polar coordinates) are given in all three cases by an equation of the form

$$\pm(nA/r_0)J_n(\chi_1r_0)J_n(\chi_2r_0)=\alpha_2J_n(\chi_1r_0)J_n'(\chi_2r_0)-\alpha_1J_n(\chi_2r_0)J_n'(\chi_1r_0),\quad (1)$$

where in case (1)

$$A=\frac{\beta^2}{\beta_0}\frac{p_2^2-p_1^2}{\zeta_1\zeta_2},\quad \alpha_s=\frac{\chi_s}{\zeta_s}[\beta^2-\beta_0^2+2\zeta_sR_pH_0-4R_p^2H_0^2]\quad (s=1,2),$$
$$\delta=(1-\mu_z/\mu),\quad R_p=(\alpha_H\beta_0/2\mu)(M_0/H_0),$$

and in cases (2) and (3)

$$A=(p_1^2-p_2^2)\beta_0,\quad \alpha_s=\chi_s[(\beta_0^2-\beta^2)\zeta_s+2\beta^2R_pH_0]\quad (s=1,2),$$
$$\delta=1-\epsilon_z/\epsilon,\quad R_p=\alpha_E\beta_0/2\epsilon,$$

and where for all three cases

$$\zeta_s=[(\beta^2-\beta_0^2)\delta-p_s^2]/2R_pH_0,\quad \chi_s^2=\beta_0^2-\beta^2+p_s^2\quad (s=1,2),$$
$$p_{1,2}^2=-2R_p^2H_0^2-\frac{\delta}{2}(\beta_0^2-\beta^2)\pm2R_pH_0\times\left\{\beta^2-\beta_0^2\delta+\frac{1}{4}\left[2R_pH_0+\frac{\delta}{2R_pH_0}(\beta_0^2-\beta^2)\right]^2\right\}^{\frac{1}{2}}.$$

Equation (1) gives the β's for both TE limit and TM limit modes. For small H_0, these β's are easily found by expansion. It turns out that the deviations $\Delta\beta\pm n;m$ from the β_{nm} of the TE_{nm} waves due to a small magnetic field H_0 [or to a small magnetization in case (1)] are proportional to H_0. The corresponding rotation $|\Delta\beta_{nm}/n|$ can, therefore, be specified by a new Verdet's constant. In case (1)

$$R_{\text{guide}}^{(nm)}=2\lambda_0R_p/[(u_{nm}^2-n^2)\lambda_g],\quad (2)$$

and in cases (2) and (3)

$$R_{\text{guide}}^{(nm)}=2\lambda_gR_p/[(u_{nm}^2-n^2)\lambda_0],\quad (3)$$

where λ_g is the guide wavelength of the TE_{nm} mode at zero H_0, λ_0 the wavelength in the infinite medium, and u_{nm} is the mth zero of $J_n'(x)$.

Formulas (2) and (3) show that the guide causes dispersion in R additional to that of R_p. They have meaning only in the loss free case. When there is loss, the $\Delta\beta\pm nm$ have opposite imaginary parts leading to progressive conversion from linear to circular polarization (accompanied by rotation). Equations (2) and (3) obviously fail very near resonances. The full equation (1) has then to be solved. Formula (3) also fails near cutoff ($\lambda_g=\infty$). These, and other matters are to be discussed in a later paper. The authors are indebted to Dr. C. L. Hogan and Dr. A. D. Perry of these Laboratories for discussion of their work on ferrites, and for acquainting the authors with their theoretical work on transverse H_0.

[1] D. Polder, Phil. Mag. **40**, 99–115 (1949); C. L. Hogan, Bell System Tech. J. **31**, 1 (1952).
[2] Goldstein, Lampert, and Heney, Phys. Rev. **82**, 956 (1951).
[3] E. R. Wicher, J. Appl. Phys. **22**, 1327 (1951).
[4] C. Kittel, Phys. Rev. **71**, 270 (1947); **73**, 155 (1948).

Radiation from an Electron in a Magnetic Field

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THIS problem was recently considered by Parzen[1] who concluded that quantum corrections to the classical results of Schwinger[2] and Schiff[3] should be appreciable at an electron energy of 200 mc^2 in a field of 10^4 gauss. The form of his correction is such that for this magnetic field the energy loss per turn by radiation would only increase as $R^{\frac{1}{3}}$ with increasing energy, instead of increasing as E^4/R, thus removing the stringent radiation limitation on synchrotron design.

An examination of Parzen's calculation reveals an invalid approximation.[4] More significantly, the assertion that on $l=0$ to $l=0$ transitions are appreciable can be shown to be incorrect by directly summing the series [Eq. (36)]. This yields

$$I(n'l|n0)=\frac{(-\alpha^2/2)^{l/2}}{\sqrt{l!}}I(n'0|n0)\quad (1)$$

in which the important values of $(\alpha^2/2)$ are of order unity. This result may be verified more easily by use of the energy eigenfunctions in Cartesian coordinates,[5] by the use of which the summation over l is implicitly performed.

If one now examines the formula for the power radiated in the orbital plane and makes use of these results, the exponential correction factor in Eqs. (26) cancels out and the classical result is obtained as sketched in the following:

$$I(n'0|n0)=(n'!/n!)^{\frac{1}{2}}(-\alpha^2/2)^{\lambda/2}\exp(-\alpha^2/2)L_{n'}{}^{\lambda}(\alpha^2/2)\quad (2a)$$
$$=\frac{(-1)^{\lambda/2}}{(n!n'!)^{\frac{1}{2}}}\int^{\infty}dte^{-t}t^{n'+\lambda/2}J_\lambda(\alpha(2t)^{\frac{1}{2}}).\quad (2b)$$

Combining Eq. (2b) with the relation[6]

$$J_\lambda(\alpha(2t)^{\frac{1}{2}})=(1+\tau/n)^{\lambda/2}\sum_{m=0}^{\infty}\frac{(-\alpha\tau)^m}{(2n)^{m/2}m!}J_{\lambda+m}(\alpha(2n)^{\frac{1}{2}}),\quad (3)$$

where $t=n+\tau$, one obtains

$$I(n'0|n0)\simeq\frac{(-1)^{\lambda/2}}{(2\pi n)^{\frac{1}{2}}}\exp(-\lambda^2/4n')\int_{-n}^{\infty}d\tau\exp(-\tau^2/2n)\times\sum_{m=0}^{\infty}\frac{(-\alpha\tau)^m}{(2n)^{m/2}m!}J_{\lambda+m}(\alpha(2n)^{\frac{1}{2}}).\quad (4)$$

The major contribution to this integral comes in the region $|\tau|<(2n)^{\frac{1}{2}}$. Also, $\alpha(2n)^{\frac{1}{2}}=\lambda\beta$ in the orbital plane. In the summation, only about α-terms contribute so that the relevant values of m are of order $\alpha=\lambda\beta/(2n)^{\frac{1}{2}}\ll\lambda$. For these m, $J_{\lambda+m}(\lambda\beta)=J_\lambda(\lambda\beta)$ for $1-\beta^2\ll1$, so that (4) becomes

$$\frac{(-1)^{\lambda/2}J_\lambda(\lambda\beta)\exp(-\lambda^2/4n')}{(2\pi n)^{\frac{1}{2}}}\int_{-\infty}^{\infty}\exp[-(\tau^2/2n)-\alpha\tau/(2n)^{\frac{1}{2}}]d\tau\simeq\exp(-\alpha^2/4)J_\lambda(\lambda\beta).$$

The power radiated in the orbital plane is proportional to the sum

$$\sum_{l=0}^{\infty}|I(n'l|n+1,0)+I(n'l|n-1,0)|^2,$$

which may now be evaluated using Eq. (1) to give $[2J_\lambda'(\lambda\beta)]^2$, the classical result.

It is a pleasure to acknowledge helpful discussions with Professor E. M. McMillan, who brought this matter to our attention. We also wish to thank Professor L. I. Schiff for sending us his wave packet arguments[7] which confirms Schwinger's criterion for validity of the classical approximation.

[1] G. Parzen, Phys. Rev. **84**, 235 (1951).
[2] J. Schwinger, Phys. Rev. **75**, 1912 (1949).
[3] L. I. Schiff, Rev. Sci. Instr. **17**, 8 (1946).
[4] Following his Eq. (23), we should have $[(n+\lambda)!/n!]^{\frac{1}{2}}\simeq n^{\frac{1}{2}\lambda}\exp(\lambda^2/4n)$, from the Stirling approximation; terms of order $\lambda^3/n^2\approx\alpha^3/n^{\frac{1}{2}}$ are neglected since $n\sim10^{14}$.
[5] M. H. Johnson and B. A. Lippman, Phys. Rev. **76**, 828 (1949), Eq. (43).
[6] G. N. Watson, *Theory of Bessel Functions* (Macmillan Company, New York, 1944), p. 141, Eq. 5.
[7] L. I. Schiff, Am. J. Phys. (to be published).

Radiation Loss of Electrons in the Synchrotron

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IN a recent paper of Parzen,[1] the radiation of electrons in uniform circular motion was calculated by means of the exact wave functions in a homogeneous magnetic field.

The result would seem to indicate a total radiation loss considerably smaller than to be expected on classical theory.[2,3]

This is very surprising since the main loss is due to the soft quanta, i.e., emissions which cause a relatively small change of the enormous quantum numbers involved in an orbit of macroscopic size at these energies. By the correspondence principle it is