produced in pairs by the decay of the neutral pions, the cross sections for the processes (1) and (2) would be $(10 \pm 4) \times 10^{-27}$ and $(20 \pm 5) \times 10^{-27} \mathrm{~cm}^{2}$. The cross section obtained for the charge exchange process is not very sensitive to the angular distribution adopted. It would be $(29 \pm 7) \times 10^{-27} \mathrm{~cm}^{2}$ for a $\cos ^{2} \theta$-distribution and $(18 \pm 4) \times 10^{-27} \mathrm{~cm}^{2}$ for a $\sin ^{2} \theta$-distribution.

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## Total Cross Sections of Positive Pions in Hydrogen*

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IN a previous letter, ${ }^{1}$ measurements of the total cross sections of negative pions in hydrogen were reported. In the present letter, we report on similar experiments with positive pions.

The experimental method and the equipment used in this measurement was essentially the same as that used in the case of negative pions. The main difference was in the intensity, which for the positives was much less than for the negatives, the more so the higher the energy. This is due to the fact that the positive pions which escape out of the fringing field of the cyclotron magnet are those which are emitted in the backward direction with respect to the proton beam, whereas the negative pions are those emitted in the forward direction. The difficulty of the low intensity was in part compensated by the fact that the cross section for positive pions turned out to be appreciably larger than for negative pions. The results obtained thus far are summarized in Table I.

In Fig. 1 the total cross sections of positive and negative pions are collected. It is quite apparent that the cross section of the positive particles is much larger than that of the negative particles, at least in the energy range from 80 to 150 Mev .

In this letter and in the two preceding ones, ${ }^{1,2}$ the three processes: (1) scattering of positive pions, (2) scattering of negative pions with exchange of charge, and (3) scattering of negative pions without exchange of charge have been investigated. It appears that over a rather wide range of energies, from about 80 to 150 Mev , the cross section for process (1) is the largest, for process (2) is intermediate, and for process (3) is the smallest. Furthermore, the cross sections of both positive and negative pions increase rather rapidly with the energy. Whether the cross sections level off at a high value or go through a maximum, as might be expected if there should be a resonance, is impossible to determine from our present experimental evidence.

Brueckner ${ }^{3}$ has recently poin ${ }^{+}$ed out that the existence of a broad resonance level with spin $3 / 2$ and isotopic spin $3 / 2$ would give an approximate understanding of the ratios of the cross sections for the three processes (1), (2), and (3). We might point out in this connection that the experimental results obtained to date are also compatible with the more general assumption that in the energy interval in question the dominant interaction responsible for the scattering is through one or more intermediate states of isotopic spin $3 / 2$, regardless of the spin. On this assumption, one finds that the ratio of the cross sections for the three

Table I. Total cross sections of positive pions in hydrogen.

|  |  |
| :---: | :---: |
| Energy (Mev) | Cross section <br> $\left(10^{-27} \mathrm{~cm}^{2}\right)$ |
| $56 \pm 8$ | $20 \pm 10$ |
| $82 \pm 7$ | $50 \pm 13$ |
| $118 \pm 6$ | $99 \pm 6$ |
| $136 \pm 6$ | $152 \pm 14$ |



Fig. 1. Total cross sections of negative pions in hydrogen (sides of the rectangle represent the error) and positive pions in hydrogen (arms of the cross represent the error). The cross-hatched rectangle is the Columbia charge exchange contribution
processes should be ( $9: 2: 1$ ), a set of values which is compatible with the experimental observations. It is more difficult, at present, to say anything specific as to the nature of the intermediate state or states. If there were one state of spin $3 / 2$, the angular distribution for all three processes should be of the type $1+3 \cos ^{2} \theta$. If the dominant effect were due to a state of spin $1 / 2$, the angular distribution should be isotropic. If states of higher spin or a mixture of several states were involved, more complicated angular distributions would be expected. We intend to explore further the angular distribution in an attempt to decide among the various possibilities.

Besides the angular distribution, another important factor is the energy dependence. Here the theoretical expectation is that, if there is only one dominant intermediate state of spin $3 / 2$ and isotopic spin $3 / 2$, the total cross section of negative pions should at all points be less than ( $8 / 3$ ) $\pi \lambda^{2}$. Apparently, the experimental cross section above 150 Mev is larger than this limit, which indicates that other states contribute appreciably at these energies. Naturally, if a single state were dominant, one could expect that the cross sections would go through a maximum at an energy not far from the energy of the state involved. Unfortunately, we have not been able to push our measurements to sufficiently high energies to check on this point.
Also very interesting is the behavior of the cross sections at low energies. Here the energy dependence should be approximately proportional to the 4th power of the velocity if only states of spin $1 / 2$ and $3 / 2$ and even parity are involved and if the pion is pseudoscalar. The experimental observations in this and other laboratories seem to be compatible with this assumption, but the cross section at low energy is so small that a precise measurement becomes difficult.

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## Conductivity of Cold-Worked Metals*

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THE change in the electrical conductivity of a metal upon cold-working was first calculated by Koehler, ${ }^{1}$ on the assumption that the change is primarily due to the dislocations themselves (rather than associated clusters of vacancies, for example). The scattering potential he used was the difference in
the sum of the shielded potentials of the ions, in the strained and unstrained positions, in a lattice containing a pair of edge-type dislocations. Mackenzie and Sondheimer ${ }^{2}$ repeated the calculation with the same potential, changing the approximation between the dislocations, and the method of solution of the transport equation. Both K and MS found changes in resistivity of the order of that observed.
In a recent paper with the above title Landauer ${ }^{3}$ computed the change in conductivity using as a scattering potential the energy of the bottom of the conduction band, calculated from freeelectron theory, in a lattice containing a single edge-type dislocation. His method is somewhat simpler mathematically, avoids convergence difficulties of the type encountered in K and MS (which required the treatment of a positive-negative dislocation pair), and avoids the problem of the discontinuity in the displacement across the plane connecting the pair of parallel dislocation axes. ${ }^{4}$ The purposes of this note are to extend and modify the results in L, and to discuss the effect of the singularity in the assumed scattering potential.
Equation (L6) gives the scattering potential (except for an obvious omission of a factor $E$ ),

$$
\begin{equation*}
V(\mathbf{r})=\frac{a E}{3 \pi e}\left(\frac{1-2 \nu}{1-\nu}\right) \frac{\sin \theta}{r}, \tag{1}
\end{equation*}
$$

where the notation is the same as in L . The square of the scattering matrix element between two plane wave states, not explicitly given in L , is here calculated to be

$$
\begin{equation*}
\left.\left|\left\langle\mathbf{k}^{\prime}\right|-e V\right| \mathbf{k}\right\rangle\left.\right|^{2}=\frac{16}{9} a^{2} E^{2}\left(\frac{1-2 \nu}{1-\nu}\right)^{2} \frac{\sin ^{2}\left(\kappa_{z} v^{3} / 2\right) \sin ^{2} \varphi}{\kappa_{2}^{2} \kappa_{s}{ }^{2} v^{2}}, \tag{2}
\end{equation*}
$$

where $\mathbf{k}=\mathbf{k}^{\prime}-\mathbf{k}, \mathbf{k}_{s}$ is the component of $\mathbf{k}$ in the $x y$ plane, $\varphi$ the angle between $\boldsymbol{\kappa}_{s}$ and the $x$ axis, and $v$ the volume of the metal.
Now the transport equation ${ }^{5}$ can be solved by the method in MS, and the fractional change in resistivity, averaged over direction, is found to be

$$
\begin{equation*}
\langle\Delta \rho / \rho\rangle_{\mathrm{Av}}=\frac{1}{27 \pi^{2}} E^{2} a^{2}\left(\frac{1-2 \nu}{1-\nu}\right)^{2} \frac{\tau m^{*}}{\hbar^{3} n} k_{0} N . \tag{3}
\end{equation*}
$$

This average change in resistivity is equal to that calculated ${ }^{3 \mathrm{a}}$ in L [Eq. (L7)] times $\left(16 \pi^{2}\right)^{-1}$. The numerical value for copper becomes

$$
\begin{equation*}
\langle\Delta \rho / \rho\rangle_{\mathrm{AV}}=2.23 \times 10^{-15} \mathrm{~N}, \tag{4}
\end{equation*}
$$

where we have used the experimental values given in $L$, and the normal resistivity at $20^{\circ} \mathrm{C}, 1.89 \times 10^{-18}$ gaussian unit (i.e., 1.69 microhm cm ). The reader is referred to $\mathrm{K}, \mathrm{MS}$, and L for comparison with experiment. The numerical value of $\langle\Delta \rho / \rho\rangle_{\text {AV }}$ given in Eq. (L9), $2.5 \times 10^{-14} N$, does not follow either from Eq. (L7) or Eq. (3).
This treatment (as well as those in K and MS) has assumed that the change in density, and hence, the scattering potential, have a singularity on the dislocation axis. It is then of interest to see how much of the scattering is contributed by the fictitious $r^{-1}$ singularity. This can be investigated semi-quantitatively in the following way.
Let us consider the hypothetical problem of a line dipole embedded in a metal, such that the potential near the axis is given by Eq. (1). Then the Thomas-Fermi equation tells us that the potential in the lattice is

$$
\begin{equation*}
V(\mathbf{r})=\frac{a E}{3 \pi e}\left(\frac{1-2 \nu}{1-\nu}\right) K_{1}\left(r / r_{0}\right) \frac{\sin \theta}{r_{0}}, \tag{5}
\end{equation*}
$$

where $K_{1}(z)$ is the modified Bessel function of the second kind, varying as $1 / z$ for small $z$ and as $(\pi / 2 z)^{\frac{1}{2}} \exp (-z)$ for large $z$; the value of $r_{0}$ will not concern us for the moment. This potential gives rise to an average change in resistivity, by the same method as above, just equal to that in Eq. (3) multiplied by a function

$$
\begin{equation*}
f(c)=1-(3 / 2 c) \tan ^{-1} c+\left[2\left(1+c^{2}\right)\right]^{-1} \tag{6}
\end{equation*}
$$

where $c=2 k_{0} r_{0}$. The function $f$ is a monotonically increasing
function of $r_{0}$, and varies from 0 to 1 as $r_{0}$ varies from 0 to $\infty$. That is, as $r_{0} \rightarrow \infty$, the potential and the resistivity change are just those of the unshielded line dipole given in Eqs. (1) and (3). However, we can now cut off the potential at any desired length by making use of the exponential dependence in $K_{1}$. Cutting off the potential at about a lattice spacing by letting $c=2.7$, corresponding to setting $k_{0}=1.37 \times 10^{8} \mathrm{~cm}^{-1}$ for copper, and $r_{0} \sim 10^{-8} \mathrm{~cm}$, we find $f$ to be 0.39 . Thus the fictitious contribution to the resistivity arising from within the nearest neighbor distance is about half of the value calculated in Eq. (3), and hence, the calculation can probably be trusted to about that accuracy. It should be emphasized that this entire calculation has been performed for a simple cubic lattice, and hence, is not directly applicable to more complex crystals, particularly since the variation of elastic constants with direction has not been included.
Finally, the anisotropy in the resistivity should be mentioned. This calculation gives a change in resistivity in the $x$-(slip) direction $1 / 3$ that in the $y$-direction, whereas K and MS found ratios of about 8 and 2 , respectively. This difference is a result of their approximations for taking account of the discontinuity in displacement across the plane connecting the dislocation axes. A later publication will discuss the anisotropy in more detail; it will also show that properly performed calculations using these two types of potential give closely related results for the magnitude of the resistivity change, as well as the same anisotropy, namely the ratio $1 / 3$.
The writer would like to express his warmest thanks to Professors J. Bardeen and J. S. Koehler for enlightening conversations, and to Mr. Paul Leurgans for checking some of the calculations.

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1 J. S. Koehler, Phys. Rev. 75, 106 (1949), to be referred to as K.
${ }^{2}$ J. K. Mackenzie and E. H. Sondheimer, Phys. Rev. 77, 264 (1950), to be referred to as MS.
Rolf Landauer, Phys. Rev. 82, 520 (1951), to be referred to as L.
3a Dr. Landauer has been kind enough to compare our treatments, and now agrees that Eq. (3) in this note is correct.
${ }_{5}$ Equation (4) (to be published).
(4) for the collision operator in reference 2 and all resulting expressions should be multiplied by the factor $\frac{1}{2}$.

## Long-Range Proton-Proton Tensor Force*

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## and

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THE molecular beam magnetic resonance experiments with molecular hydrogen ${ }^{1,2}$ provide an accurate measurement of the tensor interaction between two protons three-quarters of an angstrom apart. The magnetic part of this interaction depends only upon the magnetic moment of the proton and the mean inverse cube of the internuclear spacing. The first of these quantities is accurately known from nuclear resonance experiments ${ }^{1,3}$ while the second can be inferred from spectroscopic data ${ }^{4}$ on $\mathrm{H}_{2}$. Therefore, a comparison of the value computed in this way with the experimental value will either detect or set an upper limit to the long-range nonmagnetic tensor interaction between two protons.
To account for the observed radiofrequency spectrum of $\mathrm{H}_{2}$ one assumes, ${ }^{1}$ among other things, a proton-proton interaction of the form $K(r) S_{12}$, where $S_{12}$ is the usual tensor operator. ${ }^{5}$ The expectation value of $K(r)$ in the first rotational and zeroth vibrational state of the molecule is then the quantity that is experimentally determined. If this experimental quantity is called $\langle K(r)\rangle_{0,1}{ }^{\text {expt }}$, it is related to the usual parameters of the molecular hydrogen theory ${ }^{1,2}$ by
$\langle K(r)\rangle_{0,1}{ }^{\text {expt }}=(5 / 4) h d=\mu_{H} H^{\prime \prime}=(4.7750 \pm 0.0010) \times 10^{-22} \mathrm{erg}$ (1)
if the value ${ }^{2,6}$ for $d$ is taken to be $57,671 \pm 11 \mathrm{cps}$. The quantity $\langle K(r)\rangle_{0,1}{ }^{\text {expt }}$ can be assumed to consist of a magnetic part $\langle K(r)\rangle_{0,1} 1^{\text {mag }}$ and a nonmagnetic or nuclear part $\langle K(r)\rangle_{0,1} 1^{\text {nuc }}$, and


[^0]:    Research sponsored by the ONR and AEC.
    Institute for the Study of Metals, University of Chicago
    ${ }^{1}$ Anderson, Fermi, Long, Martin, and Nagle, Phys. Rev., this issue. ${ }^{2}$ Fermi, Anderson, Lundby, Nagle, and Yodh, preceding Letter, this issue, Phys. Rev.
    ${ }^{3}$ K. A. Brueckner (private communication).

