Meson-Nucleon Scattering and Nucleon Isobars*

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The scattering (including charge exchange) of π^- mesons in hydrogen rises from 18 millibarns at 60 Mev to a broad plateau of about 60 millibarns at 200 Mev, and is smaller than the π^+ scattering at 60 Mev in the ratio of 0.63 ± 0.09. The general features of the π^- scattering, except for the high energy plateau, are given qualitatively by pseudoscalar theory with pseudovector coupling in the weak coupling limit; the ratio of π^- to π^+ scattering predicted by this theory in the weak coupling limit is, however, 1.67, which is much higher than the experimental result. A phenomenological theory of the scattering is developed by using the methods of Wigner and Eisenbud and imposing the restrictions of charge symmetry. By using the qualitative assignment of the resonance levels parameters as given by weak and strong coupling theory, satisfactory agreement with experiment is obtained. It is concluded that the apparently anomalous features of the scattering can be interpreted to be an indication of a resonant meson-nucleon interaction corresponding to a nucleon isobar with spin $\frac{3}{2}$, isotopic spin $\frac{3}{2}$, and with an excitation of 277 Mev.

I. INTRODUCTION

THE scattering[1] of π^- mesons in hydrogen has been studied experimentally by Anderson[2] at Chicago, and of π^- and π^+ mesons in hydrogen and deuterium by Sachs and Steinberger[3] at Columbia. The striking features of the results of these scattering experiments are

(a) the very rapid rise in the π^- total scattering cross section from 18 mb at 60 Mev to a broad plateau of 60 mb at about 200 Mev;

(b) the considerably larger scattering of π^+ than of π^- mesons in hydrogen at 60 Mev;

(c) the equality of the π^+ and π^- scattering in deuterium. The equality of the scattering in deuterium supports the hypothesis of charge symmetry, i.e., the (π^+, D) scattering is charge symmetric to the (π^-, D) scattering since it differs only in the sign of the meson charge. This important result is additional evidence that the neutron and proton are different charge states of a fundamental "nucleon" state. We shall not discuss the deuterium scattering further since it is the subject of a paper (to be published) by Isaacs, Sachs, and Steinberger.

The rapid rise in the π^- scattering cross section in hydrogen can be partially understood from the viewpoint of meson theory. From other experiments it is known that the meson has spin zero and is pseudoscalar; the scattering predicted by this theory with symmetric pseudovector coupling† is[4]

$$\begin{aligned} \frac{d\sigma}{d\Omega}(p+,p+)=\frac{d\sigma}{d\Omega}(p-,p-)&=\frac{g^4k^4}{\omega^2\mu^4},\\ \frac{d\sigma}{d\Omega}(n0,p-)&=\frac{2g^4k^4}{\omega^2\mu^4}\cos^2\theta, \end{aligned} \tag{1}$$

if terms of the order ω/M are dropped; g^2 is the coupling strength, k the meson momentum, ω the meson total energy, and μ the meson mass.[5] These cross sections increase indefinitely with energy, which is of course physically inadmissable. The total cross section for π^- scattering (with the inclusion of charge exchange) is larger than the π^+ scattering in the ratio of 5 to 3, which is also in disagreement with the experimentally observed ratio of 0.63.

The failure of the weak coupling predictions suggests that an explanation of these apparently anomalous scattering phenomena may lie in a theoretical treatment which does not make the weak coupling approximation. One of the most characteristic features of the strong coupling theory of the pseudoscalar meson with pseudovector coupling[6] is the prediction of the existence of spin and charge isobars corresponding to resonant interactions of the meson with the nucleon. It will be shown that the introduction from a phenomenological viewpoint[7] of such resonance phenomena in a form consistent with both weak and strong coupling theory leads to a correct prediction of the general features of the observed scattering.[8]

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[1] The experiments discussed measure only attenuation of a meson beam so that processes other than direct scattering occur, such as, in particular, the charge exchange scattering of π^- mesons in hydrogen.

[2] This work has been reported by H. L. Anderson in an invited paper at the Chicago Physical Society meeting, October 27, 1951, and has been submitted by Nagle, Anderson, Fermi, Long, and Martin in an abstract to the New York Physical Society meeting, January 31–February 2, 1952.

[3] A. Sachs and J. Steinberger, Phys. Rev. 82, 958 (1951) and post dead-line paper at Chicago Physical Society meeting, October 29, 1951.

† This form of coupling is also called charge independent.

[4] We shall refer to the π^+ and π^- scattering in hydrogen as $\sigma(p+,p+)$ and $\sigma(p-,p-)$, respectively, and to the charge exchange scattering of π^- mesons as $\sigma(n0,p-)$.

[5] We take units such that $\hbar=c=1$.

[6] W. Pauli and S. M. Dancoff, Phys. Rev. 62, 85 (1942).

[7] A phenomenological analysis of the type made in this paper has also been carried out for low energies by M. L. Goldberger, Phys. Rev. 83, 239 (1951) and by Yoshio Yamaguchi, Osaka City University (private communication). The author is indebted to Professor Yamaguchi for a copy of his manuscript in advance of publication.

[8] Neutral-photomeson production also can be interpreted to be an indication of the existence of nucleon isobars; see K. A.

II. PHENOMENOLOGICAL DESCRIPTION OF SCATTERING

A. Cross Section for Scattering[9]

When the meson coupling is not weak, the complexity of the meson-nucleon interaction at small distances makes a detailed analysis of the phenomenon very difficult. The considerations of this region can, however, be avoided by substituting boundary conditions on the meson wave function at a radius **a** at which a simpler description is possible.[10] In particular, if this radius is chosen so that the meson-nucleon interaction is negligible in the external region, then the scattering is determined completely by the behavior of the logarithmic derivative of the meson-wave function at the radius **a**.

We shall assume that, as is characteristic of pseudoscalar theory with pseudovector coupling in nonrelativistic approximation, the meson interacts only in P-states of orbital angular momentum. This follows from any gradient coupling of the form given by pseudoscalar theory, $(\boldsymbol{\sigma}\cdot\nabla\varphi)U$ (U the nucleon source density), if the meson wavelength is large compared with the source radius (usually assumed to be about $\hbar/Mc$). In addition, because of the possible coupling of nucleon-spin and meson-orbital momentum, we can in general expect different scattering in states of total angular momentum $J=\frac{1}{2}$ and $J=\frac{3}{2}$.

The states can also be characterized by a total isotopic angular momentum I in charge or isotopic space which is a constant of the system for a symmetric or charge independent theory.[11] We denote the meson-charge states by the eigenfunctions of the meson isotopic angular momentum $\varphi_1{}^M$, where $M=-1, 0, +1$ for positive, neutral, and negative pions, and the nucleon isotopic state by the isotopic spin function $t_{\frac{1}{2}}{}^{\alpha}$ where $\alpha=+\frac{1}{2}$ for neutrons and $-\frac{1}{2}$ for protons. The meson-nucleon states then are described by the wave functions in isotopic space $\varphi_1{}^M t_{\frac{1}{2}}{}^{\alpha}$. Expanding these states in the eigenfunctions $I_I{}^{I_z}$ of I and I_z, we have

$$
\begin{aligned}
\varphi_1{}^{-1}t_{\frac{1}{2}}{}^{-\frac{1}{2}} &= (\pi^+, p) \text{ state} \\
&= I_{\frac{3}{2}}{}^{-\frac{3}{2}}, \\
\varphi_1{}^{1}t_{\frac{1}{2}}{}^{-\frac{1}{2}} &= (\pi^-, p) \text{ state} \\
&= [I_{\frac{3}{2}}{}^{+\frac{1}{2}}+\sqrt{2}I_{\frac{1}{2}}{}^{\frac{1}{2}}]/\sqrt{3}, \\
\varphi_1{}^{0}t_{\frac{1}{2}}{}^{\frac{1}{2}} &= (\pi^0, n) \text{ state} \\
&= [\sqrt{2}I_{\frac{3}{2}}{}^{\frac{1}{2}}-I_{\frac{1}{2}}{}^{\frac{1}{2}}]/\sqrt{3}.
\end{aligned}
\tag{2}
$$

Writing the matrix elements for scattering from the various initial to final states as $(p+|p+)$, $(p-|p-)$, and $(n0|p-)$, we find directly from these wave functions, using the conservation of isotopic angular momentum during the scattering process, the relations

$$
\begin{aligned}
(p+|p+) &= (I_{\frac{3}{2}}|I_{\frac{3}{2}}), \\
(p-|p-) &= [(I_{\frac{3}{2}}|I_{\frac{3}{2}})+2(I_{\frac{1}{2}}|I_{\frac{1}{2}})]/3, \\
(n0|p-) &= [(I_{\frac{3}{2}}|I_{\frac{3}{2}})-(I_{\frac{1}{2}}|I_{\frac{1}{2}})]\sqrt{2}/3.
\end{aligned}
\tag{3}
$$

From these results we obtain the simple relations among the cross sections

$$
\begin{aligned}
d\sigma(p+,p+) &= d\sigma_{\frac{3}{2}}, \\
d\sigma(p-,p-) &= (1/9)[d\sigma_{\frac{3}{2}}+4d\sigma_{\frac{1}{2}}+4(d\sigma_{\frac{3}{2}}d\sigma_{\frac{1}{2}})^{\frac{1}{2}}\cos\varphi], \\
d\sigma(n0,p-) &= (2/9)[d\sigma_{\frac{3}{2}}+d\sigma_{\frac{1}{2}}-2(d\sigma_{\frac{3}{2}}d\sigma_{\frac{1}{2}})^{\frac{1}{2}}\cos\varphi],
\end{aligned}
\tag{4}
$$

where we have defined

$$
d\sigma_{\frac{3}{2}}=|(I_{\frac{3}{2}}|I_{\frac{3}{2}})|^2, \quad d\sigma_{\frac{1}{2}}=|(I_{\frac{1}{2}}|I_{\frac{1}{2}})|^2,
\tag{5}
$$

and φ is an undetermined phase factor which may depend on the energy and angles. The combined ordinary and charge exchange scattering for negative mesons, however, is independent of the phase factor, the interference terms cancelling. It is also apparent that the condition of charge independence imposes no condition of equality of the positive and negative pion scattering. The ratio of these cross sections is

$$
[d\sigma(p-,p-)+d\sigma(n0,p-)]/d\sigma(p+,p+)=\tfrac{1}{3}+\tfrac{2}{3}d\sigma_{\frac{1}{2}}/d\sigma_{\frac{3}{2}}
$$

which can vary from $\frac{1}{3}$ to an arbitrarily large number. Some of this arbitrariness will be eliminated by imposing in the next section some additional restrictions on the cross sections.

We shall now characterize the scattering by the asymptotic phase shifts. Following the above analysis, we can expect separate phase shifts for the four states with $J=\frac{1}{2}, \frac{3}{2}$ and $I=\frac{1}{2}, \frac{3}{2}$. We assign phase shifts to the various states according to the scheme

$$
\begin{aligned}
&\delta_{++}, \quad J=\tfrac{3}{2}, \quad I=\tfrac{3}{2}, \\
&\delta_{-+}, \quad J=\tfrac{1}{2}, \quad I=\tfrac{3}{2}, \\
&\delta_{+-}, \quad J=\tfrac{3}{2}, \quad I=\tfrac{1}{2}, \\
&\delta_{--}, \quad J=\tfrac{1}{2}, \quad I=\tfrac{1}{2}.
\end{aligned}
\tag{6}
$$

The differential cross section for scattering in the state $I_{\frac{3}{2}}$ then is given by[12]

$$
\begin{aligned}
k^2 d\sigma_{\frac{3}{2}}/d\Omega = \cos^2\theta\,|2e^{i\delta_{++}}\sin\delta_{++}+e^{i\delta_{-+}}\sin\delta_{-+}|^2 \\
+\sin^2\theta\,\sin^2(\delta_{++}-\delta_{-+})^2,
\end{aligned}
\tag{7}
$$

and a similar expression for scattering in the state $I=\frac{1}{2}$. The total cross sections that are of interest are combinations of the cross sections for scattering in the states $I=\frac{1}{2}$ and $I=\frac{3}{2}$ as given in Eqs. (4).

According to Feshbach, Peaslee, and Weisskopf,[10] the phase shifts (which we denote collectively by δ_λ) are given by the formula

$$
\tan\delta_\lambda=\tfrac{1}{2}\Gamma_\lambda/(E_\lambda-E),
\tag{8}
$$

Brueckner and K. M. Case, Phys. Rev. **83**, 1141 (1951) and Y. Fujimoto and H. Miyazawa, Prog. Theoret. Phys. **5**, 1052L (1950).

[9] The considerations in this section are for scattering in the center-of-mass system.

[10] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947); Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).

[11] For a more detailed discussion of the theory of charge independence, see the paper of K. M. Watson and K. A. Brueckner, Phys. Rev. **83**, 1 (1951), and in particular of K. M. Watson, Phys. Rev. **85**, 852 (1952).

[12] See C. L. Critchfield and D. C. Dodder, Phys. Rev. **76**, 602 (1949), for a detailed derivative of a similar formula.

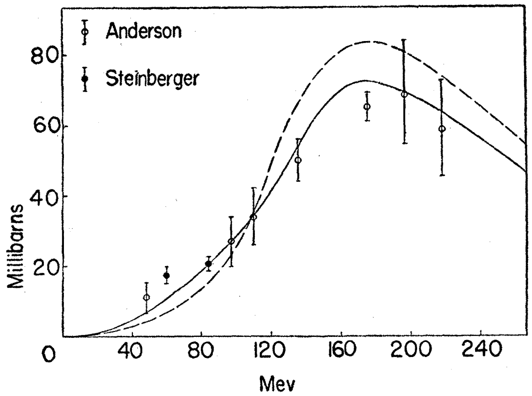

FIG. 1. The scattering of π^- mesons in hydrogen; the experimental points are those of Anderson (see reference 2) and Steinberger (see reference 3). The dashed curve is for $a^2\mu^2=0$, the solid curve for $a^2\mu^2=\frac{1}{2}$.

where

$$\Gamma_\lambda=-2k^3a^3/(1+k^2a^2)\gamma_\lambda, \tag{9}$$

and a is the radius of the boundary of the internal region. The constant γ_λ is determined by the boundary conditions at the radius a. Equations (7), (8), and (9) allow us to express the cross sections in terms of the resonance energies E_λ and the constants γ_λ.

B. Low Energy Limit and the Relation to Meson Theory

At low energies, the meson P-state interaction is weak and the weak coupling predictions for the ratios of the three scattering cross sections might be expected to hold qualitatively. This should also be true if the resonance energies are very large compared with the meson energies which, according to strong coupling theory, is equivalent to letting the coupling constant become very small. Therefore, we shall assume that in this limit the weak coupling prediction is correct and that

$$\begin{aligned} d\sigma(p+,p+)&=d\sigma(p-,p-),\\ d\sigma(n0,p-)&=2d\sigma(p+,p+)\cos^2\theta, \end{aligned} \tag{10}$$

and that $\sigma(p+,p+)$ is independent of angle. The differential cross section given in Eq. (7) reduces at low energies where, according to Eq. (9), the phase shifts approach zero, to

$$d\sigma_{\frac{3}{2}}/d\Omega=3\delta_{++}(\delta_{++}+2\delta_{-+})\cos^2\theta+(\delta_{++}-\delta_{-+})^2, \tag{11}$$

with a similar expression for $d\sigma_{\frac{1}{2}}/d\Omega$. Comparing this result with Eqs. (4) and (10), we find

$$\delta_{++}=-2\delta_{-+}=-2\delta_{+-}=-\tfrac{1}{2}\delta_{--}. \tag{12}$$

Accordingly, from Eq. (9) relating the phase shifts and resonance parameters, we have

$$\begin{aligned} \Gamma_{++}/E_{++}&=-2\Gamma_{-+}/E_{-+}=-2\Gamma_{+-}/E_{+-}\\ &=-\tfrac{1}{2}\Gamma_{--}/E_{--}\equiv 2\rho\equiv 2\rho^{\frac{3}{2}}k^3/[\mu^3(1+k^2a^2)], \end{aligned} \tag{13}$$

where the parameter ρ_0 defined by this equation is independent of the state and of energy. In this limit the low energy cross sections are

$$\frac{d\sigma}{d\Omega}(p+,p+)=\frac{9}{4}\frac{\rho_0^2k^4}{\mu^6}, \tag{14}$$

with related expressions for the other cross sections as given by Eq. (10). A comparison of this result with Eq. (1) shows that

$$(9/4)\rho_0^2\simeq g^4, \tag{15}$$

which gives an interesting relation between the resonance parameters and the coupling constant of weak-coupling meson theory. The close connection between meson theory and the more general form of the phenomological resonance theory is also evident.

Using the restrictions of Eq. (13) on the resonance energies and widths, the total cross sections as a function of energy for meson energies not necessarily near zero are

$$\frac{\sigma(p+,p+)}{4\pi\lambda\!\!\!^{-2}\rho^2}=\frac{2}{\rho^2+(E/E_{++}-1)^2}+\frac{1}{\rho^2+4(E/E_{-+}-1)^2},$$

$$\begin{aligned} \frac{\sigma(p-,p-)+\sigma(n0,p-)}{4\pi\lambda\!\!\!^{-2}\rho^2}=&\frac{2/3}{\rho^2+(E/E_{++}-1)^2}\\ &+\frac{1/3}{\rho^2+4(E/E_{-+}-1)^2}+\frac{4/3}{\rho^2+(E/E_{+-}-1)^2}\\ &+\frac{2/3}{\rho^2+(1/4)(E/E_{--}-1)^2}. \end{aligned} \tag{16}$$

To remove the arbitrariness which remains in the choice of the three resonance energies, we turn to strong coupling meson theory for a qualitative assignment of the levels. For symmetric pseudoscalar theory, Pauli and Dancoff give for the resonance energies

$$E_J=\tfrac{3}{2}(\mu^2a_0/g^2)[J(J+1)-\tfrac{3}{4}], \tag{17}$$

where J is the total angular momentum and a_0 is the source radius. The restriction is also imposed that the isotopic angular momentum I be equal to the angular

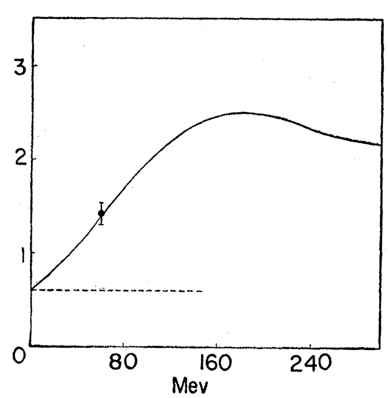

FIG. 2. The ratio of π^+ to π^- scattering, including charge exchange. The experimental point indicated is from the work of Steinberger (see reference 3).

momentum J. An isobar can therefore be excited only by scattering in the $J=\frac{3}{2}$, $I=\frac{3}{2}$ state. For the non-isobaric states, the energy E_λ is chosen to be $-\mu$, corresponding to the absence of isobaric states of the meson-nucleon system. This choice is consistent with the form of the equation given by weak or strong coupling theory. We shall make the assumption that these predictions of the strong coupling theory for the isobar energies are correct and show in the next section that satisfactory agreement with experiment can be obtained.

An additional interesting relation between the phenomenological theory and the weak and strong coupling theories is given by Eqs. (10), (14), (16), and (19) for ρ_0 and E_J. These give

$$\gamma_\lambda = -3a_0/(\mu a^3). \tag{18}$$

Wigner and Eisenbud[10] give for an approximate upper limit on γ_λ

$$-a\gamma_\lambda < (2\mu a)^{-\frac{1}{2}}. \tag{19}$$

Combining these equations and taking the source radius as the nucleon-Compton wavelength gives the result

$$\mu a > 0.74. \tag{20}$$

This approximate inequality can be satisfied if the boundary radius a is somewhat less than the meson-Compton wavelength, a distance consistent with the phenomenological treatment.

III. COMPARISON WITH EXPERIMENT

With the assignment of the resonance energies made in Sec. II, the total scattering cross sections are[9]

$$\frac{\sigma(p+,p+)}{4\pi\lambda\!\!\!^{-2}\rho^2} = \frac{2}{\rho^2+(E/E_R-1)^2} + \frac{1}{\rho^2+4(\omega/\mu)^2},$$

$$\frac{\sigma(p-,p-)+\sigma(n0,p-)}{4\pi\lambda\!\!\!^{-2}\rho^2} = \frac{2/3}{\rho^2+(E/E_R-1)^2} + \frac{5/3}{\rho^2+4(\omega/\mu)^2} + \frac{2/3}{\rho^2+(1/4)(\omega/\mu)^2}, \tag{21}$$

where E_R is the resonance energy for the $J=\frac{3}{2}$, $I=\frac{3}{2}$ state and $E+\mu=\omega$ is the meson total energy.

The variation of these cross sections with energy is given in Fig. 1, together with the experimental results. The resonance energy has been taken to be 137 Mev, corresponding to 200-Mev mesons in the laboratory system, and the boundary radius a such that $a^2\mu^2=\frac{1}{2}$. The corresponding value of ρ_0 is 0.23, corresponding to $g^2=0.35$ from Eq. (15). The general agreement of the theory with experiment is seen to be excellent except for somewhat too low values of the cross sections at low energies. The ratio of the π^+ to π^- scattering is given in Fig. 2; it is 0.60 at zero energy and then rapidly rises to nearly three at the higher energies. The value at

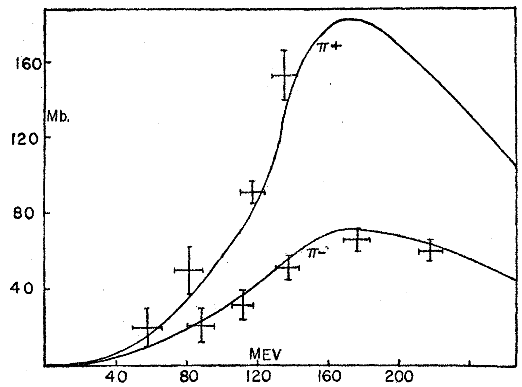

FIG. 3. Total cross sections for scattering of π^+ and π^- mesons in hydrogen (including charge exchange).

60–80 Mev varies from 1.41 to 1.77 which agrees with the ratio of 1.58 ± 0.24 reported by Steinberger for mesons in the energy rrange from 40 to 72 Mev.

IV. CONCLUSIONS

It is found that the steep variation of cross section with energy, the broad plateau suggesting a resonance peak of the π^- scattering at about 200 Mev, and the anomalous ratio of π^+ to π^- scattering can be interpreted to be an indication of the existence of an excited nucleon isobaric state. Assignment of the isobar to the state with total angular momentum $J=\frac{3}{2}$ and total isotopic angular momentum $I=\frac{3}{2}$ gives predictions in excellent agreement with experiment. Although the analysis is made phenomenologically, the parameters in the theory are adjusted to give low energy scattering which agrees with the predictions of symmetric weak-coupling theory and also to give an assignment of isobar energies which is qualitatively in agreement with the predictions of strong coupling charge-symmetric pseudoscalar theory.

The author is indebted to Professor Anderson, Professor Steinberger, and Professor Sachs for information on their experiments and to Professor K. M. Watson for valuable discussions on the theory of charge symmetry.

Note added in proof:—A number of recent measurements of the $(p+,p+)$ cross sections have been made by Anderson, Fermi, and collaborators [reported by H. L. Anderson at the Rochester Conference on Meson Physics, January 11–12, 1952] at Chicago. Their results are given in Fig. 3, together with the theoretical predictions. The charge exchange scattering of π^- mesons has also been measured at 115 Mev [Lundby, Fermi, Anderson, Nagle, and Yodh, Bull. Am. Phys. Soc. **27**, No. 1, 28 (1952)] and found to exceed the direct scattering in the ratio of 2 or 3 to one. The approximate ratios of the observed cross sections above 100 Mev, namely, $\sigma(p+,p+):\ \sigma(p-,p-):\ \sigma(n0,p-)=9:1:2$ are seen from Eq. (4) to follow directly from the assumption of predominant scattering in the $I=\frac{3}{2}$ state.