pulse height between different pieces varied by less than 10 percent. The proportionality between energy and pulse height was checked within 6 percent by using Cs^{137} gamma-rays (Fig. 2).

From this it is evident that large LiI(Sn) crystals can be grown which give good resolution and pulse height. This makes it possible to build relatively small detectors with high efficiency for thermal and epithermal neutrons. The resolution makes it also possible to use these crystals for rough neutron-energy measurements in the region where the $Li^6(n, \alpha)He^3$ cross section is large enough ($\sim$1 barn).

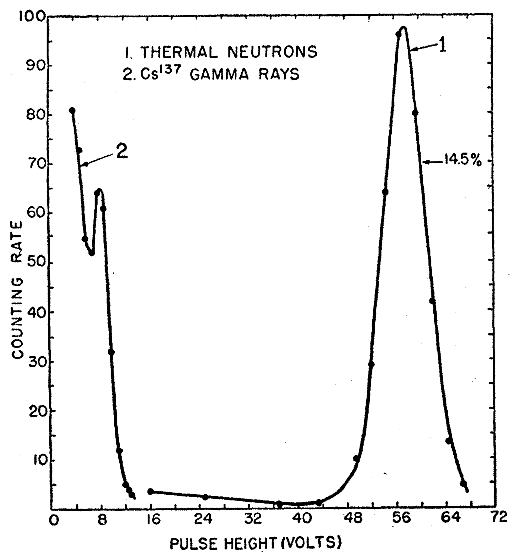

FIG. 2. LiI(Sn) 1 in.×¾ in.×⅜ in. crystal response. (1) irradiated with thermal neutrons; (2) Cs^{137} gamma-rays.

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The Graphs for the Kernel of the Bethe-Salpeter Equation

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In view of the recent discussions of Bethe and Salpeter[1] and Low and Gell-mann[2] on the derivation of an integral equation for describing bound states in field theory, it is of interest to see whether this kernel is likely to show any irregular type of convergence. Although it is the intention of these authors to escape some of the limitations of perturbation expansions, particularly when dealing with problems concerning bound states, it has not been found possible to obtain a tractable equation in a closed form, so that the kernel can only be employed in the form of a power series expansion in the coupling constant.

The kernel for the two-particle scattering process is composed of "irreducible" parts. These parts are such that it is not possible, by cutting the two spinor lines carrying the external energy and momentum, to separate the graph into two portions such that each portion has exactly four external spinor lines.

As an indication of the possible convergence of the power series describing the kernel, a lower limit to the number of irreducible graphs is given.

Let $N(n)$ denote the number of graphs of order n, whether reducible or irreducible, and $N_I(n)$ the number of irreducible graphs. Then a difference equation for $N_I(n)$ can be set up, in a manner exactly parallel to the Bethe-Salpeter equation.

$$\frac{N(n)}{n!}=\frac{N_I(n)}{n!}+\frac{N_I(n-2)}{(n-2)!}\frac{N(2)}{2!}+\cdots+\frac{N_I(2)}{2!}\frac{N(n-2)}{(n-2)!}. \quad (1)$$

This equation is obtained by requiring that the last $(n-2r)$ points, as described by the arrows on the spinor lines carrying the external energy and momentum, should form an irreducible graph, while the first $2r$ form a graph which may be reducible.

If

$$\Phi(v)=\sum_{n=0}^{\infty} v^n\frac{N(n)}{n!},\quad \Psi(v)=\sum_{n=2}^{\infty} v^n\frac{N_I(n)}{n!},$$

Eq. (1), in terms of generating functions, can be written

$$\Phi(v)=1+\Phi(v)\Psi(v), \quad (2)$$

which models the Bethe-Salpeter equation.

Now if "ladder" graphs are considered, for which only virtual boson exchange between the two spinor lines takes place, then we have

$$N(n)/n!=\tfrac{1}{2}(n/2)!,$$

and it is evident that $N(n)\geqslant N_I(n)$.

Also

$$\frac{N_I(n)}{n!}\geqslant\frac{N(n)}{n!}-\sum_{r=1}^{\frac{1}{2}(n-2)}\frac{N(2r)}{2r!}\frac{N(n-2r)}{(n-2r)!}. \quad (3)$$

Then the ratio of succeeding terms in the summation,

$$R_r=\frac{N(2r)N(n-2r)}{N(2r+2)N(n-2r-2)}\frac{(2r+2)(2r+1)}{(n-2r)(n-2r-2)}=\frac{n-2r}{2r+2}>R_{r+1}$$

and $R_1R_2\cdots R_{\frac{1}{2}(n-2)}=1$. Also $R_r=1$ for $(n-2r)/(2r+2)=1$, that is for $r=(n-2)/4$, so that

$$R_r \gtreqless 1 \quad \text{for} \quad r\lesseqgtr(n-2)/4.$$

Then the inequality can be written

$$\frac{N_I(n)}{n!}\geqslant\frac{N(n)}{n!}-\frac{2N(n-2)}{(n-2)!}\frac{N(2)}{2!}$$
$$-\frac{2N(n-4)}{(n-4)!}\frac{N(4)}{4!}\left(1+\frac{1}{R_4}+\frac{1}{R_4R_6}+\cdots+\frac{1}{R_4\cdots R_{\frac{1}{2}(n-4)}}\right)$$
$$\geqslant\frac{N(n)}{n!}-\frac{2N(n-2)}{(n-2)!}\frac{N(2)}{2!}-nC\frac{N(n-4)}{(n-4)!}\frac{N(4)}{4!},$$

where C is independent of n.

So

$$[N(n)/n!][1-O(1/n)]\leq N_I(n)/n!\leq N(n)/n!.$$

Hence $N_I(n)/n!$ increases as rapidly as $N(n)/n!$, namely $\log[N_I(n)/n!]\sim\frac{1}{2}n\log n+O(n)$. Thus there are, in this sense, as many terms in the kernel of the Bethe-Salpeter equation as in the original perturbation expansion, and so this kernel may very well not form a convergent series in the coupling constant, in the same way as the Born approximation for the S-matrix may not lead to a convergent series.

This result then raises the question of the meaning of the approximate solution of an integral equation, obtained by neglecting further terms in the kernel, when this kernel could not be approximated indefinitely with increasing order. This would be particularly difficult to justify in the case of pseudoscalar meson-nucleon interactions, for which the experimentally determined coupling constant is large.

This result can be modified trivially to fit any of the field theories so far introduced.

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