TABLE I. Relative decrease Δ of radiation loss in percent due to radiative correction for energies γMc^2 of heavy particle, $Z=18$ (argon).

γ	10	20	50	100	200	500	1000
$\Delta\times 100$	1.06	1.68	2.57	3.24	4.1	5.15	6

for the relative decrease of the collision loss per unit path length. The evaluation of this for the case of argon ($Z=18$) gives the result summarized in Table I.

The correction Δ would be a correction over and above the correction due to the density effect. It is difficult to compare this result with existing experiments. At the present time we can only conclude that if precise experiments at ultrarelativistic energies become available, the radiative correction would have to be included in the theoretical discussion. The method used here cannot claim any accuracy better than about a factor of two, and for a detailed comparison with experiment it would be necessary to refine the calculation.

Note added in proof: While this paper was in print there appeared a paper by H. D. Rathgeber [Z. Naturforsch. **6a,** 598 (1951)] on the energy loss of fast mesons in water. It is found that the energy loss of measons stays nearly constant in the range from 2 to 20×10^9 ev while according to the Bethe-Bloch theory it should increase 29 percent in this range. 16 percent of these can be accounted for as being due to the density effect, thus leaving 13 percent unexplained. The radiative correction here calculated would result in a further decrease of about 3–4 percent. Although the method of calculation used here does not claim accuracy better than about a factor of two, it seems difficult to explain the whole of the discrepancy found by Rathgeber as due to radiative corrections alone.

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The Interaction of a Charged Pi-Meson with the Deuteron*

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The radiative absorption, charge-exchange scattering, elastic and inelastic scattering of a π^+ meson by the deuteron are calculated in conventional weak-coupling theory for the pseudoscalar field with both direct and gradient coupling. The nucleon-nucleon interaction is treated phenomenologically and the "impulse" approximation employed. The behavior of the ratio $\sigma_{\text{nonradiative}}/\sigma_{\text{radiative}}$ for the *PS* theory is different from that for the *PV* theory. The charge-exchange scattering of a *PS* meson exhibits a minimum for forward scattering for both types of coupling but otherwise mirrors the behavior of the charge-exchange cross section for a neutron target. The elastic and inelastic scattering cross sections are comparable for an incident 25-Mev meson, and small energy transfers to the deuteron are favored, in contrast to the case in which the target nucleus is more complex.

INTRODUCTION

IN this paper, we shall investigate the interaction of a charged π-meson of positive energy with the deuteron. The nonradiative absorption of a π^+ meson by the deuteron ($\pi^++d\rightarrow p+p$) and its inverse ($p+p\rightarrow\pi^++d$) have been the subjects of a previous paper[1] by the author. The reactions to be considered here are:

$$\pi^++d\rightarrow\begin{cases} p+p+\gamma & \text{radiative absorption} \\ p+p+\pi^0 & \text{charge-exchange scattering} \\ d+\pi^+ & \text{elastic scattering} \\ n+p+\pi^+ & \text{inelastic scattering.}\end{cases}$$

Recent experimental results on the reactions[2,3] $\pi^++d\leftrightarrows p+p$ prove that the π^+ meson has zero spin. In addition, comparison of the theoretical results[4] with the experimental data[5] on the reactions π^- (K-shell) $+d\rightarrow n+n$ and $n+n+\gamma$ show that if the π^- meson has zero spin, it has odd parity (*PS*). If one assumes that the π^+ and π^- mesons differ only in the sign of their Coulombic charge, then one may assign zero spin and odd parity to the charged π-meson field. The calculation of the above reactions will be carried out assuming both direct (*PS*) and gradient (*PV*) coupling of the *PS* π-meson to a nucleon. Although the two nucleon interaction will be treated phenomenologically, it will be consistent to consider direct coupling for the *PS* field since we shall take account of the possibility of the meson interacting with negative energy state nucleons. The nucleons will be assumed to be Dirac particles, and transitions through intermediate negative energy states will be included. The calculation is performed assuming a positive charge for the π-meson; however, since the Coulomb force between the meson and the nucleus is ignored, the derived expressions are independent of the sign of the charge of the meson. The π^+ meson is con-

* This work was assisted by the AEC.
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1 W. Cheston, Phys. Rev. **83**, 1118 (1951).
2 Clark, Roberts, and Wilson, Phys. Rev. **83**, 649 (1951).
3 Cartwright, Richman, Whitehead, and Wilcox, Phys. Rev. **81**, 652 (1951); Crawford, Crowe, and Stevenson, Phys. Rev. **82**, 97 (1951).
4 S. Tamor, Phys. Rev. **82**, 38 (1951).
5 W. Panofsky (private communication).

sidered since the final state nucleons (two protons) in the radiative and nonradiative and charge-exchange scattering are easier to detect experimentally than those (two neutrons) which would result if the π^- meson had been considered.

GENERAL PROCEDURE

The reactions listed above will be treated as second-order processes on the assumption that the meson-nucleon coupling is weak (e.g., $g^2/\hbar c<1$). The weak coupling theory has been sufficiently successful in explaining the qualitative features of the data on the meson-nucleon reactions[6] to justify a study of the chief meson-deuteron reactions. We shall attempt to clarify in what respects the meson-deuteron and meson-nucleon reactions differ from each other and to extract those predictions which are not too sensitive to the details of the meson theory. Wherever possible, the results of the meson-deuteron calculations will be compared directly to the results of the meson-nucleon calculations.

Following Tamor[4] and Chew,[7] it will be assumed that the nucleon in the deuteron which interacts with the initial state quantum also interacts with the final state quantum. The possibility of, say, nucleon 1 interacting with the initial quantum and nucleon 2 with the final quantum will be assumed to be small and will be ignored. This is equivalent to the assumption that the characteristic time for the interaction of a nucleon with the initial and final state quanta is short compared to the period of the deuteron. The rather loose binding of the nucleons in the deuteron makes this a good approximation.

In general, the matrix element which must be evaluated in these problems may be written in this approximation:

$$M=\sum_{i=1}^{2}\int\Psi_f^*(\mathbf{R}_i,\boldsymbol{\sigma}_i,\boldsymbol{\tau}_i)\left\{\frac{\exp(-i\mathbf{k}'\cdot\mathbf{R}_i)}{(2E')^{\frac{1}{2}}}I\frac{\exp(i\mathbf{k}\cdot\mathbf{R}_i)}{(2E)^{\frac{1}{2}}}\right\}\times\Psi_0(\mathbf{R}_i,\boldsymbol{\tau}_i,\boldsymbol{\sigma}_i)d\mathbf{R}_1d\mathbf{R}_2,$$

where $\mathbf{R}_i$ = space coordinate of the ith nucleon; $\boldsymbol{\sigma}_i$ = spin coordinate of the ith nucleon; $\boldsymbol{\tau}_i$ = isotopic spin coordinate of the ith nucleon; $(\mathbf{k},E)$ = momentum-energy 4-vector of the initial quantum; $(\mathbf{k}',E')$ = momentum-energy 4-vector of the final quantum; and I = operator which transforms the initial two-nucleon state (deuteron) into the final state and represents the relativistic summation over plane-wave intermediate states. I will be evaluated for the different processes using the methods of Feynman.[8] For Ψ_0, the deuteron wave function, we shall choose the representation employed in our previous paper on the nonradiative absorption of a π^+ meson by the deuteron:

$$\Psi_0=\frac{N}{(4\pi)^{\frac{1}{2}}}\frac{e^{-\gamma R}-e^{-\beta R}}{R}|T_m\rangle\frac{|\tau_n^1\tau_p^2-\tau_n^2\tau_p^1\rangle}{\sqrt{2}}.$$

In the case of the radiative absorption reaction, plane waves will be assumed for the final two-nucleon wave function. This will be a good approximation since we shall only interest ourselves in the total cross section for the process. The validity of the plane wave approximation follows from the fact that the distribution at any angle in the gamma-ray energy k' will be highly peaked at $k_{\max}'$; little error is made if the square of the matrix element is given its value at $k_{\max}'$ and taken outside the integral over the density of states. The integral can then be extended over all energies instead of over just those allowed by the process. Since we thus sum over a complete set of functions for the two nucleons, this complete set can be the set of plane waves or the set of distorted waves insofar as the total cross section is concerned.

For the remaining reactions, the distortion of the final two-nucleon wave function will only be assumed to act in the S-state. The form of the even part of the final wave function is written:

$$\psi_{\text{even}}=\frac{e^{i\mathbf{k}\cdot\mathbf{R}}+e^{-i\mathbf{k}\cdot\mathbf{R}}}{\sqrt{2}}-\sqrt{2}\frac{\sin kR}{kR}+\sqrt{2}f(R)\frac{\sin(kR+\delta)}{kR},$$

where δ = S-phase shift, and $f(R)$ = zero energy wave function for the $n-p$ or $p-p$ systems. Since the major part of the cross section will come from that region in which the final state nucleons have below 10-Mev kinetic energy, we may use the expression for the phase-shifts which is independent of the shape of the nucleon-nucleon potential:[9]

$$k\cot\delta=-\alpha+\tfrac{1}{2}k^2r_0,$$

where α = scattering length, and r_0 = effective range of the nucleon-nucleon interaction.

The matrix element for the interaction of a π^+ meson with the deuteron may be calculated in the initial stages in the same manner as the interaction of a π^+ meson with a free nucleon. It is only in the final stages of the calculation, in which one is forced to restrict the number of final states for the nucleon, that the effect of the nucleon's presence in the deuteron will become apparent.

RADIATIVE ABSORPTION

It is assumed that only that nucleon in the deuteron which is originally a neutron can interact with the meson and photon fields:

$$\pi^+ + \begin{pmatrix} n \\ + \\ p \end{pmatrix} \begin{matrix} \to p+\gamma \\ \\ \to p. \end{matrix}$$

[6] R. E. Marshak, Revs. Modern Phys. **23**, 137 (1951).
[7] G. Chew, Phys. Rev. **80**, 196 (1950).
[8] R. Feynman, Phys. Rev. **76**, 749 (1949); Phys. Rev. **76**, 769 (1949).
[9] H. Bethe, Phys. Rev. **76**, 38 (1949); J. Blatt and J. Jackson, Phys. Rev. **76**, 18 (1949).

The matrix element for the radiative absorption of a π^+ meson by a neutron will only be calculated in the $PS(PV)$ theory. The equivalence theorem between the $PS(PV)$ and $PS(PS)$ theories[10] for this reaction is approximately valid, although conservation of energy for the reaction $\pi^+ + n \rightarrow p + \gamma$ does not hold when the absorbing neutron is in the deuteron. We shall ignore the contribution of the anomalous magnetic moments of the nucleons. Although the anomalous magnetic moments have been shown to give large contributions to PS π^0 production in photon-nucleon collisions, their contributions to charged meson production (the inverse of the process $\pi^+ + n \rightarrow p + \gamma$) are negligibly small.[11]

The three terms which enter into the matrix element may be written as follows[12] (setting $\hbar = c$ (velocity of light) $= M$ (nucleon mass) $= 1$):

$$4\pi\frac{ge}{\mu}\mathfrak{A}\frac{1}{\mathfrak{P}_0+\mathfrak{k}-1}\gamma_5\mathfrak{k}, \tag{1}$$

$$4\pi\frac{ge}{\mu}\gamma_5\mathfrak{k}''\frac{1}{k''\cdot k''-\mu^2}(A\cdot k+A\cdot k''), \tag{2}$$

$$-4\pi\frac{ge}{\mu}\gamma_5\mathfrak{A}, \tag{3}$$

where k = initial meson momentum-energy 4-vector; k'' = intermediate meson momentum-energy 4-vector; P_0 = initial nucleon momentum-energy 4-vector; and A = 4-vector polarization of the photon field. The first term represents the absorption of the π^+ meson by the neutron with the subsequent interaction of the proton current with the photon field:

$$\pi^+ + n \rightarrow p' \rightarrow p + \gamma.$$

The second term represents the interaction of the π^+ meson current with the photon field and the subsequent absorption of the π^+ meson by the neutron:

$$\pi^+ + n \rightarrow \gamma + \pi^{+\prime} + n \rightarrow \gamma + p.$$

The third term represents the "catastrophic" absorption of a π^+ meson by the neutron with the emission of a gamma-ray:

$$\pi^+ + n \rightarrow \gamma + p.$$

Throughout the calculation, it will be assumed that the terms depending on the momentum of the initial state neutron are negligible. This may be called the "target at rest" approximation. The matrix element is a slowly varying function of the target momentum; in addition, the kinematic factors which appear when we pass over to the case of the deuteron will strongly favor zero momentum for the target nucleons since the Fourier transform of the deuteron wave function is highly peaked at zero momentum.

The matrix element for the radiative absorption of a π^+ meson by a neutron is then, in nonrelativistic approximation:

$$M_n = 4\pi\frac{ge}{\mu}\frac{1}{(\mu^2+2E)}\left\{-2E\boldsymbol{\sigma} + \left(1-\frac{\mu^2+2E}{Ek'-\mathbf{k}\cdot\mathbf{k}'}\right)(\boldsymbol{\sigma}\cdot\boldsymbol{\kappa})\mathbf{k} - (\boldsymbol{\sigma}\cdot\boldsymbol{\kappa})(\boldsymbol{\sigma}\times\mathbf{k})\right\}\cdot\mathbf{A},$$

$$\mathbf{k}'-\mathbf{k}=\boldsymbol{\kappa}.$$

In the above, use has been made of the following device for obtaining the nonrelativistic approximation to the matrix element of an operator θ:

$$\langle f|\theta|i\rangle \sim \left\langle f\left|\frac{\mathfrak{P}^f+1}{2}\theta\frac{\mathfrak{P}_i+1}{2}\right|i\right\rangle.$$

We must now take care of the fact that the absorbing neutron is in the deuteron. The matrix element for the radiative absorption of a π^+ meson by the deuteron is:

$$M = \sum_{i=1}^{2}\int \Psi_f^*(\mathbf{R}_i, \boldsymbol{\sigma}_i, \boldsymbol{\tau}_i)\{M_n{}^i\tau_+{}^i\} \times \frac{\exp(i\boldsymbol{\kappa}\cdot\mathbf{R}_i)}{(4Ek')^{\frac{1}{2}}}\Psi_0(\mathbf{R}_i, \boldsymbol{\sigma}_i, \boldsymbol{\tau}_i)d\mathbf{R}_1 d\mathbf{R}_2,$$

where

$$\tau_+|\text{neutron}\rangle = |\text{proton}\rangle,$$
$$\tau_+|\text{proton}\rangle = 0.$$

Performing the indicated isotopic spin operation and passing over to the center-of-mass system, we find:

$$M = (8Ek')^{-\frac{1}{2}}\int \Psi_f^*(\mathbf{R};\boldsymbol{\sigma};\boldsymbol{\sigma}_2)\{M_n{}^{(1)}\exp(\tfrac{1}{2}i\boldsymbol{\kappa}\cdot\mathbf{R}) - M_n{}^{(2)}\exp(-\tfrac{1}{2}i\boldsymbol{\kappa}\cdot\mathbf{R})\}\psi_0(R)|T_m\rangle d\mathbf{R}.$$

For transitions to even parity plane wave final states:

$$M^{\text{even}} = \left\langle s\left|\frac{M_n{}^{(1)}-M_n{}^{(2)}}{(16Ek')^{\frac{1}{2}}}\right|T_m\right\rangle\{a_+ + a_-\},$$

where

$$a_\pm = \int \psi_0(R)\exp[i(\mathbf{K}\pm\tfrac{1}{2}\boldsymbol{\kappa})\cdot\mathbf{R}]d\mathbf{R},$$

and $\mathbf{K}$ = final nucleon momentum in the nucleon center-of-mass system. Using the expression for M_n found previously, we may write

$$M^{\text{even}} = -\frac{\pi ge}{\mu(\mu^2+2E)(Ek')^{\frac{1}{2}}}\langle s|\{\alpha\Delta\boldsymbol{\sigma}+\beta(\Delta\boldsymbol{\sigma}\cdot\boldsymbol{\kappa})\mathbf{k} + (\Delta\boldsymbol{\sigma}\times\boldsymbol{\kappa})\times\mathbf{k}\}\cdot\mathbf{A}|T_m\rangle,$$

[10] F. Dyson, Phys. Rev. **73**, 929 (1948).

[11] M. Kaplon, Phys. Rev. **83**, 712 (1951).

[12] We use the following notation: boldface roman **k** for a 3-vector, boldface italic ***k*** for a 4-vector, and lightface German $\mathfrak{k}$ for $\Sigma\gamma_\mu k_\mu$.

where

$$\alpha=2E,\quad \beta=\frac{\mu^2+2E}{Ek'-\mathbf{k}\cdot\mathbf{k}'}-1,\quad \Delta\boldsymbol{\sigma}=\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2.$$

The differential cross section in the laboratory system ignoring the odd-parity states contribution is then

$$\frac{d\sigma}{d\cos\alpha}=\frac{2N^2g^2e^2}{3\mu^2k(\mu^2+2E)^2}\int_0^{k_{\max}'(\alpha)}B^{\text{even}}K\Theta^2k'dk',$$

where

$$16\pi N^2B^{\text{even}}\equiv\int_{-1}^{+1}\{a_++a_-\}^2d(\cos\theta);\quad \theta=\measuredangle(\boldsymbol{\kappa},\mathbf{K})$$

$$B^{\text{even}}=\frac{1}{\Gamma_1^+\Gamma_1^-}+\frac{1}{\Gamma_2^+\Gamma_2^-}+\frac{1}{2\Gamma_1K\kappa}\ln\frac{\Gamma_1^+}{\Gamma_1^-}+\frac{1}{2\Gamma_2K\kappa}\ln\frac{\Gamma_2^+}{\Gamma_2^-}$$
$$-\frac{2}{K\kappa(\Gamma_2^2-\Gamma_1^2)}\left[\Gamma_2\ln\frac{\Gamma_1^+}{\Gamma_1^-}-\Gamma_1\ln\frac{\Gamma_2^+}{\Gamma_2^-}\right],$$

$$\begin{pmatrix}\Gamma_1\\\Gamma_2\end{pmatrix}=\begin{pmatrix}\gamma^2\\\beta^2\end{pmatrix}+K^2+\tfrac{1}{4}\kappa^2;\quad \Gamma_i^\pm=\Gamma_i\pm K\kappa,$$

$$\Theta^2=8E^2+4E\{k^2(1+\cos^2\alpha)-2\mathbf{k}\cdot\mathbf{k}'\}$$
$$+L^2k^2\sin^2\alpha(\mathbf{k}-\mathbf{k}')^2-4E(1+L)k^2\sin^2\alpha,$$

$$L=(\mu^2+2E)/(Ek'-\mathbf{k}\cdot\mathbf{k}'),\quad \alpha=\measuredangle(\mathbf{k},\mathbf{k}').$$

The integration over k' must be performed numerically.

The contribution to the cross section due to transitions to final nucleon states of odd parity will be small compared to the contribution from the even parity states. The major portion of each contribution will come from that region in which the final nucleons have essentially zero energy in the nucleon center-of-mass system. For absorption of a 25-Mev π^+ meson, the ratio $I^{\text{odd}}/I^{\text{even}}$ is about 0.1 where

$$I^{\text{even(odd)}}=\int B^{\text{even(odd)}}Kk'dk',$$

$$16\pi N^2B^{\text{odd}}\equiv\int_{-1}^{+1}\{a_+-a_-\}^2d(\cos\theta).$$

The spin sums, however, favor the odd-parity transitions. The total cross section for an incident 25-Mev π^+ meson is $3.6\ g^2/\hbar c\times10^{-27}$ cm². This is less than the cross section for the radiative absorption of a 25-Mev π^+ meson by a free neutron, which is calculated to be $5.9\ g^2/\hbar c\times10^{-27}$ cm² using the value of the cross section for the inverse process[11] and detailed balancing. The reduction of the cross section when the neutron is in the deuteron is due primarily to the operation of the Pauli principle.

If we define

$$R=\sigma_{\pi_++d\to p+p}/\sigma_{\pi_++d\to p+p+\gamma},$$

then, using the total cross section for the nonradiative absorption calculated in a previous paper,[1] we find

$$R_{25}=2.5.$$

Tamor[4] has shown that the ratio of the transition probabilities, R_K, in the $PS(PV)$ theory for the two competing reactions

$$\pi^-(K\text{-shell})+d\to\begin{cases}n+n\\n+n+\gamma,\end{cases}$$

is 2.1. Assuming that the π^- and π^+ mesons have the same spin and parity, we note that for the PS theory, R is essentially constant from 0 to 25-Mev meson energy. For a PV meson, on the other hand, several authors[4,13] have demonstrated that nonradiative absorption of a π^- meson from the K-shell of the mesic-deuterium atom should occur twice as frequently as radiative absorption. This prediction was based on the conservation of angular momentum and parity and the assumption that the different sub-states formed by the deuteron and spin one meson (i.e., $J=0, 1, 2$) are occupied according to their statistical weights. For the absorption of a charged spin one meson from the continuum, this result no longer holds and one must compare transition probabilities directly. This should offer a method of distinguishing between the PS and PV theories since Tamor[4] has shown that the ratio of transition probabilities for the PV theory at zero energy is about 130 compared to 2.1 for the PS theory.

CHARGE-EXCHANGE SCATTERING

As was the case in the radiative absorption of a π^+ meson by the deuteron, only that nucleon which is originally the neutron in the deuteron can interact with the π^+ and π^0 meson fields. We must therefore calculate the matrix element for the charge-exchange scattering of a π^+ meson by the neutron. Since the equivalence theorem does not hold for this reaction, we shall calculate the cross section for both the $PS(PV)$ and $PS(PS)$ theories.

The matrix element for the charge-exchange scattering of a π^+ meson by the neutron in the $PS(PV)$ theory may be written as the sum of two terms: (1) represents the absorption of the π^+ meson with subsequent emission of the π^0 meson; (2) represents the emission of the π^0 meson with the subsequent absorption of the π^+ meson:

$$-4\pi\frac{gg_p}{\mu\mu_0}\gamma_5\mathfrak{k}'\frac{1}{\mathfrak{P}_0+\mathfrak{k}-1}\gamma_5\mathfrak{k},\tag{1}$$

$$-4\pi\frac{gg_n}{\mu\mu_0}\gamma_5\mathfrak{k}\frac{1}{\mathfrak{P}_0-\mathfrak{k}'-1}\gamma_5\mathfrak{k}',\tag{2}$$

[13] Brueckner, Serber, and Watson, Phys. Rev. 81, 575 (1950).

TABLE I. Coefficients of $(g_p^2/\hbar c)$, $(g_n^2/\hbar c)$, and $(g_pg_n/\hbar c)$ in the expression for the charge-exchange differential cross section in units of $g^2/\hbar c\times 10^{-28}$ cm². The kinetic energy of the incident meson is 25 Mev. The *PS* theory with gradient coupling is assumed.

Angle of scattering	g_p^2	g_n^2	g_ng_p
0°	0.29	0.11	−0.35
30°	11.9	7.2	−7.1
60°	17.2	7.8	5.4
90°	7.9	8.9	16.7
120°	4.8	20.8	−9.2
180°	8.2	40.8	−36.2

where g_n= coupling constant of π^0 meson to the neutron, and g_p= coupling constant of π^0 meson to the proton. Those parts of the nonrelativistic approximation to the matrix elements which do not contain terms involving the Pauli spin operators are

(1)

$$4\pi\frac{gg_p}{\mu\mu_0}\frac{E'}{2E+\mu^2}\left[\mu^2+\frac{2}{E'}\mathbf{k}\cdot\mathbf{k}'\right],$$

(2)

$$4\pi\frac{gg_n}{\mu\mu_0}\frac{E}{2E'-\mu_0^2}\left[\mu_0^2-\frac{2}{E}\mathbf{k}\cdot\mathbf{k}'\right].$$

The leading spin dependent terms are

(1) (2)

$$2\pi i\frac{gg_p}{\mu\mu_0}\boldsymbol{\sigma}\cdot\mathbf{k}'\times\mathbf{k}\frac{4+\mu^2+2E}{\mu^2+2E},\quad 2\pi i\frac{gg_n}{\mu\mu_0}\boldsymbol{\sigma}\cdot\mathbf{k}'\times\mathbf{k}\frac{4+\mu_0^2-2E'}{2E'-\mu_0^2}.$$

The nonrelativistic approximations written above were obtained by the same method used for the radiative absorption reaction and are evaluated in the "target at rest" approximation.

If we let M_n represent the matrix element for the charge-exchange scattering of a π^+ meson by the neutron in the $PS(PV)$ theory, then the corresponding matrix element for the charge-exchange scattering of a π^+ meson by the deuteron for transitions to triplet plane wave final states may be written:

$$M^t=\left\langle T_m'\left|\frac{M_n^{(1)}+M_n^{(2)}}{(16EE')^{\frac{1}{2}}}\right|T_m\right\rangle\{a_--a_+\}.$$

For transitions to final singlet plane wave states:

$$M^s=\left\langle s\left|\frac{M_n^{(1)}-M_n^{(2)}}{(16EE')^{\frac{1}{2}}}\right|T_m\right\rangle\{a_-+a_+\}.$$

Using the explicit expression for M_n found above, these matrix elements become

$$M^t=\frac{4\pi g}{\mu\mu_0}\left\langle T_m'\left|\left\{\frac{g_p}{2E+\mu^2}\left[E'\left(\mu^2+\frac{2}{E'}\mathbf{k}\cdot\mathbf{k}'\right)+\frac{i}{4}(\boldsymbol{\sigma}^1+\boldsymbol{\sigma}^2)\cdot\mathbf{k}'\times\mathbf{k}(4+\mu^2+2E)\right]+\frac{g_n}{2E'-\mu_0^2}\left[E\left(\mu_0^2-\frac{2}{E}\mathbf{k}\cdot\mathbf{k}'\right)+\frac{i}{4}(\boldsymbol{\sigma}^1+\boldsymbol{\sigma}^2)\cdot\mathbf{k}'\times\mathbf{k}(4+\mu_0^2-2E')\right]\right\}\right|T_m\right\rangle\{a_--a_+\}$$

$$M^s=\frac{\pi i g}{\mu\mu_0}\langle s|\Delta\boldsymbol{\sigma}\cdot\mathbf{k}'\times\mathbf{k}|T_m\rangle\times\left\{g_p\frac{4+\mu^2+2E}{2E+\mu^2}+g_n\frac{4+\mu_0^2-2E'}{2E'-\mu_0^2}\right\}\{a_++a_-\}.$$

The differential cross section follows immediately and is

$$\frac{d\sigma}{d\cos\alpha}=\frac{2N^2}{k}\int_0^{k'_{\max}(\alpha)}\frac{K}{E'}\{B^{\text{odd}}\langle M^{t^2}\rangle+B^{\text{even}}\langle M^{s^2}\rangle\}\frac{k'^2dk'}{(4\pi)^2}$$

$$16\pi N^2B^{\text{odd}}\equiv\int_{-1}^{+1}\{a_+-a_-\}^2d\cos\theta$$

$$B^{\text{odd}}=\frac{1}{\Gamma_1^+\Gamma_1^-}+\frac{1}{\Gamma_2^+\Gamma_2^-}-\frac{1}{2\Gamma_1K\kappa}\ln\frac{\Gamma_1^+}{\Gamma_1^-}-\frac{1}{2\Gamma_2K\kappa}\ln\frac{\Gamma_2^+}{\Gamma_2^-}-\frac{2}{K\kappa(\Gamma_2^2-\Gamma_1^2)}\left[\Gamma_1\ln\frac{\Gamma_1^+}{\Gamma_1^-}-\Gamma_2\ln\frac{\Gamma_2^+}{\Gamma_2^-}\right],$$

where $\langle\ \rangle\equiv$ average over final nucleon spin states. In obtaining the differential cross section, the integration over the directions of the final state nucleons in the nucleon center-of-mass has been performed using $\boldsymbol{\kappa}$ as the polar axis. The coefficients of g_ng_p, g_n^2, and g_p^2 in the expression for the differential cross section for an incident 25-Mev meson are listed in Table I at five different angles of scattering in the laboratory system.

If the matrix element is a slowly varying function of k' over the region of integration, then we can compare the charge-exchange scattering of a π^+ meson by the deuteron to the charge-exchange scattering by a neutron. If the above condition holds, then we may write the differential cross section for the deuteron reaction as

$$\frac{d\sigma}{d\cos\alpha}=\frac{2N^2}{k}\{\langle M^{t^2}\rangle I_t(\alpha)+\langle M^{s^2}\rangle I_s(\alpha)\}.$$

The differential cross section for the reaction π^+

$+n\rightarrow p+\pi^0$ is

$$\frac{d\sigma}{d\cos\alpha}=\frac{2\pi}{k}<M_n^2>\frac{q^2}{q(1+E'')-E''k\cos\alpha},$$

where $(\mathbf{q}, E'')$ = energy-momentum 4-vector of scattered meson in the laboratory system. At 90°, the coefficients of g_n^2, g_p^2, and g_ng_p in the expression for the cross section for the deuteron reaction vary slowly for those values of k' which give the major contribution to the cross section. Therefore the square of the matrix element may be removed from the integral over the density of states at this angle. In addition, at 90°:

$$<M_t^2>\sim\tfrac{2}{3}<M_n^2>,$$
$$<M_s^2>\sim\tfrac{1}{3}<M_n^2>.$$

Therefore, for an incident 25-Mev meson,

$$R_{d/n}=|(d\sigma/d\cos\alpha)_{\text{deuteron}}/(d\sigma/d\cos\alpha)_{\text{neutron}}|_{90°}\sim 0.35.$$

This is a reasonable value since we expect the ratio to be less than 0.5. The factor $\frac{1}{2}$ arises because the two final state nucleons are identical in the deuteron reaction. Additional factors tending to decrease the ratio result from the properties of the overlap integrals:

$$\int\psi_0\exp(\tfrac{1}{2}i\boldsymbol{\kappa}\cdot\mathbf{R})[(e^{i\mathbf{K}\cdot\mathbf{R}}\pm e^{-i\mathbf{K}\cdot\mathbf{R}})/\sqrt{2}]d\mathbf{R}<1.$$

The approximation concerning the constancy of the matrix element over the allowed range of k' is not a good one as soon as $\cos\alpha$ approaches unity. However, the order of magnitude of $R_{d/n}$ at 0° can be obtained by a comparison of $I_t(0°)$ to $I_t(90°)$. We find for an incident 25-Mev meson:

$$I_t(0°)/I_t(90°)\sim 1.3\times 10^{-3}.$$

We therefore expect a minimum in the cross section for forward scattering.

The value of $R_{d/n}$ at 90° estimated above is not a good one if we assume a symmetrical meson theory (e.g., $g_n=-g_p$). This is because the matrix element at 90° will now vary rapidly with k' due to the near cancellation at this angle for the symmetrical theory. Table II shows the variation in $d\sigma/d\cos\alpha$ with angle for an incident 25-Mev meson in the case of the deuteron reaction for $g_n=-g_p$. All values are normalized to $d\sigma/d\cos 180°=1$ since the cross section exhibits no anomalies at this angle. In addition, Table II also shows the variation of $d\sigma/d\cos\alpha$ with angle for an incident 25-Mev meson in the case of the neutron for $g_n=-g_p$. These values were obtained using the analytic expressions given by Ashkin, Simon, and Marshak[14] modified[15] to take into account the mass difference between the charged and neutral mesons. These values are also normalized to $d\sigma/d\cos 180°=1$.

[14] Ashkin, Simon, and Marshak, Prog. Theor. Phys. 5, 634 (1950).
[15] T. Auerbach (private communication).

The values at 90° are quite different for the two reactions. The cancellation in the matrix element for $g_n=-g_p$ seems to be more complete in the case of the deuteron. This is due to the neglect in the matrix element of terms which are, at first thought, negligible compared to those considered. At 90°, however, the almost complete cancellation of the large terms causes the small terms to assume an important role. These small relativistic terms cannot be consistently included in the deuteron calculation since only terms up to and including $(v/c)_{\text{nucleon}}$ may be used in the matrix element; the decomposition of the matrix element for the deuteron calculation into matrix elements between plane waves is only valid if terms in $(v/c)_{\text{nucleon}}^2$ and higher can be neglected. It is therefore assumed that a more correct theory for the deuteron would give approximately 10^{-2} for the ratio $(d\sigma/d\cos 90°)/(d\sigma/d\cos 180°)$ instead of approximately 10^{-3}.

In the case of the $PS(PS)$ theory, the prediction concerning the small value of the cross section for forward scattering remains unchanged. The precipitous drop in the cross section at 90° when use is made of the symmetrical theory does not occur in the $PS(PS)$ theory since the scattering from an individual nucleon does not exhibit any cancellation at 90°. The matrix element for the charge-exchange scattering of a π^+ meson by a neutron may be written in the $PS(PS)$ theory as

$$-\frac{4\pi g}{\mu\mu_0}\left\{g_p\gamma_5\frac{1}{\mathfrak{P}_0+\mathfrak{k}-1}\gamma_5+g_n\gamma_5\frac{1}{\mathfrak{P}_0-\mathfrak{k}'-1}\gamma_5\right\}.$$

The nonrelativistic approximation to this operator is immediate:

$$M_n=\frac{4\pi g}{\mu\mu_0}\left\{g_p\frac{E}{\mu^2+2E}+g_n\frac{E'}{2E'-\mu_0^2}\right\}.$$

The terms which have been neglected in the above are of order $(v/c)_{\text{meson}}\times(v/c)_{\text{nucleon}}$ smaller than those considered. The total cross section for an incident 25-Mev π^+ meson is

$$\sigma=\{0.50g_p^2/\hbar c+1.81g_n^2/\hbar c+2.06g_pg_n/\hbar c\}(g^2/\hbar c)\times 10^{-28}\text{ cm}^2.$$

The angular distribution is determined by the expres-

TABLE II. Differential cross section for the charge-exchange scattering of a 25-Mev π^+ meson from the deuteron and from the neutron in the $PS(PV)$ theory with $g_n=-g_p$. All values normalized to $d\sigma/d\cos 180°=1$.

Angle of scattering	Deuteron	Neutron
0°	8.8×10^{-3}	1.2
30°	0.31	0.76
60°	0.23	0.45
90°	1.6×10^{-3}	1.0×10^{-2}
120°	0.41	0.59
180°	1	1

sion I_t defined previously. For $g_n=-g_p$ the $PS(PS)$ cross section becomes small. This effect is also present in the $PS(PS)$ theory for the charge-exchange scattering from a single nucleon.

ELASTIC SCATTERING

The elastic and inelastic scattering of a π^+ meson differ in one major respect from the reactions previously considered. Both the neutron and the proton in the deuteron can scatter the meson. This will cause an added interference effect not found in the reactions considered previously. Once again the approximation will be made that, as far as the nucleon which is interacting with the meson fields is concerned, the other nucleon merely "goes along for the ride."

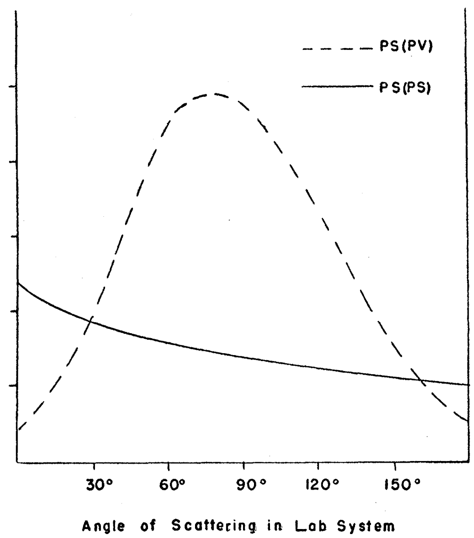

FIG. 1. Angular distribution in the laboratory system of π^+ mesons elastically scattered from the deuteron. The kinetic energy of the incident meson is 25 Mev.

The matrix element for the elastic scattering of a π^+ meson by the deuteron can be written:

$$M=\sum_{i=1}^{2}\int\Psi_f^*\left\{\frac{M_n{}^i\tau_n{}^i+M_p{}^i\tau_p{}^i}{(4EE')^{\frac{1}{2}}}\right\}\exp(i\boldsymbol{\kappa}\cdot\mathbf{R}_i)d\mathbf{R}_1d\mathbf{R}_2,$$

where M_n = matrix element for the reaction:

$$\pi^++n\rightarrow p'\rightarrow\pi^{+\prime}+n'$$

M_p = matrix element for the reaction:

$$\pi^++p\rightarrow\pi^++n''+\pi^{+\prime}\rightarrow\pi^{+\prime}+p'$$

$\tau_n|\text{neutron}\rangle=|\text{neutron}\rangle$,

$\tau_n|\text{proton}\rangle=0$,

etc.

Since the deuteron is charge antisymmetric, performing the indicated isotopic spin operation, we find

$$M=\frac{1}{(16EE')^{\frac{1}{2}}}\int\Psi_f^*(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2;\mathbf{R}_1,\mathbf{R}_2)$$
$$\times\{(M_n{}^{(1)}+M_p{}^{(1)})\exp(i\boldsymbol{\kappa}\cdot\mathbf{R}_1)$$
$$+(M_n{}^{(2)}+M_p{}^{(2)})\exp(i\boldsymbol{\kappa}\cdot\mathbf{R}_2)\}$$
$$\times\Psi_0(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2;\mathbf{R}_1,\mathbf{R}_2)d\mathbf{R}_1d\mathbf{R}_2.$$

Integration over the center-of-mass coordinates yields

$$M=1/2(2\pi)^3(4EE')^{\frac{1}{2}}\int a_{\mathrm{K}^+}a_{\mathrm{K}}\langle T_m'|M^{(1)}+M^{(2)}|T_m\rangle dK$$

$$M^{(i)}=M_n{}^{(i)}+M_p{}^{(i)};\quad \mathbf{K}^+=\mathbf{K}+\tfrac{1}{2}\boldsymbol{\kappa}.$$

As was the case in the reactions previously considered, we shall make the "target at rest" approximation:

$$M=\frac{\langle T_m'|M^{(1)}+M^{(2)}|T_m\rangle}{2(2\pi)^3(4EE')^{\frac{1}{2}}}\int a_{\mathrm{K}^+}a_{\mathrm{K}}d\mathbf{K}.$$

The indicated integration is immediate yielding

$$M=\frac{\langle T_m'|M^{(1)}+M^{(2)}|T_m\rangle}{(16EE')^{\frac{1}{2}}}\int\psi_0{}^2(R)\exp\left(\frac{i}{2}\boldsymbol{\kappa}\cdot\mathbf{R}\right)d\mathbf{R}$$
$$=\frac{\langle T_m'|M^{(1)}+M^{(2)}|T_m\rangle}{(16EE')^{\frac{1}{2}}}a'^2.$$

The matrix element for the scattering of π^+ meson by a neutron and by a proton may be calculated using the Feynman techniques. In the case of the elastic scattering of the π^+ meson by the deuteron we must form M_n+M_p. The nonrelativistic approximations to M_n and M_p may be found by setting $g=g_p=g_n$ and $\mu=\mu_0$ in the charge-exchange scattering matrix elements. If we let A be that part of the interaction operator which does not contain $\boldsymbol{\sigma}$ and B be that part which does depend on $\boldsymbol{\sigma}$, then the differential cross section in the laboratory for the elastic scattering of a π^+ meson by the deuteron is

$$\frac{d\sigma}{d\cos\alpha}=\frac{4\pi k'^2}{k}\frac{a'^2}{E'(k'-k\cos\alpha)+2k'}\{A^2+\tfrac{2}{3}B^2\}.$$

a'^2 is a maximum for forward scattering ($\boldsymbol{\kappa}=0$) and decreases as the angle of scattering increases. We know that in the $PS(PV)$ theory, M_n+M_p increases as the angle of scattering increases from 0° to 90°. The rate of increase of M_n+M_p with increasing scattering angle in this region overpowers the decrease in a'^2 for the momentum transfers which enter into this problem. For the $PS(PS)$ theory, M_n+M_p is essentially constant from 0° to 180°. Therefore, the PS theory with direct coupling predicts a maximum for forward scattering whereas the *PS* theory with gradient coupling predicts a maximum at approximately 90°.

The differential cross section for elastic scattering of a 25-Mev π^+ meson by the deuteron in the *PS* theories is shown in Fig. 1. The total cross sections at 25 Mev are

$$\sigma_{PS(PS)}=3.6\times10^{-27}\ (g^2/\hbar c)^2\ \text{cm}^2,$$

$$\sigma_{PS(PV)}=2.4\times10^{-26}\ (g^2/\hbar c)^2\ \text{cm}^2.$$

The total cross sections for the scattering of a 25-Mev meson by a nucleon have been calculated by Auerbach[15] and are

$$\left.\begin{aligned}\sigma_{PS(PS)}&=0.86\times10^{-27}\ (g^2/\hbar c)^2\ \text{cm}^2\\ \sigma_{PS(PV)}&=1.67\times10^{-26}\ (g^2/\hbar c)^2\ \text{cm}^2\end{aligned}\right\}\pi^++n\rightarrow n+\pi^+,$$

$$\left.\begin{aligned}\sigma_{PS(PS)}&=1.21\times10^{-27}\ (g^2/\hbar c)^2\ \text{cm}^2\\ \sigma_{PS(EV)}&=1.53+10^{-26}\ (g^2/\hbar c)^2\ \text{cm}^2\end{aligned}\right\}\pi^++p\rightarrow p+\pi^+.$$

If $M_n=M_p$ and did not contain terms which depended on the nucleon spin, we would expect a cross section for elastic scattering by the deuteron to be approximately four times the cross section for the scattering from an individual nucleon. Since a'^2 is less than unity except in the forward direction, and in addition, since M_n is only approximately equal to M_p even in the *PS*(*PS*) theory, we would expect some reduction in the factor four. In the *PS*(*PV*) theory, destructive interference in certain regions as well as terms depending on the nucleon spin cause a further decrease in the factor four. The actual numbers quoted above bear out this general argument. The energy dependence of the deuteron cross section mirrors the scattering from a single nucleon as calculations at 50 and 100 Mev have shown.

The cross sections for the elastic scattering of a π^+ meson at 50 and 100 Mev in the *PS*(*PV*) theory are

$$\sigma(50\ \text{Mev})=6.2\times10^{-26}\ (g^2/\hbar c)^2\ \text{cm}^2,$$

$$\sigma(100\ \text{Mev})=11.2\times10^{-26}\ (g^2/\hbar c)^2\ \text{cm}^2.$$

Several other authors[16,17] have investigated the elastic scattering of a π^+ meson by the deuteron in the *PS* theories. Blair[16] has found results in general agreement with the results quoted above. However, Ferretti and Gallone[17] find that the ratio of the square of the matrix element for the scattering by the deuteron to the square of the matrix element for the scattering from a proton is about 1/10 at 25 Mev. This result is in disagreement with the results of Blair and those quoted here and is difficult to understand since destructive interference effects only play a role for forward scattering in the *PS*(*PV*) theory and do not exist at all in the case of the *PS*(*PS*) theory.

[16] J. Blair, Phys. Rev. 83, 1246 (1951).
[17] B. Ferretti and S. Gallone, Phys. Rev. 77, 153 (1950).

INELASTIC SCATTERING

The inelastic scattering of a π^+ meson by the deuteron is carried out in a manner similar to that employed for elastic and charge-exchange scattering. In contrast to the previous reactions, no states are excluded from the final state nucleons by the Pauli principle or by the requirement that the final state be bound. For reasons which will become evident, it is convenient to group the final states into the following categories:

SE singlet spin—even parity,
SO singlet spin—odd parity,
TE triplet spin—even parity,
TO triplet spin—odd parity.

SE

The matrix element for this transition is

$$M^{SE}=\sum_{i=1}^{2}\int\frac{\langle\tau_n{}^1\tau_p{}^2+\tau_p{}^1\tau_n{}^2|}{\sqrt{2}}\times\left\{\frac{M_n{}^i\tau_n{}^i+M_p{}^i\tau_p{}^i}{(4EE')^{\frac{1}{2}}}\right\}\frac{|\tau_n{}^1\tau_p{}^2-\tau_n{}^2\tau_p{}^1\rangle}{\sqrt{2}}\times\exp(i\boldsymbol{\kappa}\cdot\mathbf{R}_i)\Psi_{SE}{}^*(\mathbf{R}_1,\mathbf{R}_2;\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)\times\Psi_0(\mathbf{R}_1,\mathbf{R}_2;\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)d\mathbf{R}_1d\mathbf{R}_2.$$

Carrying out the indicated isotopic spin operation and considering plane wave final states, M_{SE} becomes

$$M_{SE}=\frac{1}{2\sqrt{2}}\left\langle S\left|\frac{\theta_1'-\theta_2'}{(4EE')^{\frac{1}{2}}}\right|T_m\right\rangle\{a_++a_-\},$$

where $\theta_i'=M_n{}^i-M_p{}^i$.

The matrix element for a *SE* transition will be small for two reasons: (1) destructive interference between M_n and M_p; (2) only the spin dependent terms will contribute. This may be seen in still another way which is perhaps more general. The intermediate state nucleons can only occupy *SE* or *TO* states since they are identical. If we consider only the *S*-part of the initial meson wave function, then the transition must proceed through the *TO* intermediate state by conservation of parity (the *PS* meson has an intrinsic odd parity). However, conservation of angular momentum and parity between initial and final states requires the nucleons and the meson to be in states of $l=2$ in the final state. This will make transitions through *TO* intermediate states vanishingly small. The transition to *SE* final states can be carried out through the *SE* intermediate states, but for this transition the odd angular momentum states of both the initial and final state mesons must contribute if parity and angular momentum are to be conserved. This reduces M_{SE} in comparison to those matrix elements in which the *S*-part of one of the mesons can contribute.

If we define B' as the spin dependent part of $\theta_1'-\theta_2'$, then the differential cross section for transitions to SE final states is

$$\frac{d\sigma}{d\cos\alpha}=\frac{2N^2}{3k}\int_0^{k'_{\max}(\alpha)}|B'|^2\frac{K}{E'}B^{\text{even}}\frac{k'^2dk'}{(4\pi)^2}.$$

TE

In this case, the matrix element is formally the same as in the elastic scattering reaction:

$$M^{TE}=\sum_{i=1}^{2}\int\frac{\langle\tau_n^1\tau_p^2-\tau_p^1\tau_n^2|}{\sqrt{2}}\times\left\{\frac{M_n^i\tau_n^i+M_p^i\tau_p^i}{(4EE')^{\frac{1}{2}}}\right\}\frac{|\tau_n^1\tau_p^2-\tau_n^2\tau_p^1\rangle}{\sqrt{2}}\times\exp(i\boldsymbol{\kappa}\cdot\mathbf{R}_i)\Psi_{SE}^*(\mathbf{R}_1,\mathbf{R}_2;\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)\times\Psi_0(\mathbf{R}_1,\mathbf{R}_2;\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)d\mathbf{R}_1d\mathbf{R}_2.$$

Carrying out the indicated isotopic spin operation and considering plane wave final states,

$$M^{TE}=\frac{1}{2\sqrt{2}}\left\langle T_m'\left|\frac{\{\theta_1+\theta_2\}}{(4EE')^{\frac{1}{2}}}\right|T_m\right\rangle\{a_++a_-\},$$

$$\theta_i=M_n^i+M_p^i.$$

In contrast to the case of M^{SE}, here transitions through TO intermediate states are not small and can proceed through the S-part of either the final or initial meson. If A and B are given the same definition as in the section on elastic scattering, then the differential cross section for transitions to TE final states is

$$\frac{d\sigma}{d\cos\alpha}=\frac{2N^2}{k}\int_0^{k'_{\max}(\alpha)}\frac{K}{E'}\{A^2+\tfrac{2}{3}B^2\}B^{\text{even}}\frac{k'^2dk'}{(4\pi)^2}.$$

SO

M^{SO} is identical to M^{TE} with Ψ_{TE}^* replaced by Ψ_{SO}^*. M^{SO} can therefore be written:

$$M^{SO}=\frac{1}{2\sqrt{2}}\left\langle S\left|\frac{\theta_1-\theta_2}{(4EE')^{\frac{1}{2}}}\right|T_m\right\rangle\{a_+-a_-\}.$$

The differential cross section for transitions to SO final states is then

$$\frac{d\sigma}{d\cos\alpha}=\frac{2N^2}{3k}\int_0^{k'_{\max}(\alpha)}\frac{K}{E'}B^{\text{odd}}B^2\frac{k'^2dk'}{(4\pi)^2}.$$

Transitions through SE intermediate states are possible by means of the P-part of one of the meson wave functions and the S-part of the other meson wave function. The P-part of one of the meson wave functions is needed for the transition from the initial to the intermediate SE state. However, the S-part of the wave function of the remaining meson is all that is needed for the transition from the SE intermediate state to the SO final state. Even though $B^{\text{even}}\sim 16B^{\text{odd}}$ for a 25-Mev meson, M^{SO} will be larger than M^{SE}.

TO

M^{TO} is identical to M^{SE} with Ψ_{SE}^* replaced by Ψ_{TO}^*. M^{TO} can therefore be written:

$$M^{TO}=\frac{1}{2\sqrt{2}}\left\langle T_m'\left|\frac{\theta_1'+\theta_2'}{(4EE')^{\frac{1}{2}}}\right|T_m\right\rangle\{a_+-a_-\}.$$

If A' is defined as that part of θ_i' which does not depend on $\boldsymbol{\sigma}$ and B' is defined as that part of θ_i' which does depend on $\boldsymbol{\sigma}$, then the differential cross section for transitions to TO final states is

$$\frac{d\sigma}{d\cos\alpha}=\frac{2N^2}{k}\int_0^{k'_{\max}(\alpha)}\frac{K}{E'}B^{\text{odd}}\{A'^2+\tfrac{2}{3}B'^2\}\frac{k'^2dk'}{(4\pi)^2}.$$

Destructive interference between the neutron and proton in addition to the small amplitude of the final nucleon states makes this transition small compared to the TE and SO transitions.

The angular distribution in the laboratory for the PS theories is shown in Fig. 2 for an incident 25-Mev meson. The total cross sections at this energy are

$$\sigma_{PS(PS)}=2.4\times10^{-27}\ (g^2/\hbar c)^2\ \text{cm}^2,$$

$$\sigma_{PS(PV)}=1.6\times10^{-26}\ (g^2/\hbar c)^2\ \text{cm}^2.$$

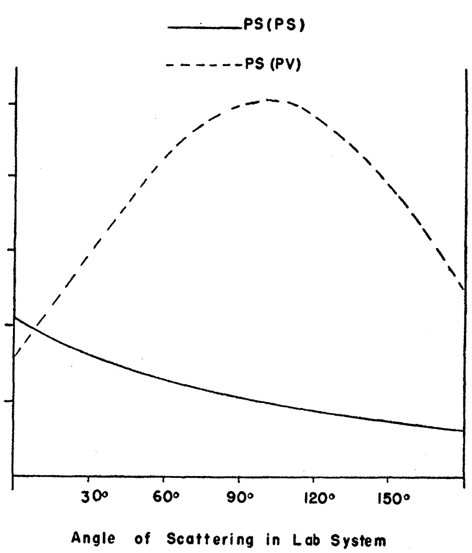

FIG. 2. Angular distribution in the laboratory system of π^+ mesons inelastically scattered from the deuteron. The kinetic energy of the incident meson is 25 Mev.

The effect of the distortion of the final state $n-p$ wave function is small. At 90°, the value of the cross section is increased by approximately 2 percent over its value for plane wave final states. However, as is the case for those reactions in which a photon is the final state quantum, the distribution in energy of the final state meson at a given angle is greatly modified. Figure 3 shows the energy distribution for the final state meson for 90° scattering with and without taking into account the $n-p$ interaction. The small value of the meson energy half-width (2 Mev), in addition to the fact that the major transition is to *TE* final states, causes the inelastic scattering differential cross section to exhibit the same properties as the differential cross section for elastic scattering, in which the final state meson energy distribution is a delta-function at a given angle and in which the transition is solely to *TE* states.

The ratio of the cross section for elastic scattering to that for inelastic scattering is 1.5 for both couplings at 25 Mev. Both the energy distribution of the final state nucleons and the ratio of the elastic and inelastic cross sections are similar to those associated with the reactions $p+p\rightarrow\pi^++d$, $p+p\rightarrow\pi^++n+p$ which have been measured experimentally at a comparable meson energy.

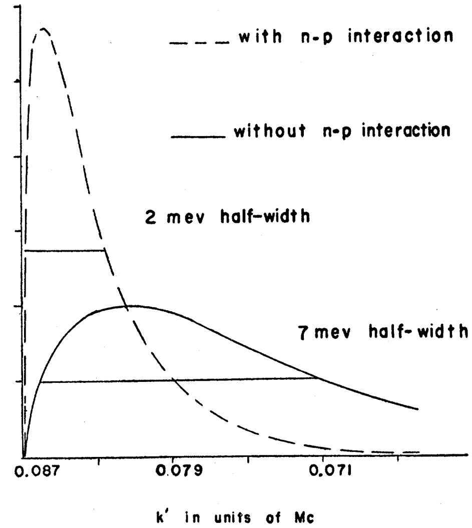

FIG. 3. Energy distribution of π^+ mesons inelastically scattered from the deuteron through 90°. The kinetic energy of the incident meson is 25 Mev. The effect of considering the interaction of the neutron and proton in the final state is shown.

DISCUSSION

Examination of the nonradiative absorption of a π^+ meson by the deuteron and the inverse reaction has made possible the determination of the spin of the π^+ meson. However, with our present knowledge of the deuteron wave function at small distances or, what is equivalent, the behavior of the nucleon-nucleon potential at small nucleon separations, the *PS* theory does not seem to be able to explain the large angular anisotropy[3] in the differential cross section for the nonradiative absorption reaction. In fact, assuming that the charged π-meson is *PS*, the differential cross section for the nonradiative absorption reaction suggests a method of investigating the extremely short-range properties of the $n-p$ and $p-p$ potentials. The radiative absorption reaction affords a method of choosing between the *PS* and *PV* theories for the charged π-meson since the behavior of the ratio $\sigma_{\text{nonradiative}}/\sigma_{\text{radiative}}$ is completely different for the two theories. This result is independent of the weak coupling theory and is based only on the form of the interaction operators for the two theories. The charge-exchange scattering reaction affords a method of investigating the charge-exchange scattering of a π^+ meson from a neutron, since the presence of the proton in the deuteron only restricts the number of states available to the final nucleons. The interference between the scattering of a π^+ meson from a neutron and from a proton is exhibited in the elastic and inelastic scattering of a π^+ meson by the deuteron. The theoretical cross sections for elastic and inelastic scattering from the deuteron at 25 Mev are comparable, and high energy for the final state meson is favored in the inelastic scattering reaction. This is in contrast to the experimental results[18] on the inelastic scattering of a π^+ meson from a more complex nucleus in which large energy transfers from the incident meson to the nucleus seem to be involved. In the case of the deuteron, the theoretical results predict low relative momenta for the final nucleons because of the strong interaction of the final state nucleons and the high Fourier amplitude for the low momentum components of the deuteron.

ACKNOWLEDGMENTS

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[18] G. Bernardini, Phys. Rev. **82**, 313 (1951).