Relative Pair Production Cross Sections in the Energy Region 50 to 300 Mev*

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A cloud-chamber survey has been made of pairs produced in Al, Ag, Au, and Th foils by an x-ray beam from the 300-Mev betatron. By comparing the pairs produced in two of the foils mounted to intercept the same x-ray beam, values of the relative pair production cross sections for Ag to Al, Au to Al, and Th to Al were found to be, respectively, (3.4 ± 1.4), (7.9 ± 1.5), and (11.7 ± 1.7) percent low compared with the Bethe-Heitler theory predictions. The results are consistent with a correction factor to the theoretical pair cross section given by $[1-(1.4\pm0.1)10^{-5}Z^2]$. The calculated number of Compton electrons between 50 and 200 Mev expected from the Al foil is 1201, from the Ag foil is 355, and from the Au foil is 113. The observed numbers with statistical errors indicated are, respectively, 1170 ± 40, 354 ± 30, and 70 ± 18.

INTRODUCTION

THE Bethe-Heitler theory[1] for pair production in the field of the nucleus, the Wheeler-Lamb theory[2] for pair production in the field of the atomic orbital electrons, and the Klein-Nishina equation[3] for the Compton scattering are generally used in accounting for the absorption of gamma-rays for energies much larger than mc^2. It is sometimes necessary to take into account the absorption caused by photonuclear disintegration and the photoelectric effect. The usual methods for studying the absorption of gamma-rays are measurement of the beam intensity before and after its passage through an absorber and measurement of the beam intensity alternately with and without an absorber placed in the beam between the source of gamma-rays and the detector. It has been well established by such experiments that the absorption coefficients for the heavy elements are low compared with the values predicted by the above theories. This discrepancy has been blamed on the failure of the Born approximation in the development of the pair production theory when applied to the heavy elements. It is the purpose of this experiment to measure unambiguously the Z dependence of the pair production cross section. In addition, by measuring the energies of the pair members, the cross sections for the distribution of energy between the pair members are studied for Al, Ag, and Au.

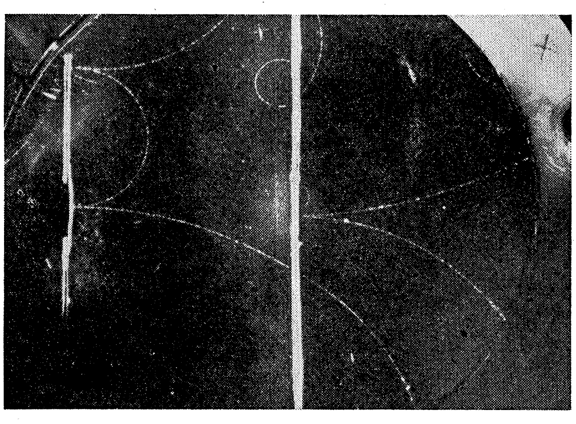

FIG. 1. Photograph of the cloud chamber showing pairs created in Au and Al foils by a beam of photons traversing the chamber from left to right. Note Pb absorbers on which the foils were mounted.

Using the information obtained from the pair spectrum observed, the quantum spectrum from the 300-Mev betatron is calculated and compared with the quantum spectrum which one might expect from the thick W target of the betatron. Also, by using the calculated quantum spectrum, the energy distribution of the Compton electrons as calculated by the Klein-Nishina formula is compared with the observed Compton electron spectrum.

THE EXPERIMENT

The foil arrangement was such that two foils of different atomic numbers were placed parallel to one another and spaced 14 cm apart in a 35-cm cloud chamber. A horizontal x-ray beam from the betatron entered the chamber through a 5-mil Al window and passed through both foils, thus making sure that the quantum spectrum for each foil was practically the same. The foils were approximately 1 inch high and $1\frac{1}{2}$ inches long. Their thicknesses were 0.6295 ± 0.0005, 0.2844 ± 0.004, 0.1008 ± 0.0001, and 0.1274 ± 0.0002 grams per cm^2, respectively, for Al, Ag, Au, and Th. They were mounted by small pieces of black Scotch electrical tape on Pb foil frames which were 1 inch high, $\frac{1}{8}$ inch thick and extended completely across the chamber, except for a $1\frac{1}{2}$ inch gap where the foils were mounted in the beam. These Pb absorbers degraded the energy of any pair member whose energy was low enough to curve around and hit one of the absorbers. This prevented the particles from forming tracks which would have been complete circles, obliterating the origin of the pair. A clear origin was necessary to determine with certainty the two tracks which constitute a

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[1] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) A146, 83 (1934).
[2] J. Wheeler and W. Lamb, Phys. Rev. 55, 858 (1939).
[3] O. Klein and Y. Nishina, Z. Physik 52, 852 (1929).

pair. Figure 1 is a photograph showing a pair created in the Au foil (left) and a pair created in the Al foil. The tracks which are deflected up are made by electrons.

The experiment was divided into two parts. In the first part two foils of different Z's were placed in the chamber. The ratio of the number of pairs of total energy greater than 50 Mev produced in one foil to those produced in the other was determined. The direct ratios for Au to Al, Ag to Al, and Au to Ag were obtained by successive cloud chamber runs. An indirect ratio for Au to Al was determined from the Ag to Al and Au to Ag ratios. The final ratio of Au to Al was calculated by a statistically weighted average of the direct and indirect ratios. In a similar manner, the ratio for Ag to Al was determined.

The second part of the experiment was to determine the ratio for Th to Al. This was done by the direct method only. The Th was wrapped in a thin sheet of Al to absorb the emitted alpha-particles and very low energy beta-particles which would otherwise obliterate the origin of the pairs. The Al shield required a correction of about 10 percent to be made on the number of pairs observed from the shielded Th.

In order to eliminate any systematic error in measuring the radii of curvature due to the relative positions of the foils and also to compensate for the almost negligible degradation of the x-ray beam through the first foil, the foils were interchanged in position. Approximately an equal number of pairs was measured in each position. The foil thicknesses were chosen to produce about the same number of pairs when bombarded by the same x-ray beam.

The apparatus alignment is shown in Fig. 2. The chamber was placed directly in the x-ray beam at about 7 meters from the betatron target. Because the lighted region of the cloud chamber was only an inch deep, an x-ray beam $\frac{1}{4}$ inch high at the foil was used. The width of the x-ray beam at the foil was approximately $\frac{3}{4}$ inch long. The collimation of the beam was obtained primarily by a 12-inch long, 4-inch square lead brick containing a vertically and horizontally tapered slot. The collimator was placed at about 3 meters from the betatron target and the taper of the slot was matched to the divergence of the beam. A crotch collimator was used to restrict the size of the beam coming out of the betatron crotch and with a secondary collimator reduced the background in the chamber caused by scattered secondary electrons and photons. As the betatron target did not appear to act as a point source, the crotch collimator also helped to define the beam at the foils by reducing the apparent size of the target. Behind the collimation system at about 4 meters a large magnet with a 5000-gauss field was used to sweep out all electrons and positrons which were produced by the beam in coming through the collimation system.

The pulse lights and cameras were arranged so that the illumination used to expose the film was the light which was scattered by the track droplets through an angle of approximately 90 degrees from the incident light. The pulse lights were mounted and collimated so as to give a horizontal light beam 1 inch high, of the same width as the chamber, and which illuminated the region between the foils and absorbers without forming any shadows. The camera shutters were always open and the amount of illumination used for proper exposure was determined by the intensity of the light from the flash tubes.

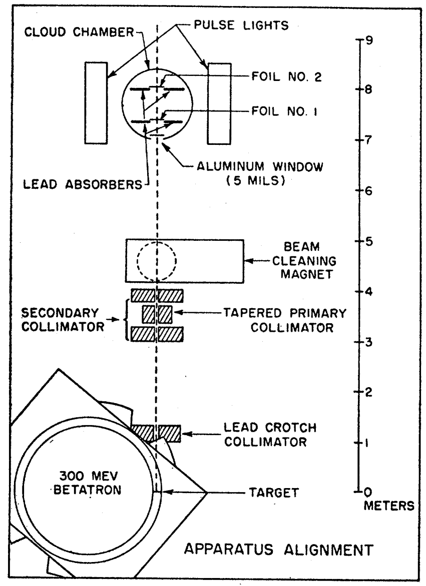

FIG. 2. The collimation scheme for obtaining a small beam in the chamber.

Operational time sequencing of the cloud-chamber functions, the operation of a diesel-driven generator which supplied a pulsed field for the cloud chamber, and the operation of the betatron were controlled by a magnetic pulse time delay circuit.[4] The cloud chamber was expanded when the magnetic field reached its maximum value of $10{,}000\pm100$ gauss. The betatron was triggered 0.04 second later. The track droplets were allowed to grow for approximately 0.15 second. After this time had elapsed the pulse lights were triggered and the film exposed. The chamber was compressed 0.01 second after the pictures were taken, thus readying the chamber for the next expansion. At this same time the camera motors were operated to advance the 35-mm film one frame. This sequence had a cycle time of 10 seconds.

The analyzing consisted of measuring the radii of curvature of all the electron and positron tracks which were photographed. Using the radius of curvature and the relativistic equation for the motion of an electron in a magnetic field, the total energy of each pair member was determined. A special curvature measuring device enabling a very rapid accumulation of good statistics

[4] C. R. Emigh, Rev. Sci. Instr. **21**, 142 (1950).

was used. This device will be described in a paper to be published later.

THE THEORY

The pairs which are observed in a cloud chamber when a thin foil is bombarded by x-rays can be separated theoretically into two groups. If the nucleus takes part in the pair creation, the phenomenon is referred to as pair production. On the other hand, when the orbital electron takes up the recoil momentum of pair production, the process is more often referred to as triplet production. However, in the energy region 50 to 300 Mev practically all the triplets formed will have their recoil electrons of such low energy that they will not be observed in the cloud chamber and the triplets will be analyzed as pairs. Therefore, a correction due to triplet production must be made.

Another correction is due to the Compton effect. In this case only an electron is seen coming from the bombarded foil, and it is impossible to distinguish this Compton electron from a pair whose positron has such a low energy that it is not observed in the cloud chamber. However, the number of pairs of this type can be estimated from the number of pairs in which the electron has such a low energy that only the positron is observed. Therefore, it is possible to make a comparison between the experimental and theoretical Compton cross sections. The following sections outline the three theories—pair production, triplet production, and the Compton effect—in a manner which is most useful in the analysis of the pair production data.

The theory of pair production which takes into account the screening of the nucleus by the orbital electrons was published by Bethe and Heitler in 1934. Their theory is based on a first-order Born approximation. The condition for the applicability of the Born approximation is that $(Z/137)\ll(v_1/c)$ and $(Z/137)\ll(v_2/c)$, where v_1 and v_2 are the velocities of the electron and the positron, respectively. For energies greater than mc^2 this condition is probably satisfied for Al. For the heavier elements, such as Ag, Au, and Th, the condition is not fulfilled. However, the condition for validity, although sufficient, may be a much too stringent one, because even for the heaviest elements the correction to the theory is only of the order of 10 percent.

Bethe and Heitler consider the screening effect of the orbital electrons by assuming a Fermi-Thomas distribution of the atomic electrons. Their calculations show that screening is very important for energies greater than 15 Mev. Because the Fermi-Thomas electron density distribution function leads to a numerical integration, the cross section for pair creation is given only numerically. A convenient form for the cross section, $\phi_p(E_1)$, for the creation of a pair whose electron has an energy in the range E_1 to E_1+dE_1, and for an incident quantum whose energy is k_0, is given by the relation

$$\phi_p(E_1)dE_1=\frac{Z^2e^4dE_1}{137(mc^2)^2k_0^3}\left\{(E_1^2+E_2^2)\left[\phi_1(\gamma)-\frac{4}{3}\ln Z\right]+\frac{2}{3}E_1E_2\left[\phi_2(\gamma)-\frac{4}{3}\ln Z\right]\right\},\quad(1)$$

where $\phi_1(\gamma)$ and $\phi_2(\gamma)$ are given graphically by Bethe and Heitler as functions of $\gamma=100\ mc^2k_0/E_1E_2Z^{\frac{1}{3}}$. The total integral cross sections are obtained by graphical integrations of the above expression.

The triplet production cross section which includes the effects of screening is given by Wheeler and Lamb. The results of their calculations for the cross section per atom, $\phi_t(E_1)$, for the creation of a pair whose electron has an energy in the range E_1 to E_1+dE_1, when the primary quantum energy is k_0 is given by the relation

$$\phi_t(E_1)dE_1=\frac{Ze^4dE_1}{137(mc^2)^2k_0^3}\left\{(E_1^2+E_2^2)\left[\psi_1(\epsilon)-\frac{8}{3}\ln Z\right]+\frac{2}{3}E_1E_2\left[\psi_2(\epsilon)-\frac{8}{3}\ln Z\right]\right\},\quad(2)$$

where $\psi_1(\epsilon)$ and $\psi_2(\epsilon)$ are given graphically as functions of $\epsilon=100\ mc^2k_0/E_1E_2Z^{\frac{2}{3}}$.

The cross section for both pair production and triplet production can be expressed in the following form:

$$\phi_p(E_1)+\phi_t(E_1)=\phi_p(E_1)[1+\phi_t(E_1)/\phi_p(E_1)].\quad(3)$$

The fraction $\phi_t(E_1)/\phi_p(E_1)$ is small compared with 1 for $Z>13$ and to the degree of accuracy for which this experiment is designed, we are justified in equating the numerical functions $\phi_1(\gamma)$ to $\phi_2(\gamma)$ in Eq. (1) and $\psi_1(\epsilon)$ to $\psi_2(\epsilon)$ in (2). The term $\phi_t(E_1)/\phi_p(E_1)$ reduces to $[\psi_1(\epsilon)-(8/3)\ln Z]/[Z\phi_1(\gamma)-(4/3)\ln Z]$. This term is practically independent of the sharing ratio E_1/k_0, and the incident quantum energy k_0, and it varies as $1/Z$. The average value for this term is $1.1/Z$ in the energy range 50 to 300 Mev. The cross section for both pair production and triplet production becomes

$$\phi_p(E_1)+\phi_t(E_1)=\phi_p(E_1)[1+1.1/Z].\quad(4)$$

The Compton effect contributes an appreciable number of tracks observed in the cloud chamber when a foil is bombarded by x-rays. The number of Compton electrons which one expects to observe in a certain energy region E_1 to E_2 is dependent upon the intensity of the incident spectrum for all energies of the primary quanta greater than E_1. In order to calculate this number, the Klein-Nishina formula is used. The differential cross section per atom can be expressed in terms of the scattered quantum, k, for values of primary radiation $k_0 > 50$ Mev as follows:

$$\frac{Z d\sigma_c}{\bar{\phi}} = \frac{137\pi}{a}\left[\frac{k_0}{k} + \frac{k}{k_0}\right]\frac{dk}{k_0}; \quad \bar{\phi} = Z^2 r_0/137, \quad a = k_0/mc^2. \tag{5}$$

Because the pair electrons and the Compton electrons have been analyzed into energy bins which are 10 Mev wide the next step is to calculate the cross section per atom for Compton scattering of the electron into the energy bin, E_1 to E_2, due to an incident quantum whose energy is greater than E_1. Integrating the expression (5) we obtain

$$\frac{Z\sigma_c}{\bar{\phi}} = \frac{137\pi}{a}\left[\ln k + \frac{1}{2}\left(\frac{k}{k_0}\right)^2\right]_{k_1}^{k_2}. \tag{6}$$

We consider first the cross section per atom, σ_c', for producing a Compton electron whose total energy lies in the range, E_1 to E_2, due to an incident quantum whose energy, k_0, is such that $E_1 \leqslant k_0 \leqslant E_2$. From the Compton formula for the frequency shift of the scattered radiation the minimum value for the scattered quantum, k_1, is given to a very good approximation by $k_0/2a$. The maximum value from energy conservation is $k_2 = k_0 - E_1 + mc^2$. Making these substitutions for the limits of the relation (6) and noting that the term $\frac{1}{2}(k/k_0)^2$ contributes practically nothing, we obtain

$$\frac{Z\sigma_c'}{\bar{\phi}} = \frac{137\pi}{a}\ln\left\{2a\left[\frac{k_0 - E_1 + mc^2}{k_0}\right]\right\}. \tag{7}$$

Next consider the cross section, σ_c'', for producing a Compton electron whose total energy lies in the range, E_1 to E_2, resulting from an incident quantum $k_0 > E_2$. From energy conservation $k_1 = k_0 - E_1 + mc^2$ and $k_2 = k_0 - E_2 + mc^2$. Making these substitutions into (6) we obtain

$$\frac{Z\sigma_c''}{\bar{\phi}} = \frac{-137\pi}{a}\left\{\ln\left(\frac{k_0 - E_2 + mc^2}{k_0 - E_1 + mc^2}\right) + \frac{1}{2}\left[\frac{k_0 - E_2 + mc^2}{k_0} - \frac{k_0 - E_1 + mc^2}{k_0}\right]\right\}. \tag{8}$$

In order to calculate the number of Compton electrons expected in a bin E_1 to E_2, we must know, in addition to the cross sections σ_c' and σ_c'', the quantum spectrum in the energy range 50 to 300 Mev. The only information on this spectrum is the number of pairs recorded in each bin. By using the pair data obtained and dividing the number of pairs in each bin by the pair production cross section for that energy, an accurate estimate of the quantum spectrum is obtained. The pair cross sections are corrected for the discrepancies observed in this experiment. However, as the spectrum of the incident quanta is not constant across the 10-Mev wide bin, E_3 to E_4, we make the assumption that its variation across the bin is proportional to $1/k_0$. This is a reasonable assumption, as the spectrum is generated by the bremsstrahlung process when the betatron target is bombarded by a mono-energetic beam of electrons. The results of this experiment also justify the assumption. Stating this assumption mathematically the number of quanta is given by

$$dN = (A/k_0)dk_0, \tag{9}$$

where A is a normalization constant equal to $N_3/\ln(E_4/E_3)$, and N_3 is the number of quanta calculated for the bin E_3 to E_4.

By multiplying Eq. (7) by (9) and integrating over k_0 a new function, ϕ_c', is defined as the cross section per atom for finding a Compton electron in the energy interval, E_1 to E_2, due to quanta in the same energy interval, provided the number of quanta varies as $1/k_0$ across that energy interval. Also by multiplying Eq. (8) by (9) and integrating over k_0, we can define another new function, ϕ_c'', as the cross section per atom for finding a Compton electron in the energy interval, E_1 to E_2, due to quanta in the energy interval, E_3 to E_4, provided the number of quanta varies as $1/k_0$ across the energy interval, E_3 to E_4, and where $E_3 \geqslant E_2$. The integral expressions for ϕ_c' and ϕ_c'' are

$$\phi_c' = \int_{E_1}^{E_2} \frac{\sigma_c' dk_0}{\ln(E_2/E_1)k_0}; \quad \phi_c'' = \int_{E_3}^{E_4} \frac{\sigma_c'' dk_0}{\ln(E_4/E_3)k_0}. \tag{10}$$

These expressions have been normalized to a single incident quantum. The integrations are rather long but straightforward.

Assuming the corrected pair and triplet cross sections are valid, the number of Compton electrons expected in any single energy bin, i, due to quanta in the same bin plus those due to quanta in all the higher energy bins, is given by the relation

$$N_c(i) = \frac{N_p(i)\phi_c'}{\phi_p(i)} + \sum_{j=i+1}^{26} \frac{N_p(j)\phi_c''(j)}{\phi_p(j)}, \tag{11}$$

where $N_c(i)$ is the number of Compton electrons in bin i, $N_p(i)$ is the number of pairs (including triplets) in bin i, ϕ_c' and ϕ_c'' are the Compton cross sections previously derived, and $\phi_p(i)$ is the average pair-plus-triplet cross section for the bin i. The running index, i, indicates the number of the bin on an increasing energy scale, 50 to 60 Mev is bin 1, 60 to 70 Mev is bin 2, and so forth up to bin 26. As $N_p(i)$ is not known exactly, only the total number of pairs-plus-triplets, plus Compton electrons are observed, Eq. (11) can be written

$$N_c(i) = \frac{N_T(i)F'(i)}{1 + F'(i)} + \frac{1}{1 + F'(i)} \sum_{j=i+1}^{26} N_p(j)F''(j), \tag{12}$$

where $N_T(i) = N_p(i) + N_c(i)$, $F'(i)$ is the ratio of the Compton cross section to the pair-plus-triplet cross section for the bin i, and $F''(j)$ is the same for the bin j.

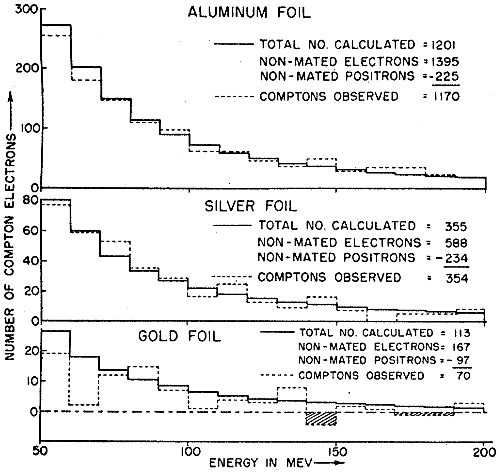

FIG. 3. Histogram showing both the calculated and observed number of Compton electrons from the Al, Ag, and Au foils.

This is the form of the relation used in computing the expected Compton spectra from the Al, Ag, and Au foils. Although the quantities $N_p(j)$'s are not known to start with, they can be found by calculating the $N_c(i)$'s in the order of $i_{\max}$ to $i=1$. The $N_p(i)$ can be calculated from the relation $N_T=N_p(i)+N_c(i)$, and the resulting $N_p(i)$ becomes one of the $N_p(j)$'s for the next calculation.

THE RESULTS

The data compiled by analyzing 46,592 pairs can be summarized in several ways. Each way is designed to compare most easily the experimental results with certain features of the theory of gamma-ray absorption. The interesting features of the theory investigated in this experiment are the Compton effect, the energy sharing in pair creation, and the Z dependence on the pair production cross section.

Using all the data from the Al foil, the calculated number of Compton electrons which one might expect in each 10-Mev wide bin is obtained by applying formula (12). This assumes that the pair and triplet cross sections for Al are given accurately by the Bethe-Heitler and the Wheeler-Lamb theories, respectively. A direct comparison between the calculated number of Compton electrons expected in each bin can be made if it is assumed that in a given energy bin the number of pairs in which only the positron is observed is equal to the number of pairs in which only the electron is observed. Then the observed number of Compton electrons can be obtained by subtracting the number of nonmated positrons observed from the number of nonmated electrons observed. The results of these calculations are shown by the histograms in Fig. 3. In the case of Ag and Au, the pair cross sections used in formula (12) for the calculations of the number of Compton electrons expected in each bin are corrected for the observed low values. For Al and Ag the calculated and observed values are in agreement within the statistical errors. For the case of Au the observed values seem low. The shaded portions in the histogram for Au indicate that more nonmated positrons than nonmated electrons were observed in those bins. A possible explanation for the low values observed for Au is that the Born approximation is not quite valid and therefore an asymmetry in the electron-positron energy distribution curve might be expected in the end regions. If we consider that the low energy electron is attracted to the nucleus while a low energy positron is repelled, then more nonmated positrons than nonmated electrons should be observed. A correction for this asymmetry would have the effect of bringing up the values of observed Compton electrons.

The data from all the experimental runs for each element are plotted in the histograms of Fig. 4 according to the distribution of energy between the pair members (these data include triplets). The data are divided into bins of approximately the same number of pairs: 50 to 100 Mev, 100 to 150 Mev, and 150 to 300 Mev. The calculated numbers of Compton electrons are indicated. For comparison, the Bethe-Heitler and Wheeler-Lamb theoretical curve is plotted for the distribution calculated by using the average energy of the incident quanta intensity for each bin. The theoretical curve is normalized by making the area under it equal to the area under the experimental pair distribution histogram. The relative agreement with the theory is within the statistical errors.

The observed number of pairs-plus-triplets-plus-Compton electrons, $N_a(Z)$, for each foil is corrected for the Compton electrons and triplets. The resulting number of pairs, $N_p(z)$, is normalized to one atom to give the cross section multiplied by the incident number

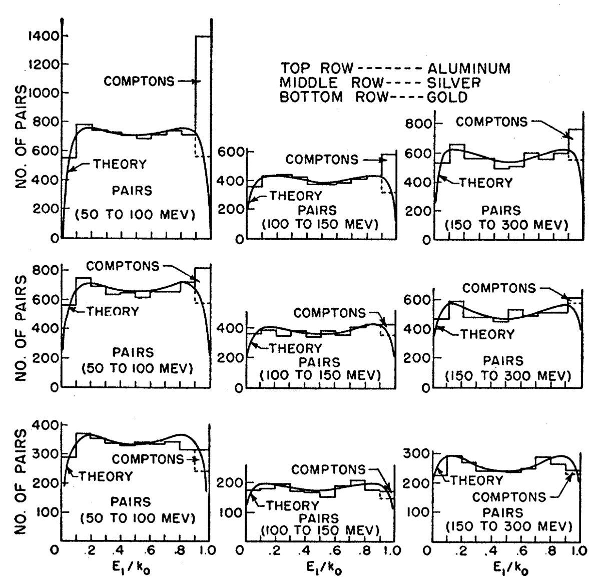

FIG. 4. Theoretical curves and experimentally observed histograms for the energy distribution among the pair members created in the Al, Ag, and Au foils.

of quanta per cm², $\phi_p(Z)N(k_0)$. As $N(k_0)$ is the same for each foil used in a particular experimental run, the ratio of the normalized $N_p(Z)$'s is a measure of the relative pair production cross section. The results of these calculations are shown in Fig. 5 for Au to Al, Ag to Al, and Th to Al. The data are divided into energy bins of approximately the same number of pairs. The cross sections are expressed in units of $Z^2r_0/137$. The Bethe-Heitler theoretical ratios are also plotted. For Th a correction was made for the pairs formed in the thin Al shield. The apparent energy dependence of the relative pair production cross sections for Th to Al, as shown by the dotted lines in Fig. 5 is probably not real. It is possible that the errors due to the bin edge resolution in making a measurement of the pair energy and the human error in making a wrong choice of pair members (only in the case of Th where the origin was sometimes obscured by the emitted beta-particles) could account for the effect. However, most errors of these types are canceled out when the pairs are counted in an over-all bin, 50 to 300 Mev. (See the solid line in Fig. 5 for Th to Al.)

By comparing the relative pair production cross sections obtained from the experiment for each 10-Mev bin with those predicted by the Bethe-Heitler theory, and calculating the weighted average (weighted according to the number of pairs in each bin) of all the bins from 50 to 300 Mev, the relative pair production cross sections for Ag to Al, for Au to Al, and for Th to Al are found to be, respectively, (3.4 ± 1.4), (7.9 ± 1.5), and (11.7 ± 1.7) percent low compared with the values predicted by the Bethe-Heitler theory.

From all the data on pairs produced in the Al foil we can find the quantum spectrum from the 300-Mev betatron's 0.02-inch W target. The theoretical cross sections for Al were used in the calculations. The results are shown in Fig. 6. The theoretical curve for comparison was calculated by Professor Walter Aron

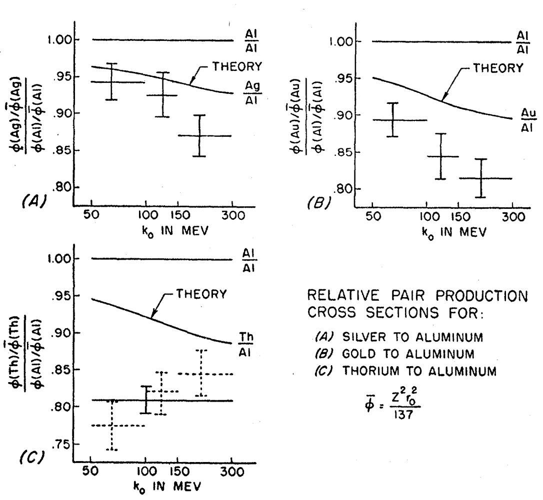

FIG. 5. Comparison of the calculated relative pair production cross sections with the experimentally observed values.

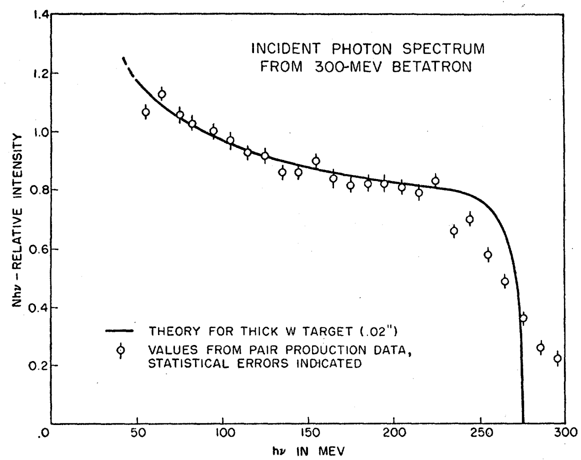

FIG. 6. Comparison of the thick target theoretical bremsstrahlung to the experimental points.

of the University of California. The calculated curve shown by Aron was for a 0.02-inch Pt target; however, the shape of the curve should be practically the same for a W target. The spectrum is in excellent agreement with the thick target calculations of Aron. The discrepancy at the high energy end of the curve is due primarily to the fact that no attempt was made during the experiment to hold the maximum energy of the betatron at a given value. Its average value was around 275 Mev. The fluctuation has practically no effect on the low energy portion of the curve.

COMPARISON WITH OTHER EXPERIMENTS

Several experiments have been reported in the literature on the absorption coefficient in the energy range 10 to 300 Mev. Dewire, Ashkin, and Beach[5] have measured the transmission through several different elements of a 280-Mev photon beam from the Cornell Synchrotron using an electron pair spectrometer. Lawson,[6] also using a pair spectrometer, measured the absorption coefficient of several elements for an 88-Mev photon beam from the G-E 100-Mev betatron. Their experimental results are compared with the theories of Bethe-Heitler for pair production in the field of the nucleus, of Wheeler-Lamb for the pairs produced in the field of the atomic electrons, and with the Klein-Nishina formula for the Compton scattering. The gamma-ray absorption cross section measurements at 17.6 Mev obtained by Walker,[7] also using a magnetic pair spectrometer, are plotted in Fig. 7 along with the results of Dewire *et al.*, and Lawson. In this case the theoretical cross section includes a small contribution from the atomic photoelectric effect in the lead absorber. This correction only amounts to a factor of 0.7 percent in the total cross section. Walker uses the results of Borsellino,[8] who gives the ratio of the pair cross section

[5] Dewire, Ashkin, and Beach, Phys. Rev. **82**, 447 (1951).
[6] J. L. Lawson, Phys. Rev. **75**, 433 (1949).
[7] R. L. Walker, Phys. Rev. **76**, 527 (1949).
[8] A Borsellino, Helv. Phys. Acta **20**, 136 (1947).

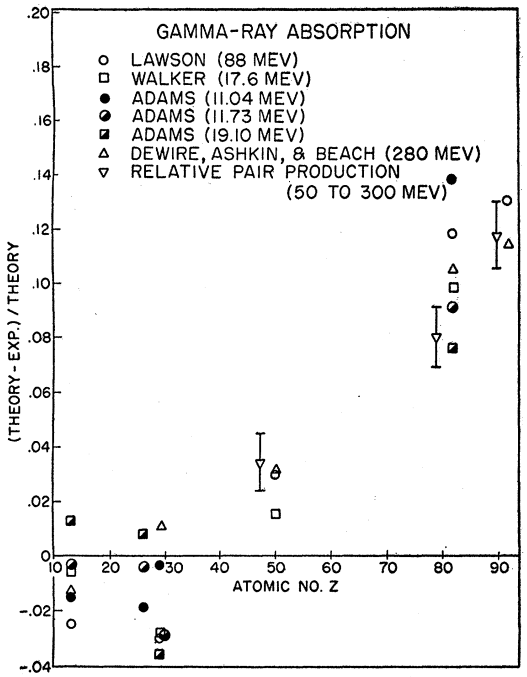

FIG. 7. A presentation of the gamma-ray absorption discrepancy between theory and experiment as found by several experimenters. Relative pair production discrepancies are also shown for comparison.

for an electron to that of a proton. However, the difference between Borsellino's results and those of Wheeler and Lamb should not make much difference in the total theoretical cross section used by Walker.

Adams[9] has measured the absorption of quanta between 11 and 20 Mev using the x-rays from the 22-Mev University of Illinois betatron. Copper, iron, and carbon threshold detectors were used to measure the absorption coefficient at energies of 11.04, 13.73, and 19.10 Mev, respectively. In comparing his results with the theory, Adams uses the modified Bethe-Heitler theory to account for pair production in the field of the atomic nucleus and its surrounding electrons by replacing Z^2 by $Z(Z+1)$, which is essentially the same as that given by the Bethe-Heitler theory and the more exact calculations by Wheeler and Lamb. He also used the form of the Bethe-Heitler theory which assumes no screening. No account is taken of the atomic photoelectric effect in the heavier elements, or the photoneutron effect in the lighter elements. The results of his experiment are also plotted in Fig. 7.

Figure 7 gives a comparative view of all the reliable data on gamma-ray absorption published to date. There is a consistent agreement among all the data presented that the total absorption cross section for the heavy elements are less than that predicted by theory. In the region of Pb the discrepancy from absorption measurements is of the order of 10 percent. The experi-

[9] G. D. Adams, Phys. Rev. 74, 1707 (1948).

TABLE I.

	Al	Cu	Ag	Sn	Au	Pb	Th	U
Experimental error	−0.01	1.1	3.4[a]	3.1	7.9[a]	10.3	11.7[a]	11.4
Correction term	0.02	1.18	3.09	3.5	8.7	9.4	11.3	11.9

[a] These data from the relative pair production experiment. Other data are taken from experiment by Dewire *et al.*

mental discrepancy found in this experiment is (3.4 ± 1.4) percent for the relative pair production between Ag and Al, (7.9 ± 1.5) percent between Au and Al, and (11.7 ± 1.5) percent between Th and Al. If we assume that the pair and triplet production cross sections are given correctly for Al by the Bethe-Heitler and Wheeler-Lamb theories, respectively, then the experimental discrepancies found in this experiment can be compared with the discrepancies found from absorption experiments.

To summarize, the discrepancy found in the absorption measurement can be ascribed to the failure of the pair production cross section theory to give the correct results. If the blame is placed on the 1st-order Born approximation then one might expect a second-order Born approximation would bring in a correction term proportional to Z^2. The results of this experiment can be fitted to a correction term of this type. A good approximation is obtained by replacing the Z^2 factor in the theoretical pair production cross section by $Z^2[1-(1.4\pm 0.1)10^{-5}Z^2]$. Table I compares this approximation with the results of this experiment and the data of Dewire *et al.* Dewire's data are used because at the enegy of 280 Mev the Compton and photoelectric effects can be ignored.

In a very recent experiment by A. I. Berman at Los Alamos, very accurate absorption measurements at 20 Mev for 19 elements from H to U were made using a carbon threshold detector. His experimentally determined correction for pair production is $(1.5\pm 0.1)10^{-5}Z^2$.

In conclusion it might be noted that the data representing the energy sharing distribution for Au (see Fig. 4) are in agreement[10] within the statistical errors with that predicted by Bethe-Heitler. Thus it leads us to believe that the discrepancy may be caused primarily by an error in the magnitude of the total cross section as was assumed above, and that the error is not a function of the energy distribution among the pair members. If this is so, then the same discrepancy may appear in the bremsstrahlung cross section which is almost the reverse process of pair production. The results of Curtis[11] on the measurement of the absolute brems-

[10] The sharing distribution has been verified also by J. W. Dewire and L. A. Beach, Phys. Rev. 83, 476 (1950), measuring the pairs formed in a $\frac{1}{2}$-mil Au foil when bombarded by a 270-Mev photon beam. A statistical error of about 10 percent for the experimental points were obtained.

[11] C. D. Curtis, Phys. Rev. 81, 308 (1951), and by private communications.

strahlung cross sections for 60-Mev electrons incident on a thin Pb foil give results which are (8±3) percent low compared with the Bethe-Heitler theory for bremsstrahlung.

The results obtained by Walker[12] on the relative pair production cross sections for incident quanta of 17.6 Mev as presented give discrepancies which are somewhat larger than those given by recent absorption measurements and the results of this experiment. However, the results of Walker can be made compatible by using the Wheeler and Lamb theory for triplet production in making the correction to the number of pairs observed to obtain the number of pairs created in the field of the nucleus. The triplet correction term obtained experimentally by Walker is $0.8/Z$, although his data are not inconsistent with a correction term $1.1/Z$ which is given by the Wheeler and Lamb theory. By applying this latter correction to his data, the relative pair production cross section discrepancy for Pb to Al, Sn to Al, and Cu to Al becomes respectively (13±3), (8±4), and (3±3) percent. Further correction to Walker's results should be made to account for the fact that he measured only the pairs produced in the central half of the differential cross section ($0.25<E_1/k_0<0.75$), where the effects of screening are greater in proportion to the effects of screening where $E_1k_0<0.25$ and $E_1/k_0>0.75$, especially in the case of the lower Z elements. The effect of a correction of this type would be to reduce his values of relative pair production cross sections by another percent or two. If one makes these corrections, the results of Walker should be compatible with the results of absorption measurements and those of this experiment.

[12] R. L. Walker, Phys. Rev. **76**, 1440 (1949).

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An Investigation into the Nuclear Scattering of High Energy Protons

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The angular distributions of high energy protons, elastically scattered from various nuclei, have been considered theoretically. For this purpose, the optical model of the nucleus, hitherto developed on the basis of the nuclear scattering of high energy neutrons, has been suitably modified to account for the proton scattering. Appropriate proton wave equations have been established and solved exactly. However, these solutions have been found unsuitable for numerical computations. Next, the WKB and the Born approximations have been used, within their respective regions of validity, to yield appropriate expressions for the differential scattering cross section. Finally, the theoretical and the experimental diffraction patterns for the nuclear scattering of 340-Mev protons have been compared. It has been found that the above nuclear model gives only an approximate account of such a scattering. It is further indicated that the effective nuclear radii, appropriate for this energy, are about 10 percent smaller than those calculated on the basis of the nuclear scattering of 90-Mev neutrons.

I. INTRODUCTION

THE experimental observations and the corresponding theoretical considerations on the scattering of high energy nucleons by nuclei have provided valuable information about nuclear structure. In this connection, the theoretical investigations have been carried out by making use of the optical model of the nucleus, employing the concept of a partially transparent nucleus as introduced by Serber.[1] Fernbach, Serber, and Taylor[2] have accounted for the nuclear scattering of 90-Mev neutrons on this basis and Fernbach[3] has extended the same considerations to

[1] R. Serber, Phys. Rev. **72**, 1114 (1947).
[2] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[3] S. Fernbach, *The Scattering of High Energy Neutrons by Nuclei* (unpublished Ph.D. dissertation, Physics Department, University of California, 1951), pp. 16–18.

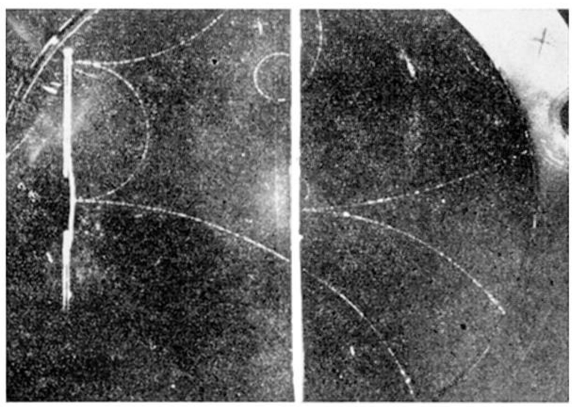

Fig. 1. Photograph of the cloud chamber showing pairs created in Au and Al foils by a beam of photons traversing the chamber from left to right. Note Pb absorbers on which the foils were mounted.