unless the spins are somehow oriented. If the final state is a deuteron, then it is a mixture of $S$ and $D$ states and there can be some spin orientation, the reference direction being provided by the initial $D$ state. This leads the angular distribution of the emitted mesons to be anisotropic. Whether the correct amount of anisotropy is obtained, depends on the fraction of $D$ state in the deuteron, as well as on the details of the transition matrix elements to the high energy $S$ and $D$ states. If the final state is a double neutron, resulting from $n+p\rightarrow 2n+\pi^+$, then at low energy it must be in a $^1S_0$ state; then the initial state can only be $^3S_1$, and an anisotropy can only exist to the extent that the high energy $^3S_1$ state spins are oriented by the tensor force coupling to $^3D_1$. In the notation of Rohrlich and Eisenstein,[3] the resultant average square matrix element becomes

$$\begin{aligned}&[U_\alpha^2+(\eta_1^\alpha)^2U_\gamma^2][1+(\eta_1^\alpha)^2]^{-1}+\tfrac{1}{2}(3\cos^2\theta-1)[1+(\eta_1^\alpha)^2]^{-2}\\&\times\Big\{(\eta_1^\alpha)^2[U_\alpha^2+U_\gamma^2-2U_\alpha U_\gamma\cos(\delta_\alpha-\delta_\gamma)]\\&-8^{\frac{1}{2}}\Big[\eta_1^\alpha U_\alpha^2+\eta_1^\gamma U_\gamma^2\Big(\frac{1+(\eta_1^\alpha)^2}{1+(\eta_1^\gamma)^2}\Big)^2\\&\qquad+(\eta_1^\alpha+\eta_1^\gamma)U_\alpha U_\gamma\cos(\delta_\alpha-\delta_\gamma)\Big(\frac{1+(\eta_1^\alpha)^2}{1+(\eta_1^\gamma)^2}\Big)\Big]\Big\}, \quad (1)\end{aligned}$$

where

$$U_{\alpha,\gamma}\equiv\int_0^\infty u_{\alpha,\gamma}u_F dr, \quad \text{and} \quad \eta_1^\alpha\eta_1^\gamma=-1,$$

and $u_F$ is the "radial" wave function of the low energy $^1S$ state. The above expression is sensitive to the nuclear dynamics but does not appear to lead to more than about a twenty percent $\cos^2\theta$-term in the angular distribution, if reasonable dynamical guesses are made for the various quantities in it.

For Theory I an almost isotropic distribution must therefore be expected for the charged mesons resulting from an $n$-$p$ collision, contrary to the distribution observed for $p$-$p$ collisions. The isotropy will hold particularly for the fastest mesons which can be produced for a given energy of the incident neutron. Fortunately, the energy spectrum of the mesons will have the now-familiar high energy peak. This peak is expected because of the attraction which two low energy neutrons bear for each other and will be reinforced because the meson production matrix element rises with meson energy. The peak itself will not be visible, however, without the use of a fairly monochromatic neutron beam, such as the beam which may be obtained from charge-exchange $p+d$ scattering.[4,5]

In Theory II, the angular distribution of the mesons will be essentially the same for $n$-$p$ as for $p$-$p$ production, because here the meson momentum is directly coupled to the nucleon momentum. To take into account both types of coupling let us consider a transition matrix of the form

$$T'=(\{\boldsymbol{\sigma}_1\tau_1^++\boldsymbol{\sigma}_2\tau_2^+\}\cdot\mathbf{q})+A(\{\boldsymbol{\sigma}_1\tau_1^++\boldsymbol{\sigma}_2\tau_2^+\}\cdot\mathbf{p}/\mu c)(\mathbf{q}\cdot\mathbf{p}/\mu c). \quad (2)$$

Taking the nucleon final state to be the $^1S$ state and the high energy state to be the plane wave, $e^{i(\mathbf{k}\cdot\mathbf{r})}$, the matrix element is

$$\begin{aligned}T_{AF}'&=(e^{i(\mathbf{k}\cdot\mathbf{r})}\chi_1^m,\,T'u_F\chi_0^0)\\&=\tfrac{1}{2}(e^{i(\mathbf{k}\cdot\mathbf{r})}\chi_1^m,\,[(\{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2\}\cdot\mathbf{q})+A(\hbar/\mu c)^2\\&\qquad\times(\{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2\}\cdot\mathbf{k})(\mathbf{k}\cdot\mathbf{q})]u_F\chi_0^0).\end{aligned}$$

Then

$$\begin{aligned}\Sigma_m|T_{AF}'|^2&=U^2[1+2A(\hbar k/\mu c)^2\cos^2\theta+A^2(\hbar k/\mu c)^4\cos^2\theta],\\U&\equiv\int_0^\infty u_F\sin kr dr. \quad (3)\end{aligned}$$

While field theory leads to a coefficient $A$ which is small, of the order $(\mu/M)^2$, it is in the spirit of Theory II to treat $A$ as a free parameter. Indeed, if II is the only mechanism responsible for the $p$-$p$ anisotropy, then the above expression represents an exactly equal $n$-$p$ anisotropy.

Production of charged mesons by collisions between neutrons and protons thus provides a sensitive test for the type of coupling between meson field and nucleons. Near-isotropy would favor Theory I, while a strong anisotropy would indicate some such theory as II. It would probably be helpful to have an approximately monoenergetic neutron beam and to observe the angular distribution of the high energy mesons.

The total cross section can be estimated from the $p$-$p$ cross section, in terms of the relative amplitudes which the final state nuclear wave functions have at the origin.

$$\sigma_{np}\approx\int dE_F\,\rho(E_F)\left[\left(\frac{\partial u_F}{\partial r}\right)_{r=0}\bigg/\left(\frac{\partial u}{\partial r}\right)_{r=0}\right]^2\sigma_{pp}.$$

For a total kinetic energy of 50 Mev in the final state, the integrated $n$-$p$ production cross section is about equal to the $p$-$p$ production cross section. This estimate is probably more satisfactory for Theory I, which does not involve the uncertain nucleon momenta so strongly as does Theory II.

* AEC Postdoctoral Fellow.


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## Faraday Rotation of Guided Waves


H. Suhl and L. R. Walker
*Bell Telephone Laboratories, Murray Hill, New Jersey*


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RECENTLY several means of causing microwave Faraday rotation (F.R.) have been proposed: (1) the gyromagnetic resonance in ferrites,[1] (2) the cyclotron resonance of electrons in a discharge plasma,[2] and (3) the Hall effect in conductors or semiconductors.[3] Experimental evidence of the rotation is most readily obtained by placing these agents into a circular waveguide carrying the lowest order transverse electric mode ($TE_{11}$). A steady magnetic field $H_0$ along the guide axis $0z$ then causes rotation of the field pattern about the guide axis.

However, the standard treatment of F.R. deals with plane waves of infinite extent, hence is inapplicable to guided waves. We have established the natural modes of propagation in a guide filled with any one of the proposed media. These modes are circularly polarized and are never purely transverse electric or magnetic, but can be classified according to whether their limits as $H_0\rightarrow 0$ are $TE$ or $TM$. Right-circular and left-circular waves proceed down the guide with different propagation constants given by a transcendental equation. For small $H_0$, and for waves with $TE$ limit, the difference, as in the elementary theory, leads to a rotation of the plane of polarization of an initially plane polarized wave. The rotation in radians per gauss per unit length (Verdet's constant $R$) equals that in the unbounded medium $R_p$, times a factor depending on the propagation characteristics of the guide at $H_0=0$. The rotation of $TM$ limit modes varies as a higher power of $R_pH_0$ for small $H_0$, so that there is no Verdet's constant in the usual sense.

The permeability in case (1), and the dielectric constant in cases (2) and (3) are tensors of the form

$$\begin{bmatrix}\mu & -i\alpha_H M_0 & 0\\ i\alpha_H M_0 & \mu & 0\\ 0 & 0 & \mu_z\end{bmatrix};\quad\begin{bmatrix}\epsilon & -i\alpha_E H_0 & 0\\ i\alpha_E H_0 & \epsilon & 0\\ 0 & 0 & \epsilon_z\end{bmatrix}$$

respectively. $M_0$ is the magnetization due to $H_0$. The $\mu$'s, $\epsilon$'s and $\alpha$'s may depend on the frequency $\omega$, and except for $\mu_z$, $\epsilon_z$ may be function of $|H_0|$. They may have imaginary parts to allow for losses. In case (1), the $\mu$'s and $\alpha$ may also depend on sample shape.[4] All these quantities are assumed independent of position.

Verdet's constant for a plane wave in the unbounded medium progressing along $0z$ is given by $R_p=\alpha_H\beta_0M_0/2\mu H_0$ in case (1) and $R_p=\alpha_E\beta_0/2\epsilon$ in cases (2) and (3), where $\beta_0=\omega(\mu\epsilon)^{\frac{1}{2}}$. In general $R_p$ may be field and frequency dependent.

The possible propagation constants $\beta$ for the modes that vary as $e^{\pm in\varphi-i\beta z}$ with polar angle $\varphi$ and axial distance $z$ (in cylindrical polar coordinates) are given in all three cases by an equation of the form

$$\pm(nA/r_0)J_n(\chi_1 r_0)J_n(\chi_2 r_0)=\alpha_2 J_n(\chi_1 r_0)J_n'(\chi_2 r_0)-\alpha_1 J_n(\chi_2 r_0)J_n'(\chi_1 r_0), \quad (1)$$

where in case (1)

$$A=\frac{\beta^2}{\beta_0}\frac{p_2^2-p_1^2}{\zeta_1\zeta_2}, \quad \alpha_s=\frac{\chi_s}{\zeta_s}[\beta^2-\beta_0^2+2\zeta_s R_p H_0-4R_p^2H_0^2] \quad (s=1,2),$$
$$\delta=(1-\mu_z/\mu), \quad R_p=(\alpha_H\beta_0/2\mu)(M_0/H_0),$$

and in cases (2) and (3)

$$A=(p_1^2-p_2^2)\beta_0, \quad \alpha_s=\chi_s[(\beta_0^2-\beta^2)\zeta_s+2\beta^2R_pH_0] \quad (s=1,2),$$
$$\delta=1-\epsilon_z/\epsilon, \quad R_p=\alpha_E\beta_0/2\epsilon,$$

and where for all three cases

$$\zeta_s=[(\beta^2-\beta_0^2)\delta-p_s^2]/2R_pH_0, \quad \chi_s^2=\beta_0^2-\beta^2+p_s^2 \quad (s=1,2),$$
$$p_{1,2}^2=-2R_p^2H_0^2-\frac{\delta}{2}(\beta_0^2-\beta^2)\pm 2R_pH_0 \times\left\{\beta^2-\beta_0^2\delta+\frac{1}{4}\left[2R_pH_0+\frac{\delta}{2R_pH_0}(\beta_0^2-\beta^2)\right]^2\right\}^{\frac{1}{2}}.$$

Equation (1) gives the $\beta$'s for both $TE$ limit and $TM$ limit modes. For small $H_0$, these $\beta$'s are easily found by expansion. It turns out that the deviations $\Delta\beta\pm n;m$ from the $\beta_{nm}$ of the $TE_{nm}$ waves due to a small magnetic field $H_0$ [or to a small magnetization in case (1)] are proportional to $H_0$. The corresponding rotation $|\Delta\beta_{nm}/n|$ can, therefore, be specified by a new Verdet's constant. In case (1)

$$R_{\text{guide}}{}^{(nm)}=2\lambda_0R_p/[(u_{nm}^2-n^2)\lambda_g], \quad (2)$$

and in cases (2) and (3)

$$R_{\text{guide}}{}^{(nm)}=2\lambda_gR_p/[(u_{nm}^2-n^2)\lambda_0], \quad (3)$$

where $\lambda_g$ is the guide wavelength of the $TE_{nm}$ mode at zero $H_0$, $\lambda_0$ the wavelength in the infinite medium, and $u_{nm}$ is the $m$th zero of $J_n'(x)$.

Formulas (2) and (3) show that the guide causes dispersion in $R$ additional to that of $R_p$. They have meaning only in the loss free case. When there is loss, the $\Delta\beta\pm nm$ have opposite imaginary parts leading to progressive conversion from linear to circular polarization (accompanied by rotation). Equations (2) and (3) obviously fail very near resonances. The full equation (1) has then to be solved. Formula (3) also fails near cutoff ($\lambda_g=\infty$). These, and other matters are to be discussed in a later paper. The authors are indebted to Dr. C. L. Hogan and Dr. A. D. Perry of these Laboratories for discussion of their work on ferrites, and for acquainting the authors with their theoretical work on transverse $H_0$.

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## Radiation from an Electron in a Magnetic Field

D. L. Judd, J. V. Lepore, M. Ruderman, and P. Wolff
*Radiation Laboratory, Department of Physics, University of California, Berkeley, California*
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THIS problem was recently considered by Parzen[1] who concluded that quantum corrections to the classical results of Schwinger[2] and Schiff[3] should be appreciable at an electron energy of 200 $mc^2$ in a field of $10^4$ gauss. The form of his correction is such that for this magnetic field the energy loss per turn by radiation would only increase as $R^{\frac{1}{2}}$ with increasing energy, instead of increasing as $E^4/R$, thus removing the stringent radiation limitation on synchrotron design.

An examination of Parzen's calculation reveals an invalid approximation.[4] More significantly, the assertion that on $l=0$ to $l=0$ transitions are appreciable can be shown to be incorrect by directly summing the series [Eq. (36)]. This yields

$$I(n'l|n0)=\frac{(-\alpha^2/2)^{l/2}}{\sqrt{l!}}I(n'0|n0) \quad (1)$$

in which the important values of $(\alpha^2/2)$ are of order unity. This result may be verified more easily by use of the energy eigenfunctions in Cartesian coordinates,[5] by the use of which the summation over $l$ is implicitly performed.

If one now examines the formula for the power radiated in the orbital plane and makes use of these results, the exponential correction factor in Eqs. (26) cancels out and the classical result is obtained as sketched in the following:

$$I(n'0|n0)=(n'!/n!)^{\frac{1}{2}}(-\alpha^2/2)^{\lambda/2}\exp(-\alpha^2/2)L_{n'}{}^{\lambda}(\alpha^2/2) \quad (2a)$$
$$=\frac{(-1)^{\lambda/2}}{(n!n'!)^{\frac{1}{2}}}\int^{\infty}dte^{-t}t^{n'+\lambda/2}J_\lambda(\alpha(2t)^{\frac{1}{2}}). \quad (2b)$$

Combining Eq. (2b) with the relation[6]

$$J_\lambda(\alpha(2t)^{\frac{1}{2}})=(1+\tau/n)^{\lambda/2}\sum_{m=0}^{\infty}\frac{(-\alpha\tau)^m}{(2n)^{m/2}m!}J_{\lambda+m}(\alpha(2n)^{\frac{1}{2}}), \quad (3)$$

where $t=n+\tau$, one obtains

$$I(n'0|n0)\backsimeq\frac{(-1)^{\lambda/2}}{(2\pi n)^{\frac{1}{2}}}\exp(-\lambda^2/4n')\int_{-n}^{\infty}d\tau\exp(-\tau^2/2n) \times\sum_{m=0}^{\infty}\frac{(-\alpha\tau)^m}{(2n)^{m/2}m!}J_{\lambda+m}(\alpha(2n)^{\frac{1}{2}}). \quad (4)$$

The major contribution to this integral comes in the region $|\tau|<(2n)^{\frac{1}{2}}$. Also, $\alpha(2n)^{\frac{1}{2}}=\lambda\beta$ in the orbital plane. In the summation, only about $\alpha$-terms contribute so that the relevant values of $m$ are of order $\alpha=\lambda\beta/(2n)^{\frac{1}{2}}\ll\lambda$. For these $m$, $J_{\lambda+m}(\lambda\beta)=J_\lambda(\lambda\beta)$ for $1-\beta^2\ll 1$, so that (4) becomes

$$\frac{(-1)^{\lambda/2}J_\lambda(\lambda\beta)\exp(-\lambda^2/4n')}{(2\pi n)^{\frac{1}{2}}}\int_{-\infty}^{\infty}\exp[-(\tau^2/2n)-\alpha\tau/(2n)^{\frac{1}{2}}]d\tau \backsimeq\exp(-\alpha^2/4)J_\lambda(\lambda\beta).$$

The power radiated in the orbital plane is proportional to the sum

$$\sum_{l=0}^{\infty}|I(n'l|n+1,0)+I(n'l|n-1,0)|^2,$$

which may now be evaluated using Eq. (1) to give $[2J_\lambda'(\lambda\beta)]^2$, the classical result.

It is a pleasure to acknowledge helpful discussions with Professor E. M. McMillan, who brought this matter to our attention. We also wish to thank Professor L. I. Schiff for sending us his wave packet arguments[7] which confirms Schwinger's criterion for validity of the classical approximation.

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## Radiation Loss of Electrons in the Synchrotron

H. Olsen and H. Wergeland
*Fysisk Institutt, Norges Tekniske Høgskale, Trondheim, Norway*
(Received January 14, 1952)

IN a recent paper of Parzen,[1] the radiation of electrons in uniform circular motion was calculated by means of the exact wave functions in a homogeneous magnetic field.

The result would seem to indicate a total radiation loss considerably smaller than to be expected on classical theory.[2,3]

This is very surprising since the main loss is due to the soft quanta, i.e., emissions which cause a relatively small change of the enormous quantum numbers involved in an orbit of macroscopic size at these energies. By the correspondence principle it is