The Differential Range Spectrum of Cosmic-Ray Mesons at Sea Level*

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The low energy region of the differential range spectrum of cosmic-ray mesons at sea level has been investigated by means of a counter controlled cloud chamber. The results obtained in the region from 18 g/cm^2 to 76 g/cm^2 of air equivalent absorber have proved to be about 20 percent higher than predictions based upon similar measurements at greater absorber thicknesses. A possible explanation of this discrepancy lies in the consideration of the effect of multiple scattering of mesons in previous experiments. The present experiment employs a geometrical arrangement of the apparatus which minimizes the corrections to be applied to the data to allow for loss of mesons from the apparatus due to scattering. The earlier work in the field has not been corrected for scattering effects and can, as a result, be expected to give too low a value of the spectrum. The average differential intensity of mesons measured in the range interval mentioned above is 6.86×10^{-6} (particles/sec/sterad/g). The differential range spectrum of protons was also measured and is in good agreement with other work which measured the spectral values at somewhat higher energies than those encountered here. The proton intensity is 2.12×10^{-6} (particles/sec/sterad/g) in the region 16.5 to 42.3 g/cm^2 of air equivalent and 0.67×10^{-6} (particles/sec/sterad/g) in the region 42.5 to 63.3 g/cm^2 of air equivalent.

INTRODUCTION

THE differential range spectrum of cosmic-ray mesons has been studied extensively by a number of workers using a variety of experimental arrangements. In a recent review article Rossi[1] has classified the various methods of measurement into four categories. They are the coincidence, anticoincidence, delayed coincidence, and magnetic deflection methods. Of these the first is perhaps the most inaccurate method in that it cannot distinguish between protons, mesons, and multiple events and cannot easily be corrected for scattering. The anticoincidence method in the modified version used by Germain[2] avoids these difficulties. The present work employs an experimental arrangement similar to Germain's. The delayed coincidence method, which depends upon the decay properties of the μ-meson, cannot be used to obtain an accurate absolute value of the range spectrum because of the difficulties in evaluating the detection efficiency of the apparatus. Also, the geometrical arrangement of the apparatus used does not lend itself to any simple scattering correction. Usually the shape of the spectrum is measured with this method, and the resulting curve is normalized to an absolute value in some appropriate fashion. The magnetic deflection method is accurate for intermediate energies, but at very high and low energies serious difficulties are encountered in the measurements. At low energies the cone of acceptance of the apparatus plays an important role in the determination of the spectral values, while at high energies the magnetic deflections are too small to give accurate determinations of the energy.

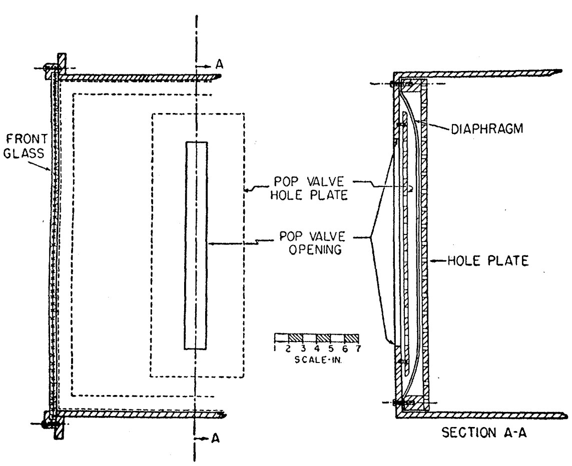

FIG. 1. Sectional views of the laterally expanding cloud chamber.

The only part of the range spectrum which has not been extensively studied lies in the low energy region below 100 g/cm^2 air equivalent of absorber. The present experiment is an investigation of this region and has been carried out by the anticoincidence method with the addition of a cloud chamber to discriminate between protons and mesons. The work was done at sea level and consisted of photographing the tracks of particles which stopped in the lead plates of the cloud chamber. The solid angle, through which particles entered the cloud chamber, was defined by a quadruple coincidence arrangement of Geiger counters. A bank of anticoincidence counters underneath the chamber prevented the apparatus from taking pictures of particles that went all the way through the chamber. To minimize the possibility of having particles scatter out of the solid angle of acceptance of the apparatus, no absorber was placed in this region. Because the rate at which the data could be collected was low, it was not feasible to acquire a statistical accuracy of greater than about 10 percent.

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[1] B. Rossi, Revs. Modern Phys. 20, 537 (1948).
[2] L. Germain, Phys. Rev. 80, 616 (1950).

APPARATUS

As mentioned above, the apparatus used to measure the differential range spectrum in this experiment consisted of a counter controlled cloud chamber. The chamber had an active volume 16 in. by 24 in. by 20 in. The comparatively great depth of 20 in. made it necessary to use direct illumination and take advantage of the light scattering properties of water droplets at small angles of incidence. These properties have been investigated theoretically by Blumer[3] and experimentally by Webb,[4] and both pieces of work are in good agreement. To facilitate this direct illumination, the front and back surfaces of the chamber were made of glass, and the expansions were made laterally with the aid of valves mounted in the sides of the chamber. The pop-valves were of a shutter type first used by Brode and Merkle[5] and are not shown in the figure. Figure 1(b) is a sectional view of the chamber. The $\frac{1}{32}$-inch Neoprene rubber diaphragm acts as a gasket to give an airtight seal between the active volume of the chamber and the back volume. It was found necessary to augment the diaphragm with a gasket of $\frac{1}{16}$-inch gum rubber around the edges to give a satisfactory seal. The pop-valve hole plate is necessary to prevent the diaphragm from closing the pop-valve opening during a fast expansion.

The chamber was operated at 30 cm of Hg pressure of argon and a water-alcohol mixture of 1:3 gave tracks with an expansion ratio of about 9 percent. Alternately charged and grounded lead plates (mounted as shown in Fig. 2) provided a clearing field. The chamber was illuminated by two General Electric flash tubes mounted vertically, 19 in. from the rear glass and in such a position that the sides of the chamber obscure them from the direct view of the cameras. The lamps were energized by discharging a condenser bank of 200 mf charged to 2100 v through each lamp. Aluminum reflectors behind the tubes gave increased illumination in the direction of the chamber.

The camera was placed 6 feet from the front glass and took stereoscopic pictures with two f/4.5 Eastman Projection Anastigmat lenses of 135-mm focal length on Super XX film 1.75 in. wide. The lenses were stopped down to f/22. To reduce the amount of diffusely reflected light in the chamber the hole plates were covered with black velvet.

To minimize the distortion of the tracks due to the lateral expansion, the two pop valves were adjusted to open as nearly simultaneously as possible. The greatest measured difference in the opening times observed after adjustment was 0.002 sec, and the average of several measurements was 0.001 sec. Pads of forty layers of cheese cloth supported by wire matting were placed in the pop-valve openings to reduce the turbulence created by fast expansions.

[3] H. Blumer, Z. Physik **39**, 195 (1926).
[4] C. G. Webb, Phil. Mag. **19**, 927 (1935).
[5] R. B. Brode and T. C. Merkle, private communication.

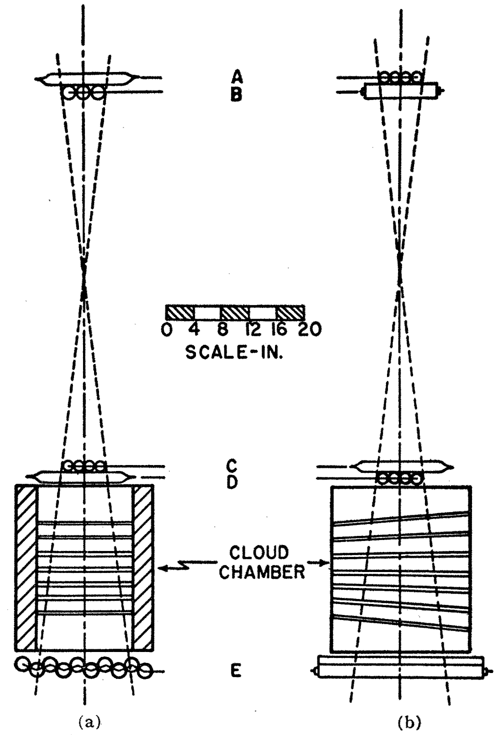

FIG. 2. Experimental arrangement used to measure the differential range spectrum.

A stereoscopic viewer was used to study the pictures. The actuating circuits used to cycle the chamber were of conventional design, with a dead time of 2 min 30 sec for each cycle of the chamber.

The coincidence circuits were of a standard type employing four coincidence channels and one anticoincidence channel. The Geiger counters used to trigger the coincidence unit were the external cathode type developed by MacKnight and Chasson.[6]

The room in which the chamber was housed was temperature controlled by means of a single heater-blower and thermostat. This simple arrangement proved very effective.

METHOD

The arrangement of the apparatus used to measure the differential range spectrum of mesons is indicated in Fig. 2. The counter trays A, B, C, and D define the solid angle of acceptance of particles into the cloud chamber. These trays are so arranged that the geometrical definition of the solid angle does not depend upon the effective length of the counters but instead upon their width. This arrangement has been used because the effective width of a cylindrical counter is a much more accurately defined quantity than its effective length. Because low energy particles tend to scatter through large angles, several precautions were

[6] M. L. MacKnight and R. L. Chasson, Rev. Sci. Instr. **22**, 700 (1951).

TABLE I. Values of the experimental parameters.

$A\Omega = 3.24$ sterad-cm^2	$N_2 = 6579$
$t_1 = 14{,}029$ min	$\tau = 2.50$ min
$t_2 = 20{,}826$ min	$T_1 = 4039$ min
$N_1 = 3996$	$T_2 = 4379$ min

taken to minimize this effect upon the flux of particles through the counter telescope. The apparatus was covered by a thin aluminum roof of 0.151-g/cm^2 thickness. The only other material in the solid angle of the telescope was that of the counter walls and the $\frac{1}{4}$-in. plywood trays holding the counters. Particles entering the chamber must penetrate the $\frac{3}{8}$-in. steel top of the chamber. Because of the small amount of material above and in the counter telescope, it will be assumed that all particles incident upon the telescope and within its solid angle will enter the chamber. However, after entering the chamber some of the particles will be scattered out of the active volume by the lead plates and will not contribute to the measured flux. A correction for this effect was made.

To check the determination of the solid angle the fourfold rate, *ABCD*, was measured and from this the total vertical intensity of the cosmic radiation determined. The result of this determination agreed with the value given by Rossi[1] to within the probable error of the measurements.

The particles stopping in the chamber are identified by their change in ionization as they slow down and stop. For accurate discrimination between protons and mesons it is useful to see at least three segments of the particle's track. For this reason, as well as to aid in the identification of electrons, the first two plates in the chamber were $\frac{1}{4}$ in. thick and only particles stopping in the third, fourth, fifth, and sixth plates were counted.

To distinguish between mesons and electrons, the shower producing properties of the latter were used. However, as pointed out in Appendix I, there is some possibility of confusing electrons and mesons stopping in the third and fourth plates of the chamber.

The stereoscopic photographs of the chamber include the images of a clock and a message register to give the time and number of each event. Although pictures of particles stopping in the chamber were desired, many pictures showed penetrating particles which triggered the apparatus due to the inefficiency of the anticoincidence tray, *E*. In addition, pictures of particles scattering out of the illuminated region of the chamber as well as a few low energy nuclear events were taken.

It was necessary to take the data in two parts. The first part, in which the low energy region of the spectrum was measured, made use of the lead plate arrangement in Fig. 2. The second part was performed to observe a higher energy region. A single plate 2 in. thick was placed at the position formerly occupied by the $\frac{1}{4}$-in. plates at the top of the chamber. This effectively moved the sixth plate of the first arrangement up to the position formerly occupied by the third plate and allowed more points on the spectrum at higher energies to be measured. This procedure also made it possible to check empirically the scattering correction applied to the sixth plate at the two different positions in the chamber. However, it must be borne in mind that mesons and protons stopping in the sixth plate in the second part of the experiment can no longer be distinguished by their ionizing characteristics, because all particles emerging from the 2-inch plate will be at minimum ionization on entering it. Thus, the ratio of mesons to protons found in the first part of the experiment must be used to determine the number of mesons stopping in this plate during the second part of the work. Hence, no very accurate check can be expected, but the two results should be reasonably consistent.

REDUCTION OF THE DATA

The differential range spectrum is given by the number of particles stopping per unit time per unit area from unit solid angle in a given thickness of absorber, plotted as a function of the total absorber traversed by the particles. This can be expressed by the equation

$$I_v(R) = [1/A\Omega d][N(R)/T],$$

where I_v is the vertical differential intensity under R, the total absorber traversed. A is the total area over which the particles are accepted; T is the total time during which the particles could be recorded by the apparatus; and $N(R)$ is the number of particles stopping in the absorber, d, in the time, T. Ω is the solid angle of acceptance of the apparatus. All ranges are expressed in units of grams per cm^2 air equivalent to simplify comparison with other data. The conversion from lead to air was made with the aid of the range-energy relations[8] of the two substances. Table I gives the values of the various quantities listed above which were used in this experiment. Table II gives the limits in total range, R, provided by the finite thickness, d, of each lead plate.

The total time, T, during which the apparatus could record the stopping particles is just equal to the running

TABLE II. The range limits of each plate and the corresponding number of stopped mesons in each interval. Total absorber above the chamber: 10.3 g/cm^2 (of air).

Plate	Range (in air) From	To	d (g/cm^2)	$N(R)$
3	17.9	25.7	7.8	47
4	25.7	33.5	7.8	50
5	33.5	41.7	8.2	39
6	41.7	49.9	8.2	37 (53)
7	49.9	58.5	8.6	49
8	58.5	67.1	8.6	54
9	67.1	75.8	8.7	46

[8] D. J. X. Montgomery, *Cosmic Ray Physics* (Princeton University Press, Princeton, 1949), p. 349.

time, t, minus the total dead time of the machine. If τ is the dead time for each photograph, the total dead time is $N\tau$, where N is the total number of pictures taken. These numbers are included in Table II. In determining the total running time, t, it was necessary to decide upon a method of treatment of pictures which were unclear as a result of some failure of the apparatus. None of these pictures were counted, nor was the time associated with them. This time was chosen as the interval between the picture immediately preceding the one to be rejected and the rejected one. These times are not included in the values listed in Table II.

The errors associated with the time, T, are completely negligible. However, the determination of the other values, A, Ω, and d, have a total uncertainty of 1.6 percent associated with their values. Because this is a small uncertainty compared with the statistical inaccuracy of $N(R)$, it will not be included in the assignment of errors to the values of I_v.

RESULTS

The results of the measurements of the differential range spectrum at sea level are given in Table III with their associated probable errors. Table III also contains the differential intensity of protons. The number of protons observed was so small that the intensity has been calculated from the total number observed in each of the two phases of the experiment and the total range of lead used to stop them. The proton range curves calculated by Aaron *et al.*[9] were used to determine the ranges used.

The points of the differential spectrum of mesons are plotted in Fig. 3. The rectangles indicate the uncertainty in the range measurements in the horizontal direction and the statistical probable errors in the vertical direction. The solid curve drawn somewhat below the rectangles is the extrapolated curve given by Rossi.[1] The dotted rectangle is the value of the spectrum

TABLE III. Experimental values of the meson and proton intensities (the probable errors are indicated).

(a) Meson intensity	
Plate number	I_v (/sec/sterad/g) $\times 10^6$
3	7.69 ±0.75
4	8.19 ±0.77
5	6.06 ±0.66
6	5.84 ±0.66
6′	7.58 ±0.70
7	6.70 ±0.65
8	7.36 ±0.67
9	6.20 ±0.62
(b) Proton intensity	
Range (in g/cm² of air)	I_v (/sec/sterad/g) $\times 10^6$
16.5 to 42.3	2.12 ±0.24
42.5 to 63.3	0.671±0.012

[9] Aaron, Hoffman, and Williams, Range-Energy Curves, Technical Information Division, ORE 675-A20779 (1950), Oak Ridge, Tennessee.

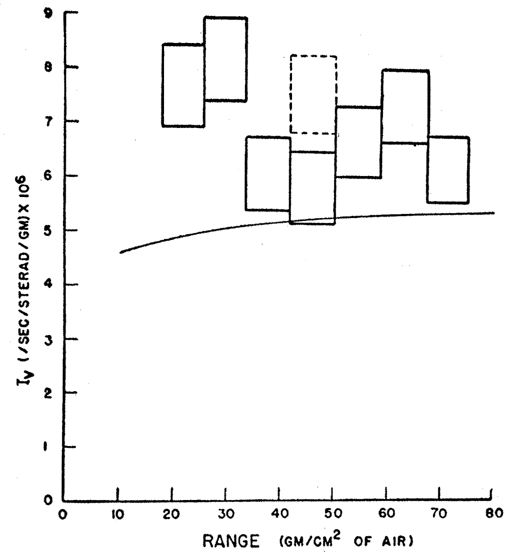

FIG. 3. Experimental results. (The solid curve is the extrapolation made by Rossi, see reference 1.)

obtained from the sixth plate in the second part of the experiment. The uncertainty in the vertical direction does not include the uncertainty in the ratio of protons to mesons which had to be used in the determination of the number of mesons from the total number of particles stopping in the sixth plate.

CONCLUSIONS

It is clear from Fig. 3 that the results of this experiment are not consistent with the extrapolated curve of Rossi. If, however, one ignores for the moment the two points at the lower energy limit of the data, the shape of Rossi's curve can be fitted to the data quite well by displacing the curve upward on the graph. The justification for ignoring the first two points lies in the consideration of the presence of electrons among the particles stopped in the first two lead plates. The detailed argument is given in Appendix I. The implication of having to move Rossi's curve upward is that the normalization point from which the curve was drawn is too low. This point was taken from the work of Sands[10] and depends for its absolute value upon the work of Greisen.[11] Greisen pointed out that his values of the flux of the penetrating component were not corrected for the effect of particles scattered out of the solid angle of his counter telescope. However, as seen in the work of Germain[2] and as is clear from the geometry of Greisen's apparatus, it is not justifiable to neglect this effect. Because the scattering correction is strongly energy dependent and because Greisen's experiment involves a large spread in energy values, it is difficult to make a meaningful correction to his result. However, it is clear that the net result is to cause

[10] M. Sands, Phys. Rev. **77**, 180 (1950).
[11] K. Greisen, Phys. Rev. **61**, 212 (1942).

TABLE IV. The calculated fraction of mesons lost by scattering in each plate (expressed in percent).

Plate number	3	4	5	6	6′	7	8	9
Percentage loss	0.34	0.6	0.9	1.0	0.23	0.5	1.0	1.7

Sands' values to be too small and hence Rossi's extrapolated curve to be too low.

As indicated in Appendix II, the present experiment is quite free from the necessity of making any sizable scattering corrections. Thus a possible explanation of the discrepancy between previous work and the present result lies in the fact that former measurements of the spectrum have not included corrections for the loss of particles by scattering in spite of the fact that they have used geometrical arrangements which make this effect appreciable. In this connection it should be mentioned that Germain[2] has made a mistake in his scattering calculations in the normalization factor of the distribution function of scattered angles that he used. This causes all of his corrected points to be too low by a factor of $(2\pi)^{\frac{1}{2}}$. When his results are corrected for this mistake, they are in good agreement with the present work.

The two values of the proton spectrum are based upon a small number of particles. However, the results are in very good agreement with the data of Mylroi and Wilson,[12] who have obtained the spectral values for protons of slightly higher energy than those observed in this experiment. However, there seems to be no agreement with the single spectral value given by Rochester and Bound[13] for protons with energy less than that observed in the present work. Indeed on the basis of this experiment there is no justification for a rapidly decreasing proton spectrum in the low energy region as Mylroi and Wilson have indicated in their work.

In conclusion the author wishes to express his indebtedness to Professor W. B. Fretter for his guidance throughout the course of the work and to Professor Robert B. Brode for his continued interest in the experiment.

APPENDIX I

The probability that an electron, which has sufficient energy to stop in the third plate of the chamber, can penetrate the steel top of the chamber and the first two lead plates without producing secondary electrons is calculated with the aid of the tables for shower theory given by Montgomery.[14] The Polya distribution can be used to estimate that 23.5 percent of the single electrons incident upon the chamber can penetrate to the third plate without producing another electron. This calculation underestimates the fraction of unaccompanied electrons because the energy loss in the first two plates is neglected. To allow for this effect one can say that the electron no longer has sufficient energy to shower when it reaches the second plate of the chamber. The fraction of particles which do not shower then becomes 41.5 percent.

To see how this might affect the meson spectrum, it is necessary to know the ratio of mesons to single electrons at the energies considered here. Lombardo and Hazen[15] have given an approximate value of 5 percent for this ratio. Thus the possible contribution to the meson spectrum is 5 percent of 41.5 percent, or 2 percent.

A similar correction for the fourth plate gives approximately the same result as that obtained above for the third plate. However, the corrections for the remaining points of the spectrum are much smaller than those just discussed.

It should be pointed out that in the above calculations we have required that more than one particle emerge from a plate. This criterion allows for a possible confusion in distinguishing an electron which is accompanied by another shower produced electron from a meson which is accompanied by a knock-on electron. If one repeats the above calculations and demands the appearance of three or more particles to identify an electron, one finds that about 10 percent of the particles stopping in the third plate and the same fraction stopping in the fourth plate could be electrons. An electron accompanied by only one other particle under one of the top plates would be erroneously called a meson with a knock-on electron.

As a possible check of this last point, the pictures of mesons stopping in the chamber were examined for the presence of knock-on electrons. It was found that of the mesons stopping in the fifth and sixth plates about 7 percent were accompanied by knock-on electrons. This is in good agreement with the results of Hazen.[16] However, of the mesons stopping in the third and fourth plates, 11 percent were accompanied by what appeared to be knock-on electrons. From this it seems reasonable to suppose that some of these latter events were indeed electrons which had only one additional shower produced electron accompanying them. Hence the points on the spectrum contributed by the third and fourth plates can be expected to be higher than they should be.

A very rough estimate of the contribution of electrons to these two points on the spectrum indicates that they should be about 10 percent too high. This is based upon the very poor statistics involved in the knock-on count mentioned above. Because this estimate is so crude, no attempt will be made to correct the results of Table III for this effect.

[12] M. G. Mylroi and J. G. Wilson, Proc. Phys. Soc. (London) **A64**, 404 (1951).
[13] G. D. Rochester and M. Bound, Nature **146**, 745 (1940).
[14] D. J. X. Montgomery, *Cosmic Ray Physics* (Princeton University Press, Princeton, 1949), pp. 63, 334, and 339.
[15] B. Lombardo and W. E. Hazen, Phys. Rev. **68**, 74 (1945).
[16] W. E. Hazen, Phys. Rev. **64**, 7 (1943).

APPENDIX II

The scattering corrections to the various points of the spectrum measured in this experiment were calculated with a modification of the method used by Germain.[2] Table IV summarizes the results of these calculations. In this table the fraction of mesons lost by scattering is expressed in percent. Because the corrections are so small, they have not been applied to the results in Table III. The author is indebted to Dr. S. Fernbach for an enlightening discussion of these calculations.

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Energy of a Bloch Wall on the Band Picture. I. Spiral Approach

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It is shown that the band or itinerant electron model of a solid is capable of accounting for the "exchange stiffness" which determines the properties of the transition region, known as the Bloch wall, which separates adjacent ferromagnetic domains with different directions of magnetization. In this treatment the constant spin function usually assigned to each running electron wave is replaced by a variable spin function. At each point of space the spin of a moving electron is inclined at a small velocity-dependent angle to the mean spin direction of the other electrons, and this gives rise to an exchange torque which makes the spin direction of the given electron precess as it moves through the transition region, the precession rate being just sufficient to keep it in approximate alignment with the macroscopic magnetization. Physical insight into the mechanisms involved is provided by a rigorous solution of the wall problem for a ferromagnetic free electron gas in the Slater-Fock approximation, although it is known that the free electron gas is not likely to be ferromagnetic in higher approximations. Rough upper limits to the exchange stiffness constants for actual ferromagnetic metals can be calculated without using any empirical constants other than the saturation moment and the lattice constant. The results are only a few times larger than the observed values.

I. INTRODUCTION

It has been shown by Bloch[1] and others that the change in the direction of magnetization of adjacent ferromagnetic domains takes place gradually, over a distance of the order of 10^{-5} cm; the transition layer is commonly known as the Bloch wall. The surface energy of this transition layer is a quantity of fundamental importance to the theory of ferromagnetic domains.[2] The occurrence of the transition layer represents a compromise between two tendencies, the tendency of the exchange effect to make the magnetization vector $\mathbf{M}$ vary as gradually as possible with position, and the tendency of anisotropy forces to force $\mathbf{M}$ into directions of easy magnetization. The former tendency can be described quantitatively by saying that when the spatial variation of the orientation of $\mathbf{M}$ is sufficiently gradual, it entails an increase ΔW in the energy per unit volume, given by

$$\Delta W = A|\nabla \mathbf{M}|^2/M^2, \tag{1}$$

where A is a numerical coefficient characteristic of the material. This coefficient A, which we shall refer to as the "Bloch wall coefficient," is also important in the theory of the temperature variation of the saturation magnetization at low temperatures (theory of spin waves),[3] and in the theory of the influence of skin effect on microwave resonance phenomena in ferromagnetics.[4]

In the papers cited under reference 1 the value of A is calculated on the Heitler-London or atomic model, according to which the electrons responsible for ferromagnetism are treated as localized on individual atoms of the crystal lattice. The calculation has not been carried out previously on the band or collective electron ferromagnetism model,[5] such as is employed in the work of Stoner and Slater, where the electrons are not localized, but are pictured as running waves moving through the lattice. In view of the well-known inadequacies of either model by itself it is rather unsatisfactory that the Bloch wall has not been treated on the running wave model, and it is with this gap that the present paper and a following one[6] are concerned. Specifically, we shall undertake to calculate A in the Slater-Fock approximation, i.e., assuming that the wave function of a crystal in which the spin direction varies slowly with position can be represented as a determinant of one-electron wave functions. Spin-orbit and other purely magnetic interactions will be neglected.

Two approaches to the problem of calculating A for

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[1] F. Bloch, Z. Physik **74**, 295 (1932); L. Landau and E. Lifshitz, Physik Z. Sowjetunion **8**, 153 (1935); E. Lifshitz, J. Phys. U.S.S.R. **8**, 337 (1944); L. Néel, Cahiers phys. **25**, 1 (1944). For a summary of wall theory and other aspects of domain theory, see C. Kittel, Revs. Modern Phys. **21**, 541 (1949).

[2] C. Kittel, Phys. Rev. **70**, 965 (1946); Williams, Bozorth, and Shockley, Phys. Rev. **75**, 155 (1949).

[3] C. Herring and C. Kittel, Phys. Rev. **81**, 869 (1951).

[4] C. Kittel and C. Herring, Phys. Rev. **77**, 725 (1950).

[5] For a discussion of the various models used in ferromagnetism the following review articles are suggested: J. H. Van Vleck, Revs. Modern Phys. **17**, 27 (1945); W. Shockley, Bell System Tech. J. **18**, 645 (1939); E. C. Stoner, Rep. Prog. Phys. **11**, 43 (1946–47).

[6] C. Herring (to be published).