently make such a test entirely feasible. Moreover, there are already in the literature some relevant measurements and these (with one possible exception), as far as they can be relied upon, appear in fact to confirm the conclusion which is reached in this note, at least in what concerns the non-existence of a low energy end cutoff. A brief discussion of this published experimental work and an indication of the possibilities for pertinent proportional counter measurements is given in Sec. III.

II. THEORETICAL CONSIDERATIONS

As follows from the discussion of the preceding section, our problem is to determine in the first approximation the probability $P(W)dW$ for the emission by a given nuclide of a negative beta-particle into the kinetic energy range dW at W, when simultaneously the system of surrounding electrons makes a transition from the ground state of the initial atom to that of the daughter atom (singly ionized in its outer shell), it being assumed that the nuclear and atomic transitions can be treated as a combined process.

A precise formulation of this problem would involve the quantized fields of nucleons and leptons (electrons and neutrinos), the latter including the filled bound states corresponding to the available atomic electrons, and subject to proper Fermi and Coulomb couplings.[9] For our present purpose it is possible that it will suffice to employ a phenomenological approach, which corresponds in some respects to the reasoning which enters in the treatment of the atomic electron screening effect. We simply assume that the Fermi interaction operator which involves the change of a neutron into a proton, serves also to change all wave functions pertaining to the nuclear charge specified by Z into the corresponding wave functions related similarly to to $(Z+1)$. It then follows that in forming the first-order matrix element corresponding to the combined nuclear and atomic transitions in question, we obtain the product of the usual matrix element for the beta-transition and of the integral

$$\int \psi_{Z+1}{}^* \psi_Z d\tau, \tag{2}$$

where ψ_Z, ψ_{Z+1} are the wave functions of the system of

[4] J. R. Reitz, Phys. Rev. **77**, 10 (1950) (contains references to previous papers).

[5] See L. Goldstein, reference 2, and M. H. Hebb, Physica **5**, 701 (1938); R. S. Ingarden, Acta Phys. Polon. **9**, 109 (1948).

[6] It is only in the second of the references in footnote 5 that the reasoning is explicit; the last reference merely contains mention of a relevant calculation, presumably unpublished, of J. Blaton "on the basis of Boltzmann's Virial Theorem"; while in the first reference there is an explicit statement of the assumption of independence of the nuclear and atomic transitions but the conclusion regarding the absorption of the energy D_Z by the departing beta-particle is announced without proof.

[7] The ionization energy of the outer shell of the daughter atom is quite neglible in comparison with D_Z for the Z values of interest to us.

[8] M. S. Freedman, Argonne National Laboratory (private communication).

[9] The transverse electromagnetic field could in general be left out of account in considering only first-order effects.

atomic electrons in the initial and final states, respectively.[10]

The conclusion stated in Sec. I, namely, that to the first order the spectral shape is not affected by the effect under discussion now follows from the following two observations. First, since Dirac perturbation theory here leads to conservation of energy in the combined transition, this energy being the sum of the excess in the nuclear energy levels and of D_Z as defined in (1), and since the atomic electron transition is between discrete states, it follows that the total energy in question is shared between the beta-particle and the neutrino. Secondly, the probability $P(W)dW$ which we are seeking will not be affected to any markedly appreciable extent by the atomic electron transition if the absolute square of the integral (2), which enters as a factor in its expression, differs sufficiently little from unity when ψ_{Z+1} corresponds to the ground state of the product atom. This can be shown indeed to be practically the case for sufficiently large Z.[11]

It is to be noticed that the absolute square of the integral (2) represents the probability for the corresponding atom electron transition on *sudden* perturbation theory. It can be seen indeed that it is consistent with the preceding considerations to think of the combined process in question as a beta-decay accompanied by the atomic readjustment in a time that is compatible with the application of sudden perturbation theory. The essential thing, however, is to account for the disposition of the energy D_Z which is involved in the transition. Sudden perturbation theory in itself does not involve directly any energy considerations and is, in fact, limited in this respect by the time-energy uncertainty relation.

It needs perhaps to be stated that the possibility cannot be ruled out that the simple treatment here given is not adequate and that a strict field-theoretical approach would bring out significant deviations from the shape predicted here, even in the first approximation. Before there is definite experimental evidence pointing in that direction, however, there would perhaps be scant justification for undertaking any highly involved investigation of this sort.

III. EXPERIMENTAL CONSIDERATIONS

There are so far three published results which are perhaps pertinent to our problem.

The cloud-chamber measurements of Richardson and Leigh-Smith[12] of the spectrum of RaD in the form of gaseous lead tetramethyl would be entirely conclusive, at least as to the existence of a low energy end beta-spectrum cutoff, provided we could safely rely on such measurements when they involve track lengths of the order of a millimeter and less. These investigators conclude by an analysis of their photographs, that more than 50 percent of the RaD beta-spectrum lies below 4 kev. Since the value of D_Z for RaD is around 15 kev,[13] the implication of this result, if valid, is quite obvious. Another published result perhaps pointing in the same direction is provided by the measurements of Langer, Motz, and Price[14] of the beta-spectrum of Pm^{147}. They find a straight Kurie plot down to 8 kev and possibly in the case of their windowless counter measurements even down to 6 kev, whereas D_Z for promethium is close to 10 kev.[13] On the other hand, the cloud-chamber investigation of the low energy end beta-spectrum of RaE by Waltner and Rogers[15] appears to confirm the "cutoff" conclusion. Again, the reliability of these low energy measurements must be left an open question.

Of the experimental techniques that are currently being perfected and which may be suitable for the study of appropriate beta-spectra at energies below D_Z, perhaps the most promising at the present time is that involving the use of proportional counters with gaseous sources. The substantial improvements in proportional counter low energy electron spectroscopy developed recently in a number of laboratories, notably at the University of Glasgow,[16] would indeed appear to make the proportional counter a suitable instrument for a completely reliable study of our problem. True, the number of nuclides which can be used to that end with this method is limited. However, unless it turns out that the negative beta-spectral shape at the low energy end is essentially distinct from that so far predicted, it would suffice for our purpose to have such reliable measurements in only a few cases. One example of a nuclide which possibly presents an optimum combination of desirable properties is provided by Ni^{63} which can be used in the form of gaseous nickel carbonyl. Plans for making proportional counter measurements of the low energy end of the spectrum of Ni^{63} are now in progress at the University of Arkansas.[17]

[10] Related discussion is to be found in Feinberg's paper listed under reference 2, although the problem treated there is not the one discussed here.

[11] This is made plausible by the calculations given in the papers listed in reference 2. Results of more extensive calculations that bear out this conclusion will be presented shortly for publication in the *Journal of Chemical Physics*.

[12] H. O. W. Richardson and A. Leigh-Smith, Proc. Roy. Soc. (London) **A160**, 454 (1937).

[13] See, for instance, L. L. Foldy, Phys. Rev. **83**, 397 (1951). An experimental check sufficient for our purpose was obtained by using x-ray level data.

[14] Langer, Motz, and Price, Phys. Rev. **77**, 798 (1950).

[15] A. W. Waltner and F. T. Rogers, Phys. Rev. **75**, 1445 (1949).

[16] A. L. Cockroft and S. C. Currant, Rev. Sci. Instr. **22**, 37 (1951); G. M. Insch and S. C. Curran, Phil. Mag. **42**, 892 (1951). These papers contain references to earlier work.

[17] R. R. Edwards and P. E. Damon (private communication). I wish to express my appreciation of the many stimulating discussions on this subject with Professor R. R. Edwards.