

TABLE I. The observed values of P are tabulated as a function of frequency for a sample without a glass envelope. The ac amplitude for each value of P is given, as well as the calculated value of P .

Frequency (cycles/sec)	I_{ac} (ma)	P (observed)	P (calculated)
50	156	1.15	1.41
200	156	1.18	1.41
2000	165	1.21	1.42
20,000	175	1.68	1.47

markedly in height and occurred at slightly larger values of the ac. We attributed this anomalous behavior to the large values of the ratio (r/d) of radius to skin depth of the wires, and obtained experimental support for this conclusion by making measurements on samples of various conductivities and sizes and correlating the observed deviations with the values of r/d . On the other hand, Galkin and Bezuglyi suggest that the anomalies can be traced to the fact that, at high frequencies, the wires become adiabatically isolated from the liquid bath. The wires could then be cooled by the magnetocaloric effect, during part of the cycle of the ac, thereby increasing the critical current necessary to restore the sample to the normal state. The increased critical current would necessitate a larger I_{ac} at the maximum, thereby producing a larger maximum. To support this mechanism, Galkin and Bezuglyi describe an experiment in which a tin specimen of diameter 0.15 mm was drawn in a glass capillary tube, and then cut into two pieces. The glass was removed from one of the pieces with hydrofluoric acid, but the other specimen was left in the glass envelope. Runs were then made on both samples. For the sample without glass envelope the character of the curves $V_{dc}(I_{ac})$ was independent of frequency up to 20,000 cycles/sec. At the same temperature, the curves $V_{dc}(I_{ac})$ for the sample in the glass envelope was independent of frequency up to 2000 cycles/sec, but at larger frequencies the maxima increased in size up to 10,000 cycles/sec. Further increase in frequency did not affect the size of the maxima.

The above experimental result is in disagreement with all of our data. In our first investigations, it was standard procedure to remove the glass from the samples with hydrofluoric acid. Contrary to the result of Galkin and Bezuglyi, we always observed a monotonic increase with frequency of the values of the maxima of the curves $V_{dc}(I_{ac})$ in the range 50 cycles/sec to 20,000 cycles/sec. A typical set of values of $P = [(V_{dc})_{max}/I_{dc}R_n]$ for various frequencies is given in Table I, where R_n is the resistance of the sample in the normal state and I_{dc} is the value of the dc. The values of the ac amplitudes at the maxima are also tabulated. This sample was about 7 cm long, had a diameter of about 0.13 mm and was prepared from spectrographically pure tin supplied by the Johnson, Mathey Company. The data were taken at 3.71°K, with $I_{dc} = 40$ ma and the critical current, $I_c \approx 120$ ma. The bases of the calculated values of P are given below.

Data of the general type shown in Table I were obtained in seven different samples of high purity tin from which the glass had been removed; at least two runs were made on each sample. However, for these samples the curves of restoration of resistance with direct current (Silsbee effect) were quite broad. We felt that the broadening was the result of the strains unavoidably introduced into the samples by handling. To reduce the possibility of

TABLE II. The observed values of P are tabulated as a function of frequency for a sample with a glass envelope. The ac amplitude for each value of P is given, as well as the calculated value of P .

Frequency (cycles/sec)	I_{ac} (ma)	P (observed)	P (calculated)
30	382	1.07	1.24
300	382	1.15	1.24
3000	382	1.23	1.24
5000	406	1.30	1.25
8000	425	1.36	1.28
10,000	440	1.39	1.30
15,000	453	1.42	1.31

straining the samples, the expedient was then adopted of leaving the samples in the glass envelope, except at the ends where the glass was removed, so that the sample could be mounted in clamps which served as current and potential leads. The thickness of the glass never exceeded 0.1 mm. The samples with glass envelopes exhibited quite sharp curves of Silsbee effect. A set of values of the pertinent quantities for a glass covered sample is given in Table II. This sample, prepared from pure tin, had a diameter of 0.164 mm and was 4.0 cm long. The data were taken at $T = 3.70^\circ\text{K}$, with $I_{dc} = 145$ ma and $I_c \approx 240$ ma. Similar data were obtained with five different samples of pure tin.

The values of P can be calculated for an ideal case. We assume that the resistance of the sample is zero when the total current is less than the critical value. At the critical value, I_c , the resistance jumps discontinuously to its value in the normal state and remains constant for all larger values of the current. A straight forward calculation gives

$$P = 0.5 + \frac{2\sqrt{x - \sin^{-1}[(x-1)/(x+1)]}}{\pi}, \quad x > 1,$$

where $x = I_c/I_{dc}$. Since at the maximum value of V_{dc} , $I_{ac} = I_{dc} + I_c$, it is then possible from the measured values of I_{ac} and I_{dc} to calculate P . The calculated values of P given in Tables I and II have been determined in this way. It is to be noted that at low frequencies the observed values of P are smaller than the calculated values of P . This is to be expected, since the resistance of an actual sample does not exhibit a discontinuous increase in resistance, but rather a sharp increase to about $0.8 R_n$ at the critical current followed by a very slow increase of resistance up to R_n . We would not consider it inappropriate to multiply all the calculated values of P by the factor necessary to obtain agreement between the calculated and observed values at the lowest frequency. However, we have not made this correction, since it is then clear that the calculated values of P are the largest possible values for the measured values of I_{dc} and I_{ac} , if it is assumed that the effective resistance of the samples is unchanged.

It is clear from Tables I and II that at sufficiently high frequencies the observed values of P appreciably exceed these largest possible values. This behavior was exhibited by each of our twelve pure samples. Thus it is clear that deviations exist which are greater than any that can be explained by the increase in I_{ac} due to cooling. We must conclude that at sufficiently high frequencies the effective resistance of our samples is increased above the normal value. In view of the experimental results and the analysis given above, the interpretation of Galkin and Bezuglyi is inadequate. The data is more in keeping with the skin effect interpretation given in our paper.¹

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Variational Calculation of Scattering Cross Sections*

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WE report the results of calculations attempting to assess the utility of Schwinger's variational formulation¹ of the scattering problem, when ka is so large that a phase-shift analysis, i.e., an expansion in a series of spherical harmonics, is a slowly

converging procedure. With a stationary expression¹ we compute $A(\mathbf{n}, \mathbf{n}_0)$, the scattering amplitude of waves proceeding to infinity along the direction \mathbf{n} as a result of a wave $\exp(i\mathbf{k}\mathbf{n}_0 \cdot \mathbf{r})$ incident along the direction \mathbf{n}_0 . The differential cross section is $|A(\mathbf{n}, \mathbf{n}_0)|^2$ and the total cross section is $\sigma = (4\pi/k) \text{Im}A(\mathbf{n}_0, \mathbf{n}_0)$. This formulation has been used previously with good results to estimate scattering by two-dimensional scatterers, e.g., plane slits.³ The use of variational expressions for the individual phase shifts has also been reported.⁴

For square well potentials $V(r)$ we have evaluated in closed form the integral

$$\int \int d\mathbf{r} d\mathbf{r}' \psi(\mathbf{r}, -\mathbf{n}) V(r) \cdot G(\mathbf{r}, \mathbf{r}') V(r') \psi(\mathbf{r}', \mathbf{n}_0), \quad (1)$$

for $\mathbf{n} = \mathbf{n}_0$, with $\psi(\mathbf{r}, \mathbf{n}) = \exp(i\mathbf{K}\mathbf{r} \cdot \mathbf{n}) \cdot G(\mathbf{r}, r')$ is the free space Green's function $\exp(ik|\mathbf{r} - \mathbf{r}'|)/4\pi|\mathbf{r} - \mathbf{r}'|$. We thereby obtain in closed form a variational estimate of σ . With K corresponding to the wave number inside the well, the variational formulation appears to yield significantly better results than the customary Born approximation for a square well of depth 21.3 Mev, with range $R = 2.8 \times 10^{-13}$ cm, corresponding to triplet $n-p$ scattering by central forces with no exchange (Fig. 1).⁵ There is also a signifi-

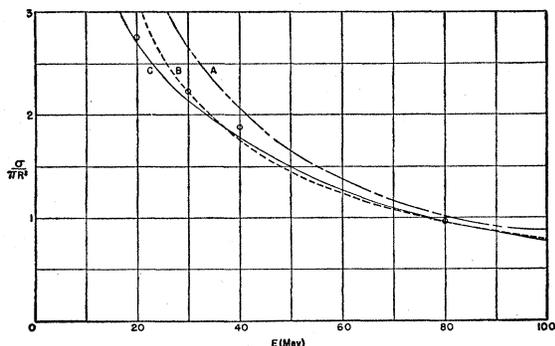


FIG. 1. Total cross sections against energy. Curve A is the Born approximation, curve B the variational result, and curve C the improved variational result. The exact points indicated by the circles are from Bethe and Camac (see reference 5).

cant improvement over the Born approximation for the case of acoustic scattering by a refractive sphere, with index of refraction 1.25. In part this improvement is ascribable to the inclusion of the integral (1), which integral appears also in second Born approximation, in part to the choice of a better trial function inside the well than the Born $\psi(\mathbf{r}, \mathbf{n}) = \exp(i\mathbf{k}\mathbf{r} \cdot \mathbf{n})$.

A further improvement can be obtained using a still better trial function inside the well, namely, a linear combination of forward and backward traveling plane waves, $\psi(\mathbf{r}, \mathbf{n}) = \exp(i\mathbf{K}\mathbf{r} \cdot \mathbf{n}) + \alpha \exp(-i\mathbf{K}\mathbf{r} \cdot \mathbf{n})$ as suggested by the results of Latter.⁶ It is unnecessary to make specific assumptions concerning the magnitude of α , since parity conservation enables us to obtain independent integral equations and thence independent variational expressions for the even and odd parts of the solution, remembering that the variational expression² for $A(\mathbf{n}, \mathbf{n}_0)$ is independent of normalizing factors in the trial function. This improved variational result is also plotted in Fig. 1.

With the use of the combination $\exp(i\mathbf{K}\mathbf{r} \cdot \mathbf{n}) + \alpha \exp(-i\mathbf{K}\mathbf{r} \cdot \mathbf{n})$, exchange can be included with little additional complication. For the square well we have been unable to evaluate (1) in closed form except for $\mathbf{n} = \mathbf{n}_0$, with $\psi(\mathbf{r}, \mathbf{n}) = \exp(i\mathbf{K}\mathbf{r} \cdot \mathbf{n})$. However, (1) can be evaluated in closed form for the ψ for arbitrary \mathbf{n} , with $V(r)$ a Yukawa well, which means that in this case closed forms can be obtained for the estimated differential and total cross sections

including exchange using a trial function which is a combination of forward and backward traveling waves. Jost and Pais⁷ have evaluated (1) in the Yukawa case, when $K = k$. Their method can also be used for $K \neq k$, but we have found it possible and equally convenient to evaluate (1) with $K \neq k$ by standard contour integral techniques, after transforming to momentum space.

Further details will be forthcoming soon.

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Critical Domain Size in Ferroelectrics

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IT is well known that small ferromagnetic particles with linear dimensions of the order of magnitude of the domain wall thickness behave as single domains.¹ The surface energy of the Bloch wall presents a splitting up of such a small particle into several domains. Similar effects have now been observed in ferroelectrics.

Colloidal KH_2PO_4 was prepared by grinding single crystals in a colloid grinding mill using a suitable protecting colloid. By means of a centrifuge different samples with homogeneous particle size ranging from 650A up to 5000A were obtained. Crystal size and shape were determined with the electron microscope. The crystals are approximately spherical. Debye-Scherrer patterns show that the lattice is not distorted by the grinding procedure.

The state of polarization below the Curie point ($\theta = 123^\circ\text{K}$) was investigated by observing the spontaneous strain with x-ray diffraction. For unstressed crystals the spontaneous strain was shown to be proportional to the spontaneous polarization.² The volume conductivity of the colloid is probably less than 10^{-16} ohm⁻¹ cm⁻¹ in the neighborhood of the Curie temperature and below.

The main results may be stated as follows: Colloidal particles with a mean diameter $D \leq 1500\text{A}$ show no measurable spontaneous strain from the Curie point down to the temperature of liquid nitrogen, whereas particles with $D \geq 4000\text{A}$ undergo about the same spontaneous strain as macroscopic domains do. The dielectric constant of the colloids has a peak value at a temperature coinciding (within the limits of error, $\pm 0.5^\circ\text{C}$) with the Curie temperature of the macroscopic crystal, indicating that the spontaneous strain is not prevented by mechanical clamping (frozen protecting colloid).³ Hence particles with no measurable strain have only a small spontaneous polarization. This result is confirmed by the fact that the decrease of the dielectric constant below the Curie point is less pronounced for colloids showing no spontaneous strain.

The influence of the electrical conductivity on the polarization effects was investigated by repeating the same experiments with a conducting colloid: Macroscopic KD_2PO_4 -crystals behave like macroscopic KH_2PO_4 -crystals, the only essential difference is that the Curie temperature of KD_2PO_4 is much higher ($\theta = 213^\circ\text{K}$).^{4,5} At this temperature the colloid is not frozen and shows a conductivity of about 10^{-9} ohm⁻¹ cm⁻¹. The KD_2PO_4 -particles were imbedded in a protecting colloid, in which the exchangeable hydrogens were replaced by deuterium.

Debye-Scherrer photographs taken at various temperatures revealed that in contrast to the nonconducting case (KH_2PO_4) even the smallest particles ($D \approx 750\text{A}$) in the conducting colloid