Radiative Correction for the Collision Loss of Fast Particles

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The correction for the collision loss of fast heavy particles because of the interaction with the radiation field is calculated approximately. The radiative corrections always result in a decrease of the collision loss. Only the collisions with energy loss $\epsilon > \gamma mc^2$ contribute significantly to this result. For $\gamma = 1000$ and $Z = 18$ (argon) the correction amounts to about 6 percent.

THE energy loss of fast particles by collisions with the atomic electrons has a minimum if the kinetic energy is approximately equal to the rest energy of the colliding particle. Beyond this value the theory predicts an expression for the specific energy loss which is essentially of the form[1]

$$-dW/dx = A \ln\gamma^2 + B, \tag{1}$$

with

$$A \simeq 4\pi ne^4/mc^2, \quad B \simeq (4\pi ne^4/mc^2) \ln(2mc^2/I), \quad \gamma = [1-(v^2/c^2)]^{-\frac{1}{2}}. \tag{2}$$

The formula (1) is derived on the basis of the assumption that the collision occurs with one single atom. If the mutual interaction of the atoms is included then the energy loss for very energetic particles decreases slightly because of the shielding effect of the closer atoms.

This effect has been calculated by a number of different workers, with slightly different results.[2] They agree, however, in predicting less ionization loss than the Bethe-Bloch formula and the gradual approach of a constant value. Recent experiments with ultrarelativistic particles in various materials tended to confirm this general behavior.[3]

A significant departure from the theoretical energy loss formula was recently reported, however, by Goodman, Nicholson, and Rathgeber.[4] They report the energy loss of cosmic-ray mesons to lie considerably below the theoretical value for values of γ up to about 10^3.

Since the collision loss depends only on the electromagnetic interaction of charged particles, the establishment of such a discrepancy would be of considerable theoretical interest. The only electromagnetic effect which could possibly be called upon to give an appreciable correction at ultrarelativistic energies would be the radiative corrections.

The radiative corrections to the collision of charged particles calculated with the methods of quantum electrodynamics are finite, if proper care is taken to remove infinities by the process of "renormalization." The rigorous calculation of the radiative corrections to the collision loss in this manner is rather involved and has not yet been done. It is, however, possible to obtain an estimate of the order of magnitude of this effect by reducing it to the problem of the radiative corrections to the scattering in a fixed Coulomb potential. This problem has been treated in detail by Schwinger.[5] The result is given in Eqs. (2.101) and (2.102) of Schwinger's paper.

In order to apply this result to the problem of collision loss we consider the problem in the center-of-mass system. If the atomic electron is for the moment considered as a free particle we could calculate the energy loss of the colliding particle by multiplying the probability for Coulomb scattering of the electron into a certain angle with the energy gain of the electron in the laboratory system associated with this scattering angle. We then sum this energy over all possible scattering angles to obtain the total energy loss per collision. A simple application of the Lorentz transformation to this problem shows that an electron which in the rest system is scattered into an angle θ by an infinitely heavy particle has in the laboratory system an energy

$$\epsilon(\theta) = mc^2\gamma^2\beta^2(1-\cos\theta), \tag{3}$$

with $\gamma^2 = 1/(1-\beta^2)$. The collision loss per unit path length is then obtained from the expression

$$-\frac{dw}{dx} = 2\pi nmc^2\gamma^2\beta^2 \int_{\theta_0}^{\pi} \sigma(\theta)(1-\cos\theta)\sin\theta d\theta, \tag{4}$$

where n is the number of electrons per unit volume. The integration over the angle θ cannot be extended to 0 for the lower limit, since for very small angle scattering the electrons cannot be considered as free.

If we calculate (4) with the differential cross section $\sigma(\theta)$ for Coulomb scattering

$$\sigma_0(\theta) = \left(\frac{e^2}{2\gamma m\beta^2 c^2}\right)^2 \frac{1}{\sin^4\frac{1}{2}\theta} \tag{5}$$

[1] See for instance W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1947), p. 218, Eq. (1).

[2] E. Fermi, Phys. Rev. **57**, 485 (1940); O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948); G. C. Wick, Nuovo cimento **1**, 302 (1943); A. Bohr, Kgl. Danske Videnskab. Selskab, Nat.-fys. Medd. **24**, No. 19 (1948).

[3] See for instance D. R. Corson and M. R. Keck, Phys. Rev. **79**, 209 (1950); W. L. Whittemore and J. C. Street, Phys. Rev. **76**, 1786 (1949); E. Hayward, Phys. Rev. **72**, 937 (1948); F. L. Hereford, Phys. Rev. **74**, 574 (1948); E. Pickup and L. Voyvodic, Phys. Rev. **80**, 89 (1950).

[4] Goodman, Nicholson, and Rathgeber, Proc. Phys. Soc. (London) **A64**, 96 (1951).

[5] J. Schwinger, Phys. Rev. **76**, 790 (1949).

we obtain

$$-dw/dx=\tfrac{1}{2}\pi(ne^4/m\beta^2c^2)J, \qquad (6)$$

where

$$J=\int_{\theta_0}^{\pi}\frac{1-\cos\theta}{\sin^4\frac{1}{2}\theta}\sin\theta d\theta=8\ln\left(\frac{1}{\sin\frac{1}{2}\theta_0}\right). \qquad (7)$$

Formula (6) thus gives

$$-dw/dx=4\pi(ne^4/m\beta^2c^2)\ln(2/\theta_0). \qquad (8)$$

The minimum scattering angle may be obtained by comparison with the rigorous formula or by arguments for the limit of energy transfer for bound electrons.[6]

$$\theta_0\sim I/mc^2\beta^2\gamma^2, \qquad (9)$$

from which we obtain

$$-dw/dx=4\pi(ne^4/m\beta^2c^2)\ln(2mc^2\beta^2\gamma^2/I). \qquad (10)$$

In order to obtain an order of magnitude estimate of the radiative correction we replace $\sigma_0(\theta)$ by

$$\sigma(\theta)=\sigma_0(\theta)[1-\delta(\theta)], \qquad (11)$$

where $\delta(\theta)$ is the radiative correction for the scattering in a Coulomb field calculated by Schwinger.

$$\delta(\theta)=(4\alpha/\pi)[A(\ln(2\gamma\sin\tfrac{1}{2}\theta)-\tfrac{1}{2})+B+C\sin^2\tfrac{1}{2}\theta f(\theta)], \qquad (12)$$

with

$$A=\ln(E/\Delta E)-13/12,\quad B=17/72,\quad C=\tfrac{1}{2}.$$

Here ΔE is the uncertainty of the energy change of the scattered electron, subject only to the condition

$$0<\Delta E\ll E-mc^2. \qquad (13)$$

$f(\theta)$ is a slowly varying function of the angle θ, which we do not need to know since the contribution from this term turns out to be entirely negligible. Formula (12) is only valid for energies and scattering angles such that

$$\gamma\sin\tfrac{1}{2}\theta\gg 1. \qquad (14)$$

For the extreme relativistic case to be considered here, the limiting angle θ_1 defined by

$$\gamma\sin\tfrac{1}{2}\theta_1=1,\quad \theta_1\simeq 2/\gamma, \qquad (15)$$

turns out to be very much larger than θ_0, although it is still small compared to one. Thus we have

$$\theta_0\ll\theta_1\ll 1. \qquad (16)$$

For angles $\theta<\theta_1$ we must use another limiting formula for δ. In this case δ is proportional to $\theta^2\ll 1$ and thus the contribution to the effect from this region turns out to be negligible.

Thus we may use formula (12) throughout and limit the scattering angle for the correction term to $\theta=\theta_1$. Comparing with (3) we see that radiative corrections to collision loss are only important for collisions such that the energy transferred to the electron is of order $mc^2\gamma$ or more.

If we write for the energy loss per unit length,

$$-dw/dx=F(1-\Delta), \qquad (17)$$

with F given by (10), we find with the help of (11), (12), and (13) for the relative decrease of the radiation loss

$$\Delta=\frac{\alpha}{2\pi}\int_{\theta}^{\pi}[A\{\ln(2\gamma\sin\tfrac{1}{2}\theta)-\tfrac{1}{2}\}+B]\times\frac{1-\cos\theta}{\sin^4\frac{1}{2}\theta}\sin\theta d\theta\Big/\ln\left(\frac{2mc^2\beta^2\gamma^2}{I}\right). \qquad (18)$$

The integration involves only the two integrals

$$\int_{\theta_1}^{\pi}\frac{1-\cos\theta}{\sin^4\frac{1}{2}\theta}\sin\theta d\theta=8\ln\left(\frac{1}{\sin\frac{1}{2}\theta_1}\right)=8\ln\gamma, \qquad (19)$$

$$\int_{\theta_1}^{\pi}\frac{\ln(2\gamma\sin\frac{1}{2}\theta)}{\sin^4\frac{1}{2}\theta}(1-\cos\theta)\sin\theta d\theta=8\ln\gamma(\ln\gamma+\ln2-\tfrac{1}{2}). \qquad (20)$$

Substituting these expressions in (18), we find for Δ

$$\Delta=\frac{(4\alpha/\pi)\ln\gamma[\{\ln(E/\Delta E)-13/12\}\times(\ln\gamma-1+\ln2)+17/72]}{\ln(2mc^2\beta^2\gamma^2/I)}. \qquad (21)$$

This represents the relative decrease of collision loss resulting from radiative corrections.

In this formula there occurs still the as yet undetermined quantity $E/\Delta E$. Here ΔE represents the energy loss of the electron resulting from the emission of low energy photons. In the theory which includes radiative corrections the cross section for elastic scattering ($\Delta E\rightarrow 0$) would actually vanish. In an actual experiment this is not what is observed, however. There is always in the nature of the experiment a limitation ΔE on the accuracy of the energy determination and this is the value to be inserted in Eq. (12).

In our case the limitation on the energy determination arises from the fact that because of the finite extension of the electron in an atom, the kinetic energy E of the electron before collision is uncertain by an amount

$$\Delta E/E\simeq\Delta p/p=(\gamma\hbar/a)(1/\gamma mc)=Z^{\frac{1}{3}}/137, \qquad (22)$$

where we have used $a\simeq a_0Z^{-\frac{1}{3}}$, $a_0=\hbar^2/me^2$, for the extension of the electron at rest.

If we insert this expression for $\Delta E/E$ in (17), we obtain the final result

$$\Delta(Z,\gamma)=\frac{(4\alpha/\pi)\ln\gamma[\{\ln(137/Z^{\frac{1}{3}})-13/12\}\times(\ln2\gamma-1)+17/72]}{\ln(2mc^2\beta^2\gamma^2/I)}, \qquad (23)$$

[6] See for instance E. J. Williams, Revs. Modern Phys. **17**, 217 (1945), especially p. 223 ff.

TABLE I. Relative decrease Δ of radiation loss in percent due to radiative correction for energies γMc^2 of heavy particle, $Z=18$ (argon).

γ	10	20	50	100	200	500	1000
$\Delta\times 100$	1.06	1.68	2.57	3.24	4.1	5.15	6

for the relative decrease of the collision loss per unit path length. The evaluation of this for the case of argon ($Z=18$) gives the result summarized in Table I.

The correction Δ would be a correction over and above the correction due to the density effect. It is difficult to compare this result with existing experiments. At the present time we can only conclude that if precise experiments at ultrarelativistic energies become available, the radiative correction would have to be included in the theoretical discussion. The method used here cannot claim any accuracy better than about a factor of two, and for a detailed comparison with experiment it would be necessary to refine the calculation.

Note added in proof: While this paper was in print there appeared a paper by H. D. Rathgeber [Z. Naturforsch. **6a**, 598 (1951)] on the energy loss of fast mesons in water. It is found that the energy loss of measons stays nearly constant in the range from 2 to 20×10^9 ev while according to the Bethe-Bloch theory it should increase 29 percent in this range. 16 percent of these can be accounted for as being due to the density effect, thus leaving 13 percent unexplained. The radiative correction here calculated would result in a further decrease of about 3–4 percent. Although the method of calculation used here does not claim accuracy better than about a factor of two, it seems difficult to explain the whole of the discrepancy found by Rathgeber as due to radiative corrections alone.

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The Interaction of a Charged Pi-Meson with the Deuteron*

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The radiative absorption, charge-exchange scattering, elastic and inelastic scattering of a π^+ meson by the deuteron are calculated in conventional weak-coupling theory for the pseudoscalar field with both direct and gradient coupling. The nucleon-nucleon interaction is treated phenomenologically and the "impulse" approximation employed. The behavior of the ratio $\sigma_{\text{nonradiative}}/\sigma_{\text{radiative}}$ for the *PS* theory is different from that for the *PV* theory. The charge-exchange scattering of a *PS* meson exhibits a minimum for forward scattering for both types of coupling but otherwise mirrors the behavior of the charge-exchange cross section for a neutron target. The elastic and inelastic scattering cross sections are comparable for an incident 25-Mev meson, and small energy transfers to the deuteron are favored, in contrast to the case in which the target nucleus is more complex.

INTRODUCTION

In this paper, we shall investigate the interaction of a charged π-meson of positive energy with the deuteron. The nonradiative absorption of a π^+ meson by the deuteron ($\pi^++d\rightarrow p+p$) and its inverse ($p+p\rightarrow\pi^++d$) have been the subjects of a previous paper[1] by the author. The reactions to be considered here are:

$$\pi^++d\rightarrow\begin{cases} p+p+\gamma & \text{radiative absorption}\\ p+p+\pi^0 & \text{charge-exchange scattering}\\ d+\pi^+ & \text{elastic scattering}\\ n+p+\pi^+ & \text{inelastic scattering.}\end{cases}$$

Recent experimental results on the reactions[2,3] $\pi^++d\leftrightarrows p+p$ prove that the π^+ meson has zero spin. In addition, comparison of the theoretical results[4] with the experimental data[5] on the reactions π^- (*K*-shell) $+d\rightarrow n+n$ and $n+n+\gamma$ show that if the π^- meson has zero spin, it has odd parity (*PS*). If one assumes that the π^+ and π^- mesons differ only in the sign of their Coulombic charge, then one may assign zero spin and odd parity to the charged π-meson field. The calculation of the above reactions will be carried out assuming both direct (*PS*) and gradient (*PV*) coupling of the *PS* π-meson to a nucleon. Although the two nucleon interaction will be treated phenomenologically, it will be consistent to consider direct coupling for the *PS* field since we shall take account of the possibility of the meson interacting with negative energy state nucleons. The nucleons will be assumed to be Dirac particles, and transitions through intermediate negative energy states will be included. The calculation is performed assuming a positive charge for the π-meson; however, since the Coulomb force between the meson and the nucleus is ignored, the derived expressions are independent of the sign of the charge of the meson. The π^+ meson is con-

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[4] S. Tamor, Phys. Rev. **82**, 38 (1951).
[5] W. Panofsky (private communication).