# Multiple Scattering of Fast Particles in Photographic Emulsions

L. VOYVODIC AND E. PICKUP
*Physics Division, National Research Council, Ottawa, Canada*

(Received July 9, 1951)

The multiple scattering theory of Williams is applied to photographic emulsion techniques, and the "scattering constant" $K$, which is commonly used in determining particle energies from mean scattering deflections, is evaluated for various experimental conditions. For fast particles $K$ varies from 19 to 30 for scattering cell lengths between 10 and $10^4$ microns of emulsion. The scattering theories of Snyder and Scott and of Molière are also compared with that of Williams. The theories of Molière and Williams agree closely for both photographic emulsions and pure substances when the Molière $\gamma$-factor, covering the transition from the classical case to the Born approximation case, is introduced into Williams' theory. For fast particles the theory of Snyder and Scott agrees with the other theories within about 1 percent for most cases for the mean scattering angle between tangents.

A simple formula, based on the theory of Williams with the Molière $\gamma$-factor, is derived for $K$ for photographic emulsions, applying over a wide range of velocities and scattering thicknesses within about 1 percent.

The results of a calibration experiment using electron pairs from $Be^8$ gamma-rays seem to confirm the validity of theoretical values of $K$ in the region $K=22$. The mean gamma-ray energy for 100 electron pairs was found to be $17.4\pm0.5$ Mev, which is in fairly good agreement with an expected mean energy of 16.7 $\pm0.3$ Mev. Results on the energy resolution of the scattering technique, and on the distribution of scattering deflections are also found to be in reasonable agreement with theory.

Finally, comparison is made between theory and other recently published emulsion calibration experiments.

## I. INTRODUCTION

THE multiple scattering method has now become widely used for determining the energies of fast, ionizing particles in photographic emulsions, particularly in cosmic-ray investigations. Limited at first to low energy particles using the projection microscope techniques,[1–4] the multiple scattering method was extended to the region of high energy physics by an important advance in technique described by Fowler.[5] A somewhat different scattering technique which is also applicable to high energy particles has been perfected by the Brussels group.[6]

Numerous publications have appeared using Fowler's technique, notably the work of the Bristol group on energies and masses of penetrating shower particles,[5,7] and on neutral meson gamma-rays,[8] whilst the method has also been used in our laboratory[9] and elsewhere. Cosmic-ray studies have also been reported[10,11] using the Brussels experimental method.

The energy-scattering relationship commonly used with Fowler's technique is of the form

$$p\beta=[Kz/\bar{\alpha}(t)]\times(t/100)^{\frac{3}{2}}\text{ Mev} \qquad (1)$$

where $p$ is the particle momentum in units Mev/$c$, $\beta$ its velocity in units of the velocity of light $c$, $z$ the particle charge, and $\bar{\alpha}(t)$ degrees is the mean projected deflection of the particle over a scattering length of $t$ microns of emulsion. For Ilford G5 emulsions, the numerical value of the "scattering constant" $K$ was given as 32.7 from early grain counting calibrations[3,5] with cosmic-ray protons. From the results of scattering observations in this laboratory, it was felt that this value of $K$ appreciably over-estimated particle energies, at least in some cases.[9]

In order to obtain a direct calibration for $K$, we have used the emulsion scattering technique as an electron pair spectrometer for the well-known[12] 14- and 17-Mev gamma-rays from $Be^8$. From a preliminary analysis of the scattering observations, a value of $K=21\pm1$ was reported[13] for the conditions of this experiment. Recently calibration experiments using proton and electrons of known energies have also been reported by Corson,[14] Berger, Lord, and Schein,[15] and Gottstein *et al.*[16]

In this paper we give the results of a theoretical study of emulsion scattering techniques. Theoretical values of the "scattering constant" $K$, based on Williams' theory[17,18] are presented in convenient graphical form, and an approximate formula is derived, which will be valid under most experimental conditions for photographic emulsions. A detailed description of the analysis and results of the $Be^8$ gamma-ray calibration experiment is then given. Since the gamma-rays have a fairly simple spectrum it has been possible to check the energy

[1] S. Lattimore, Nature **161**, 518 (1948).
[2] Goldschmidt-Clermont, King, Muirhead, and Ritson, Proc. Phys. Soc. (London) **61**, 183 (1948).
[3] Davies, Lock, and Muirhead, Phil. Mag. **40**, 1250 (1949).
[4] See also method proposed by S. A. Goudsmit and W. T. Scott, (Phys. Rev. **74**, 1537 (1948)), and later note by W. T. Scott (reference 25).
[5] P. H. Fowler, Phil. Mag. **41**, 169 (1950).
[6] Y. Goldschmidt-Clermont, Nuovo cimento **7**, 331 (1950).
[7] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950).
[8] Carlson, Hooper, and King, Phil. Mag. **41**, 701 (1950).
[9] E. Pickup and L. Voyvodic, Phys. Rev. **80**, 89 (1950); 80, 1100 (1950); Can. J. Phys. **29**, 263 (1951).
[10] G. P. S. Occhialini, Nuovo cimento **6**, Supp. 3, 411 (1949).
[11] Dilworth, Goldschmidt-Clermont, Goldsack, and Levy, Phil. Mag. **41**, 1032 (1950).
[12] R. L. Walker and B. D. McDaniel, Phys. Rev. **74**, 315 (1948).
[13] L. Voyvodic and E. Pickup, Phys. Rev. **81**, 471 (1951); 81, 890 (1951).
[14] D. R. Corson, Phys. Rev. **83**, 217 (1951).
[15] Berger, Lord, and Schein, Phys. Rev. **83**, 850 (1951).
[16] Gottstein, Menon, Mulvey, O'Ceallaigh, and Rochat, Phil. Mag. **42**, 708 (1951); Menon, O'Ceallaigh, and Rochat, Phil. Mag. **42**, 932 (1951).
[17] E. J. Williams, Proc. Roy. Soc. (London) **A169**, 531 (1939).
[18] E. J. Williams, Phys. Rev. **58**, 292 (1940).

resolution as well as the validity of energy determinations from the emulsion scattering technique.

## II. SUMMARY OF WILLIAMS' THEORY

Williams' multiple scattering theory was used since it is somewhat simpler mathematically than the later theories of Goudsmit and Saunderson,[19] Molière,[20] and Snyder and Scott,[21] and gives results identical within a few percent, as will be indicated later.

All angles are expressed in units of a natural angle of scattering $\delta$ defined[17] by

$$\delta = 2Zze^2(Nt)^{\frac{1}{2}}/p\beta \tag{2}$$

where $ze$, $p$, and $\beta$ are the charge, momentum, and velocity of the scattered particle and $Z$, $t$, and $N$ are the atomic number, thickness, and number of atoms per unit volume for the scattering material respectively.

In units of $\delta$, the arithmetic mean projected deflection is given by the statistical relation

$$\bar{\alpha} = 0.80(\log_e M)^{\frac{1}{2}} + 1.45 = 0.80\bar{\alpha}_1 + 1.45 \tag{3}$$

where $M$ measures the average number of collisions experienced by the scattered particle.

The distribution of projected deflections is closely approximated[17] by the probability function $P(\alpha) = G(\alpha) + S(\alpha)$. Here $G(\alpha)$ is a gaussian with arithmetic mean $\alpha_m$, and $S(\alpha)$ is a single scattering contribution starting at an angle $\phi_2$ and equal to $\pi/\alpha^3$ for angles greater than $\phi_2$, where

$$\alpha_m = (\bar{\alpha} - \pi/\phi_2)/(1 - \pi/2\phi_2^2), \quad \phi_2 = 5.1\bar{\alpha}_1 - 4.0. \tag{4}$$

The transition from multiple to approximately single scattering takes place in the region $\alpha = \alpha_{ms}$, where $\alpha_{ms}$ is the intersection of $G(\alpha)$ and $S(\alpha)$, and is given by

$$\alpha_{ms} = 5.1\alpha_m - 4.0. \tag{5}$$

The distribution functions and $\phi_2$, $\alpha_{ms}$ are illustrated in Fig. 7 for the case $M = 21$.

Using a Thomas-Fermi electronic screening field, Williams[17,22] has given two limiting expressions for $M$, which are designated as $M_B$ (wave-mechanical solution with the Born approximation), and $M_{cl}$ (classical dynamics solution):

$$M_B = 0.64\pi Ntz^2 Z^{4/3}\beta^{-2}(\hbar/mc)^2 \quad \text{if} \quad \gamma \ll 1 \tag{6a}$$

$$M_{cl} = 0.20\pi NtZ^{-\frac{2}{3}}(\hbar^2/me^2)^2 \quad \text{if} \quad \gamma \gg 1 \tag{6b}$$

where $\gamma = zZ/137\beta$, $\hbar$ is Planck's constant divided by $2\pi$, and $m$ is the electronic mass. This factor $M$ corresponds to the factor $\nu$ in the theory of Goudsmit and Saunderson, and to $\Omega_b$ in Molière's theory. Goldschmidt-Clermont[6] has given an expression in which Molière's factor $(1.13 + 3.76\gamma^2)$, governing the transition from the classical to the Born approximation case, is incorporated into Williams' relation for the mean deflection, using the latter's expression for $M_{cl}$. Similarly, using $M_B$, $M$ can be expressed in the form

$$M = M_B/(1 + \gamma^2/0.31) \tag{7}$$

so that $M = M_B$ and $M_{cl}$ for $\gamma \ll 1$ and $\gamma \gg 1$ respectively.

The essential conditions underlying the applicability of the results of Williams' statistical theory are that the average deflection, $\bar{\alpha}\delta$, should be small (an error of a few percent may be involved when $\bar{\alpha}\delta = 10°$), and that $M$ is $\gg 1$. It is only when $M$ is less than about 10 that the results are expected to become appreciably wrong.[17] Molière estimates that there may be an error $\sim 1$ percent when $\Omega_b \simeq 20$.

An upper limit for the validity of relation (3) is determined by nuclear dimensions. The finite size of the nucleus imposes an upper limit $(\sim \lambda\!\!\!^{-}/b)$ on the permissible Rutherford scattering angle, where $\lambda\!\!\!^{-}$ is the particle's de Broglie wavelength divided by $2\pi$, and $b$ is the radius of the scattering nucleus. For $\lambda\!\!\!^{-}/b \ll \alpha_{ms}$, where $\alpha_{ms}$ is given by (5), the effective value of $M$ becomes independent of scattering thickness, the single scattering tail is eliminated almost completely, and the resultant distribution is practically gaussian with an arithmetic mean $\alpha_b$, which is given[17] in $\delta$-units by the relation

$$\alpha_b = (19.5 - 3.1 \log_{10} Z)^{\frac{1}{2}}. \tag{8}$$

Rossi and Greisen[23] have considered this condition, and thus their results are chiefly applicable to the scattering of very energetic particles by large thicknesses of material.

## III. APPLICATION TO EMULSION TECHNIQUES

Because the photographic emulsion consists of a mixture of elements with atomic numbers $Z_i$, each with $N_i$ atoms per unit volume the resultant mean deflection must be obtained by a summation process from the individual scattering contributions. This is further complicated because of the nature of the expression for the mean deflection, and its dependence on $\beta$ and $t$.

For a pure substance, from Eq. (7),

$$\log M = \log(\text{constant} \times tNZ^2) - \log\beta^2 - \log(1 + \gamma^2/0.31)Z^{\frac{2}{3}}. \tag{9}$$

Molière's $\log\Omega_b$ can be expressed in a similar form, and by analogy with his method of summation for mixtures we get

$$\log M = \log(\text{constant} \times t\textstyle\sum_i(N_i Z_i^2)) - \log\beta^2 - \{\sum_i N_i Z_i^2 \log Z_i^{\frac{2}{3}}(1 + \gamma_i^2/0.31)\}/\sum_i(N_i Z_i^2). \tag{10}$$

In the $\delta$-factor $(NZ^2)^{\frac{1}{2}}$ is replaced by $(\sum_i N_i Z_i^2)^{\frac{1}{2}}$. Thus $\bar{\alpha}$ may be calculated using Eq. (3).

Equation (10) may be written as

$$\log M = \log(\text{constant} \times t\textstyle\sum_i(N_i Z_i^2)) - \log\beta^2 - \log P. \tag{11}$$

[19] S. A. Goudsmit and J. L. Saunderson, Phys. Rev. **57**, 24 (1940).
[20] G. Molière, Z. Naturforsch. **3a**, 78 (1948).
[21] H. S. Snyder and W. T. Scott, Phys. Rev. **76**, 220 (1949).
[22] E. J. Williams, Revs. Modern Phys. **17**, 217 (1945).
[23] B. Rossi and K. Greisen, Revs. Modern Phys. **13**, 241 (1941).

It can be shown graphically that $\beta^2 P$ is closely approximated ($\sim$1 percent) by the relation

$$\beta^2 P = 11.8(\beta^2+0.31)^{24}$$

for Ilford G5 emulsions, using the data on the components of the emulsion as supplied by the maker. Substituting in Eq. (11) and evaluating the various factors, gives

$$\log M = \log(0.94t/(\beta^2+0.30)) = \log M't. \quad (12)$$

Also

$$\delta = 1.006t^{\frac{1}{2}}/p\beta \text{ degrees} \quad (13)$$

where the scattering thickness, $t$ (henceforth referred to as cell length) is in microns of emulsion and $p\beta$ is in Mev, and the scattered particle has unit charge.

Substituting (12) and (13) in Eq. (3), we have for singly charged particles,

$$\bar{\alpha} = \frac{10.06}{p\beta}\left(\frac{t}{100}\right)^{\frac{1}{2}} \times \left\{1.45+0.80\left(\log_e \frac{0.94t}{\beta^2+0.30}\right)^{\frac{1}{2}}\right\} \text{degrees} \quad (14)$$

where $\bar{\alpha}$ is the mean angle between successive tangents.

In general $p\beta = \beta^2(E+\mu c^2)$, which reduces to $p\beta \simeq 2E$ for very slow particles and to $p\beta \simeq E$ for extremely relativistic particles, where $E$ is kinetic energy, $\mu c^2$ is rest mass energy, and the same symbol $\beta$ is used[23] for particle velocity and for the dimensionless ratio of particle velocity to the velocity of light $c$.

## A. Theoretical Mean Deflections

The variation of mean deflections with cell length for relativistic, singly charged particles (i.e., $z=1$, $\beta \simeq 1$) in emulsions has been calculated using Eqs. (3), (4), and

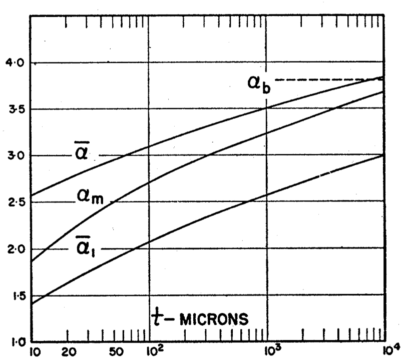


FIG. 1. Theoretical mean deflections, in $\delta$-units, for singly charged relativistic particles traversing $t$ microns of emulsion. $\bar{\alpha}$—arithmetic mean projected deflection; $\alpha_m$—arithmetic of gaussian $G(\alpha)$; $\bar{\alpha}_1$—$(\log M)^{\frac{1}{2}}$; $\alpha_b$—limiting deflection due to finite size of nucleus.

[24] It is interesting to note that $11.8(\beta^2+0.31)$ is approximately the same as using the factor $\beta^2(1+\gamma^2/0.31)Z^{\frac{1}{3}}$ in (9) with an effective $Z=41$. This probably arises from the fact that the final mean scattering angle is mainly determined by contributions from the silver and bromine nuclei in the emulsion.

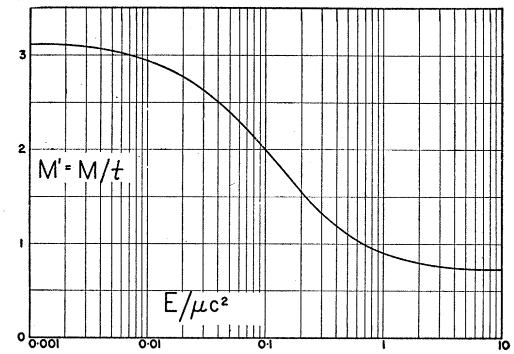


FIG. 2. Variation of $M'$, a measure of the average number of collisions per micron of emulsion, with particle energy $E$ in rest mass units $\mu c^2$, for singly charged particles.

(12), and the results are shown in Fig. 1. It is expected that the mean deflection $\bar{\alpha}$ as given in Fig. 1 should be valid for cell lengths down to about 15 microns, where $M \simeq 11$. For very large cell lengths ($t \gtrsim 10^4$ microns, $M \gtrsim 10^4$) the correct mean deflection should be the limiting value $\alpha_b$ due to finite nuclear dimensions as given by Eq. (8), or by the corresponding treatment of Rossi and Greisen.[23] The value $\alpha_b = 3.81$ units shown in Fig. 1 was derived from (8) using an effective $Z=41$.

The mean deflections for nonrelativistic particles can be obtained by using the results shown in Fig. 1 with the velocity variation of $M'$ as given by Eq. (12), or with the energy variation shown in Fig. 2. Here $M'$ measures the average number of collisions per micron of emulsion, and $E$ is the kinetic energy of the scattered particle whose rest mass energy is $\mu c^2$. The data for Fig. 1 are based on the value $M'=0.723$ for $\beta=1$. For a cell length $t$ for nonrelativistic particles, the appropriate mean deflections are those given in Fig. 1 at a cell length of $t \times M'/0.723$ micron.

## B. Geometrical Features of Emulsion Techniques

For a continuous track of a particle undergoing multiple scattering in the emulsion, the general scattering theory applies directly if one measures the angles between successive tangents to the projected track at intervals of $t$ microns. In the direct microscope method described by Fowler,[5] and in the projection microscope method of Lattimore,[1] the experimental observations are equivalent to measurements of the angles between successive chords described along the projected track. The relation between the arithmetic mean deflections for chords and tangents depends on the scattering distribution function, but we have used the simple $(2/3)^{\frac{1}{2}}$ relationship. More detailed considerations bearing on this point are given by Scott[25,26] Scott and Snyder,[27] and Snyder.[28]

[25] W. T. Scott, Phys. Rev. **75**, 1763 (1949).
[26] W. T. Scott, Phys. Rev. **76**, 212 (1949).
[27] W. T. Scott and H. S. Snyder, Phys. Rev. **78**, 223 (1950).
[28] H. S. Snyder, Phys. Rev. **83**, 1068 (1951).

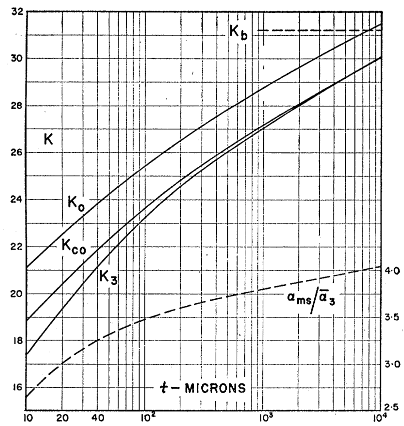


FIG. 3. Theoretical values of the "scattering constant" $K$ for singly charged particles, and Ilford G5 emulsions as a function of cell length $t$. $K_0$—no cutoff; $K_{co}$—cutoff at four times mean; $K_3$—cutoff at $\alpha_{ms}$ (angle defining transition from multiple to single scattering); $K_b$—limiting $K$ due to finite size of nucleus. The lower dashed curve shows the cut-off ratio $\alpha_{ms}/\bar{\alpha}_3$ as a function of cell length.

## C. Cutoff for Large-Angle Deflections

The principal reason for a cutoff in the experimental scattering distribution is to minimize the effects of the nongaussian single scattering tail. The convenient procedure of cutting off all angles greater than four times the resultant mean has now become standard in emulsion measurements.[2] If we call this resultant $\bar{\alpha}_{co}$, then analogous to Eq. (4) for the gaussian mean deflection $\alpha_m$, we find the approximate relation

$$\bar{\alpha}_{co}=(\bar{\alpha}-\pi/4\bar{\alpha})/(1-\pi/32\bar{\alpha}^2). \qquad (15)$$

Williams[17] has suggested that experimental scattering data be cut off at the angle $\phi_2$ as defined by (4), giving a resultant mean deflection $\bar{\alpha}_2$ which is readily calculable. For typical scattering observations on tracks in emulsions this procedure is difficult to apply without ambiguity, whilst a cutoff at the angle $\alpha_{ms}$, as defined by (5) appears to be a more practical choice. This procedure is less convenient than cutting off at $4\bar{\alpha}_{co}$, but $\alpha_{ms}$ is a more meaningful angle in the general scattering theory, and the resultant distribution truncated at $\alpha_{ms}$ is always close to gaussian. If $\bar{\alpha}_3$ is the resultant mean deflection after all events with scattering angles greater than $\alpha_{ms}$ have been deleted, then

$$\bar{\alpha}_3=(\bar{\alpha}-\pi/\alpha_{ms})/(1-\pi/2\alpha_{ms}^2). \qquad (16)$$

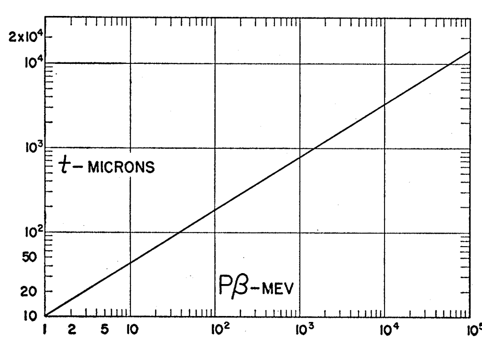


FIG. 4. Relationship between particle "energy" $p\beta$ and cell length $t$, which is calculated to give a mean deviation of 1.0 microns with Fowler's technique. ($\beta\simeq 1$.)

## D. The "Scattering Constant" $K$

Results of the preceding discussion on multiple scattering theory, geometrical features of emulsion methods, and cut-off procedures are translated here into numerical values for the "scattering constant" $K$ for use with the energy-scattering relationship (1).

(i) For chord angles without experimental cutoff, $K=K_0=10.06\times(2/3)^{\frac{1}{2}}\times\bar{\alpha}=8.21\bar{\alpha}$, where $\bar{\alpha}$ is given by (14). Substituting for $\bar{\alpha}$ we find

$$K_0=11.9\{1+0.837(\log_{10}0.94t/(\beta^2+0.30))^{\frac{1}{2}}\}\times\text{degrees}\times\text{Mev}/(100\text{ microns})^{\frac{1}{2}}. \qquad (17)$$

The variation of $K_0$ with cell length for fast particles ($\beta\simeq 1$) is shown in Fig. 3. Values of $K$ given in Fig. 3 may be somewhat in error due to the $(2/3)^{\frac{1}{2}}$ factor. Very recently Snyder[28] has given the results of more complicated estimates of $K$ (without cutoff) from the distribution for chord angles in the calculations of Snyder and Scott. These give results about 5 percent greater than the corresponding values in Fig. 3.

(ii) For chord angle measurements and a cutoff at $4\bar{\alpha}_{co}$, $K=K_{co}=8.21\bar{\alpha}_{co}$, where $\bar{\alpha}_{co}$ is given by (15). Numerical values of $K_{co}$, which is equivalent to Fowler's[5] constant $k_2$, are shown in Fig. 3 for fast particles. For a nonrelativistic particle, the appropriate value of $K_{co}$ is that given in Fig. 3 at cell length $t\times M'/0.723$ micron, where $M'$ is obtained from Fig. 2.

(iii) For chord angle measurements and a cutoff at $\alpha_{ms}$, which could be used as an alternative to $4\bar{\alpha}_{co}$, $K=K_3=8.21\bar{\alpha}_3$, where $\bar{\alpha}_3$ is given by (16). Numerical values for $K_3$ and the cutoff ratio $\alpha_{ms}/\bar{\alpha}_3$ are also shown in Fig. 3 for relativistic particles. As for $K_{co}$, the appropriate values of $K_3$ and $\alpha_{ms}/\bar{\alpha}_3$ for nonrelativistic particles are those given in Fig. 3 at cell lengths $t\times M'/0.723$ micron.

(iv) For very energetic particles such that cell lengths greater than $10^4$ microns are required to obtain measurable deviations, then $K_0$, $K_{co}$, and $K_3$ all approach a limiting value $K_b$ due to finite dimensions of the scattering nuclei. Using the value $\alpha_b=3.81$ units shown in Fig. 1, $K_b=8.21\times 3.81=31.2$ which is also shown in Fig. 3.

(v) For the "tangential" emulsion scattering method[2,3,6] a factor $0.96\pm 0.02$ has been given[2] to allow for the departure from exact tangents due to an experimental smoothing procedure adopted in deflection measurements. Hence for this technique the corresponding values of $K_0$, $K_{co}$, $K_3$, and $K_b$ are greater than those given in Fig. 3 by a factor $0.96/(2/3)^{\frac{1}{2}}=1.18\pm 0.02$.

(vi) The effect of inelastic electronic scattering, which has been neglected so far, may be included[17] to a close approximation by replacing $Z$ by $(Z^2+Z)^{\frac{1}{2}}$. This results in a correction of about 2 percent, which should be added to all values of $K$ in Fig. 3.

(vii) The wide scope of the emulsion scattering technique for fast particle energy determinations is illustrated in Fig. 4. This curve shows the cell length $t$ required in order to obtain a mean deviation $\bar{D}(t)=1.0$ microns using Fowler's technique, for particle energies ranging from 1 Mev to 100 Bev. In conjunction with Fig. 3, this curve is also useful for estimating the appropriate value of $K$ for any particle energy. The numerical data in Fig. 4 were calculated from (1) with values of $K=K_{co}$ from Fig. 3, and the geometrical relation $\bar{\alpha}(t)=(\bar{D}(t)/t)\times 180/\pi$ degrees.[29] The criterion $\bar{D}(t)=1.0$ microns has been found generally to give consistent energy determinations in the absence of severe emulsion distortion, as discussed by Fowler.[5]

## IV. COMPARISON BETWEEN THE RESULTS OF DIFFERENT THEORIES

The different scattering theories are based on the same fundamental principles although the theories of Molière and Snyder and Scott are more rigorous than that of Williams. However, as is indicated below, the results are essentially in agreement.

TABLE I. Comparison of results of different theories for scattering of singly charged particles in metal foils.

| Scatterer | $t$ (cm) | $\beta^2$ | Williams | $\bar{\alpha}$ ($\delta$-units) Williams with Molière $\gamma$-factor | Molière |
|---|---|---|---|---|---|
| Aluminum | 0.01 | 1 | 3.06 | 3.06 | 3.05 |
| Iron | 0.002 | 1 | 2.99 | 2.97 | 2.97 |
| Lead | 0.0007 | 1 | 2.89 | 2.70 | 2.71 |
| | 0.015 | 1 | 3.45 | 3.32 | 3.31 |
| | 2.0[a] | 0.867 | 4.14 | 4.03 | 4.05 |
| | 4.0[a] | 0.867 | 4.18 | 4.08 | 4.09 |

[a] The theoretical angles calculated here are actually greater than the limit imposed by the finite size of the nucleus, according to Eq. (8).

A graphical comparison of our present calculations (using Figs. 2 and 3) with those of Gottstein *et al.*,[16] based on the exact Molière theory (their Figs. 1 and 2) shows agreement between the values for $K_0$ within 1 percent over a wide range of $\beta$ and $t$. There is similar agreement with the calculations of Goldschmidt-Clermont[6] for photographic emulsions. It may be noted here, however, that the present values for $K_{co}$ differ somewhat from the corresponding values given by Gottstein *et al.* We also give, in Table I, the results of a few calculations for pure substances which show the extent of the agreement between the Williams and Molière theories.

The theory of Snyder and Scott,[21] which applies only to fast particles ($\beta\simeq 1$), cannot be compared so directly as the above two theories. It has been indicated by other authors[16] that the agreement is satisfactory. However, as a check, we have calculated the mean deflection for a number of cases, and compared the results with those given by Williams' theory with and without the Molière $\gamma$-factor, for $\beta=1$. The results are given in Table II. The values of Snyder and Scott were obtained graphically making use of the tables issued by these authors, and using their suggested method of summation for mixtures. Cell lengths were chosen so that these tables could be used conveniently. The angles compared are those between successive tangents.

[29] See Sec. VI.

TABLE II. Comparison of results of different theories for scattering of fast, singly charged particles in lead and photographic emulsions.

| $\bar{\alpha}\times p\beta$ degrees $\times$Mev ($\bar{\alpha}$ = projected angle between successive tangents) | | | | |
|---|---|---|---|---|
| Scatterer | $t$ (microns) | Snyder and Scott | Williams | Williams with Molière $\gamma$-factor |
| Emulsion | 32 | 17.0 | 16.7 | 16.3 |
| | 385 | 66.3 | 66.9 | 66.0 |
| | 4330 | 248 | 248 | 246 |
| Lead | 107 | 86.7 | 86.7 | 85.4 |

It will be seen that the results are in satisfactory agreement for photographic emulsions, especially for the longer cell lengths. Some values for lead are also included, and the discrepancy noted is mainly due to a combination of a fairly short cell length and the importance of the Molière $\gamma$-factor in this case.

## V. COMPARISON OF THEORY WITH EXPERIMENTS ON THICK AND THIN FOILS

Experiments on the scattering of fast electrons by thin metallic foils have been made by Lyman *et al.*[30] and by Kulchitsky and Latyshev,[31] and the results in terms of $1/e$ widths for the distribution are shown in Table III together with corresponding theoretical angles. The results are in agreement with theory within

TABLE III. Comparison of experimental and theoretical $1/e$ widths for distribution due to scattering of electrons by thin foils.

| Scatterer | $t$ g/cm² $\times 10^{-3}$ | cm | Energy | $\theta(1/e)$ degrees Williams with Molière $\gamma$-factor | Expl. |
|---|---|---|---|---|---|
| Lyman *et al*[a] | | | | | |
| Beryllium | 257 | 0.14 | 15.7 Mev | 3.50 | 3.06 |
| Beryllium | 491 | 0.27 | | 4.73 | 4.25 |
| Gold | 18.7 | 0.001 | | 2.71 | 2.58 |
| Gold | 37.3 | 0.002 | | 4.10 | 3.76 |
| Kulchitsky and Latyshev[b] | | | | | |
| Aluminum | 26.6 | 0.01 | 2.25 Mev | 9.65 | 9.15 |
| Iron | 15.4 | 0.002 | | 9.55 | 9.35 |
| Lead | 7.9 | 0.0007 | | 9.2 | 9.35 |

[a] See reference 30.
[b] See reference 31.

[30] Lyman, Hanson, Lanzl, and Scott, Phys. Rev. **81**, 309 (1951).
[31] L. A. Kulchitsky and D. G. Latyshev, Phys. Rev. **61**, 254 (1942).

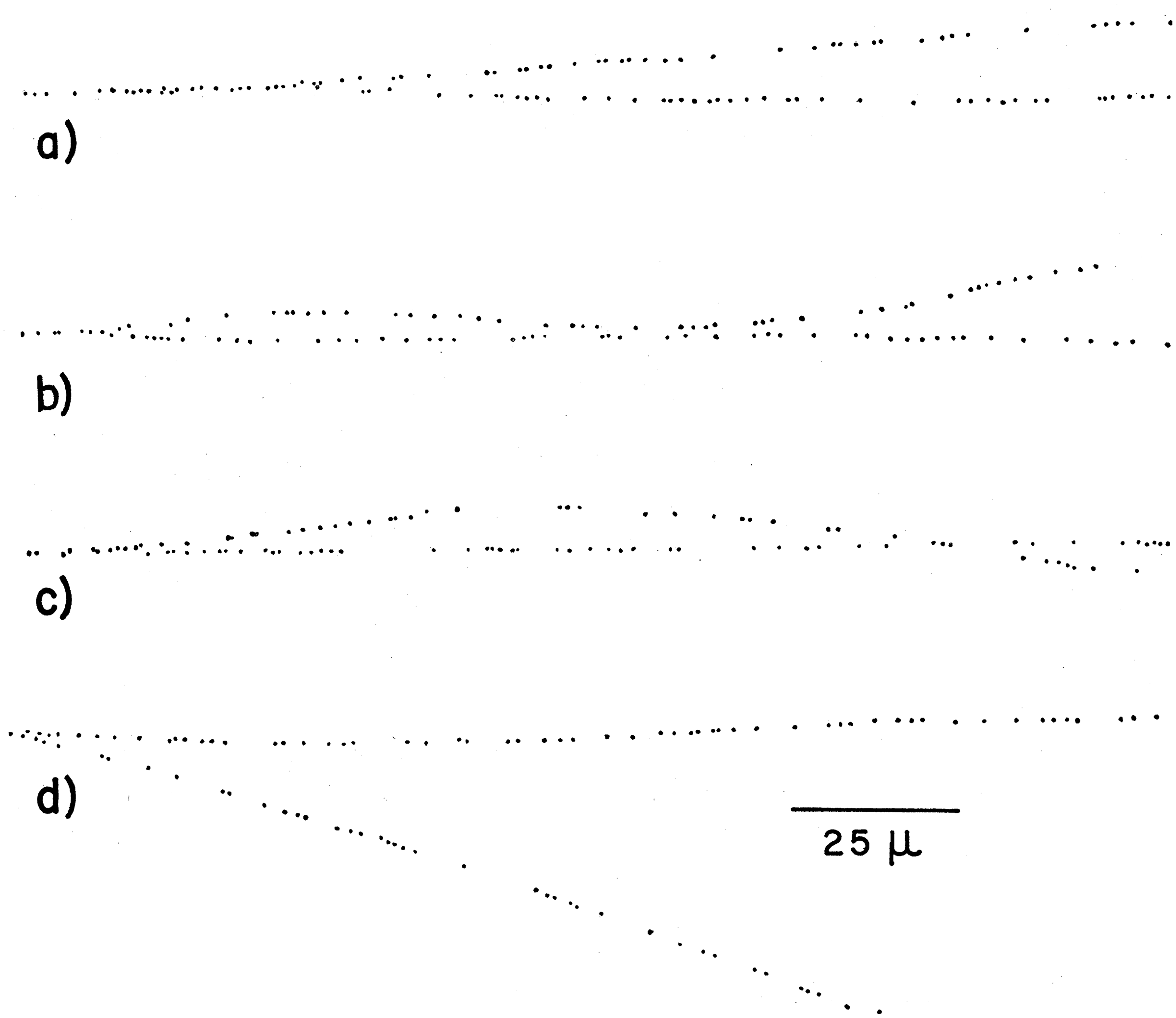


FIG. 5. Projection microscope tracings, showing four typical electron-positron pairs in Ilford G5 emulsion which has been exposed to $Be^8$ gamma-radiation.

5–10 percent, although in the case of Lyman *et al.* the measured widths are all smaller. The discrepancy between theory and experiment for lead reported by Kulchitsky and Latyshev is largely removed by the inclusion of the Molière $\gamma$-factor in the theory, but it must be noted that, for this particular case, $M(\simeq 12)$ and $t$ were small. Earlier experiments[32–34] using a spectrum of electrons with energies in the 5–10 Mev region were reported to give values smaller than theory for lead foils.

Recently Crewe[35] has reported an experiment on the scattering of cosmic-ray $\mu$-mesons (with energies $\sim$184 Mev) by one inch of lead in which the measured mean deflection $\bar{\theta}$ was $3.7\pm 0.4$ degrees, to be compared with a theoretical $\theta_b = 5.7$ degrees. In this case the scattering angle is limited by the finite size of the nucleus. Sinha[36] has previously reported a low result for lead. On the other hand experiments made, using cosmic-ray $\mu$-mesons in the energy region 0.5–1 Bev, by Code[37] and Wilson[38] give reasonable agreement with theory within a few percent. Code used a tungsten sheet (3.8 cm) and Wilson used copper (2.0 cm), gold (2.0 cm), and lead (0.3 cm, 1.0 cm), and the results were compared with theoretical ones derived from Williams' equation for the nuclear limiting angle [our Eq. (8)].

Generally, it would appear that the results are in reasonable agreement with theory except perhaps for lead, but further experimental work would seem necessary.

[32] W. A. Fowler and J. R. Oppenheimer, Phys. Rev. **54**, 22 (1938).
[33] Oleson, Chao, Halpern, and Crane, Phys. Rev. **56**, 482 (1939).
[34] C. W. Sheppard and W. A. Fowler, Phys. Rev. **57**, 273 (1939).
[35] A. V. Crewe, Proc. Phys. Soc. (London) **A64**, 660 (1951).
[36] M. S. Sinha, Phys. Rev. **68**, 153 (1945).
[37] F. L. Code, Phys. Rev. **59**, 229 (1941).
[38] J. G. Wilson, Proc. Roy. Soc. (London) **A174**, 73 (1940).

## VI. EXPERIMENTAL TEST OF SCATTERING THEORY USING $Be^8$ GAMMA-RAYS

### A. Method

Ilford G5 plates 400 microns in thickness were irradiated with about 10 milliroentgens of $Be^8$ gamma-radiation from an evaporated lithium target bombarded with a resolved proton beam at about 450-kev proton energy. The plates were aligned approximately parallel to the gamma-ray direction at about 15 cm from the lithium target. Using the normal temperature development method[39] with Elon developer, the plates recorded relativistic particle tracks with a mean grain density of 33 grains per 100 microns, and a mean grain diameter of 0.6 micron. Examples of four typical electron pairs are shown in the projection microscope tracings reproduced in Fig. 5.

Multiple scattering observations were carried out by the method described by Fowler,[5] using a standard Leitz Ortholux binocular microscope. A 90× oil immersion objective was used in conjunction with 6× eyepieces for searching, and 15× compensated eyepieces with calibrated scales for scattering measurements. When a plate was mounted on the mechanical stage of the microscope, the tracks of pair electrons were roughly parallel to the $x$-axis. It was found from preliminary observations that those pairs which could be followed for about 700 microns along the $x$-axis under these conditions gave fairly representative events.

In the pairs accepted for gamma-ray energy determinations, the $y$ coordinates of each track were recorded at intervals of 20 scale divisions along the $x$-axis, where 1 scale division = 0.88 micron for both the $x$ and $y$ coordinate eyepiece scales. The second differences of the $y$ coordinates, $D_n=(y_n-y_{n+1})-(y_{n+1}-y_{n+2})$, measure the deviation between successive chords along a projected track. The mean projected deflection between successive chords at intervals, or cell lengths, $t$ is then $\bar{\alpha}(t)=(\bar{D}/t)\times 180/\pi$ degrees, where $\bar{D}$ is the arithmetic mean of the individual deviations $D_n$, taken without regard to sign.

The values of $p\beta$ for each track were determined from the energy scattering relation given by Eq. (1), i.e., $p\beta=(K/\bar{\alpha}(t))\times(t/100)^{\frac{1}{2}}$ Mev. Corresponding to experimental cutoffs at $\alpha_{ms}$ and $4\bar{\alpha}_{co}$ for the observed deflection on each track, theoretical values of the "scattering constant" $K$ were taken from the curves for $K_3$ and $K_{co}$ shown in Fig. 3 to obtain the particle energy for each of these two types of cutoff. After correcting for the effect of inelastic electronic scattering, spurious experimental errors in scattering measurements, and energy losses along the tracks, $p\beta_1+p\beta_2$ was determined for each pair. Since $p\beta=E+mc^2$ for these relativistic electrons, $p\beta_1+p\beta_2$ is also equal to the energy of the gamma-ray responsible for the pair.

[39] Dilworth, Occhialini, and Payne, Nature **162**, 102 (1948).

Table IV. Energy determinations on electron pair 15.

| Track | Cell length $t$ divs. | No cutoff $\bar{D}$ divs. | Cutoff $\bar{D}_{co}$ divs. | $4\bar{\alpha}_{co}$ $p\beta$ Mev | Cutoff $\bar{D}_3$ divs. | $\alpha_{ms}$ $p\beta$ Mev |
|---|---|---|---|---|---|---|
| | 20 | 0.46 | 0.42 | 7.0 | 0.42 | 6.6 |
| 1 | 40 | 0.83 | 0.74 | 12.1 | 0.74 | 11.6 |
| | 60 | 1.47 | 1.47 | 11.6 | 1.47 | 11.3 |
| | 80 | 2.05 | 2.05 | 13.1 | 2.05 | 12.9 |
| | 20 | 0.89 | 0.77 | 3.8 | 0.72 | 3.9 |
| 2 | 40 | 1.97 | 1.78 | 5.0 | 1.64 | 5.3 |
| | 60 | 3.54 | 3.54 | 4.8 | 3.00 | 5.5 |
| | 80 | 5.50 | 5.50 | 4.9 | 5.50 | 4.8 |

### B. Examples of Energy Determinations

To illustrate the method of analysis, data for two typical pairs are shown in Tables IV and V, which represent examples of pair production with asymmetric and approximately symmetric energy division.

For each track in the two pairs, the tables show "energy" values $p\beta$ (uncorrected for noise level or ionization losses) for the two types of cutoff at $\alpha_{ms}$ and $4\bar{\alpha}_{co}$ and for cell lengths varying from 20 to 80 divisions. For this range of cell lengths, $K_{co}$ varies from 20.1 to 23.0 and $K_3$ from 19.0 to 22.5, as obtained from Fig. 3.

### C. Noise Level and Energy Losses

At short cell lengths very large corrections are required to obtain the true energy of a particle because of spurious experimental scattering, or noise level,[10,16] whereas with long cell lengths this correction is small, but the statistical accuracy is usually poor. Thus, because of the importance of noise level corrections, a somewhat complicated choice of cell lengths was adopted for the final energy determinations. The value accepted for the energy $p\beta$ for each track was the mean of two determinations: the first being derived from the minimum cell length $t$ for which $\bar{D}\geqslant 1.0$ divisions, and the second determination from the next longer cell length, i.e., at $t+20$ divisions. A correction of 3 percent was added to each electron energy, based on an estimated mean noise level between 0.15 and 0.2 division

Table V. Energy determinations on electron pair 58.

| Track | Cell length $t$ divs. | No cutoff $\bar{D}$ divs. | Cutoff $\bar{D}_{co}$ divs. | $4\bar{\alpha}_{co}$ $p\beta$ Mev | Cutoff $\bar{D}_3$ divs. | $\alpha_{ms}$ $p\beta$ Mev |
|---|---|---|---|---|---|---|
| | 20 | 0.53 | 0.53 | 5.5 | 0.47 | 5.9 |
| 1 | 40 | 1.08 | 0.99 | 9.0 | 0.92 | 9.4 |
| | 60 | 1.89 | 1.89 | 9.0 | 1.58 | 10.4 |
| | 80 | 2.82 | 2.82 | 9.6 | 2.62 | 10.1 |
| | 20 | 0.44 | 0.44 | 6.7 | 0.38 | 7.3 |
| 2 | 40 | 1.10 | 1.10 | 8.1 | 0.96 | 9.0 |
| | 60 | 1.90 | 1.90 | 9.0 | 1.90 | 8.7 |
| | 80 | 2.90 | 2.90 | 9.3 | 2.90 | 9.1 |

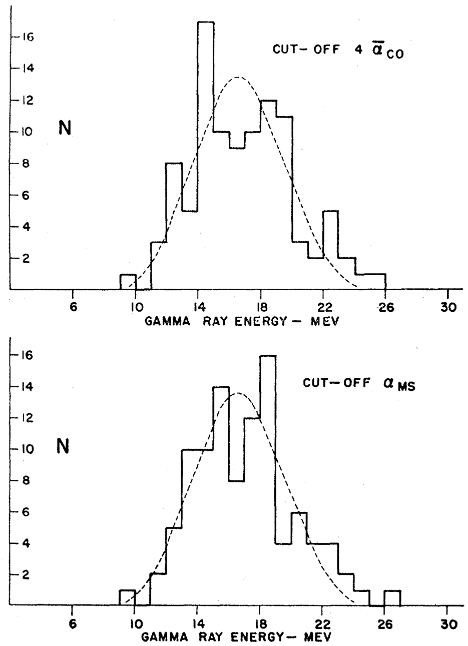


FIG. 6. Gamma-ray energies from 100 electron-positron pairs. The ordinates, $N$, show the number of events per 1-Mev energy interval. The dashed curve in both cases is the expected distribution for 10 percent statistical probable error in measurement.

and an average deviation $\bar{D}$ about 2.0 divisions using two cell lengths. A further correction of 0.2 Mev was added to each electron pair for energy losses along the track length ($\simeq$1.4 millimeters of emulsion). This may be rather low, although it should be noted that any large energy losses giving noticeable deflections are excluded from the observations.

Applying the procedure described above to the data listed in Table IV, the gamma-ray energy responsible for this electron pair was obtained as 17.6 Mev with an experimental cutoff at $4\bar{\alpha}_{co}$, and 18.2 Mev with a cutoff at $\alpha_{ms}$. Similarly, for the pair in Table V, the gamma-ray energy was obtained as 18.3 Mev and 19.4 Mev for the two types of cutoff.

### D. Statistics

The statistical probable error in each mean deviation $\bar{D}$ is expected to be nearly $50/n^{\frac{1}{2}}$ percent,[2,6] where $n$ is the number of independent deviations. Thus for the pair described in Table V there are 24 independent observations on each track, based on overlapping cells of 40 divisions and a track length of 800 divisions for each electron. This yields a statistical probable error very close to 10 percent of the resultant gamma-ray energy. By taking longer track lengths on the high energy component in cases of extremely asymmetric energy division, an attempt was made to achieve uniform statistics which would yield a 10 percent probable error in total energy for every electron pair.

## VII. RESULTS AND CONCLUSIONS OF $Be^8$ GAMMA-RAY EXPERIMENT

### A. $Be^8$ Gamma-Ray Energies

The results of gamma-ray energy determination for 100 electron-positron pairs are shown in Fig. 6 for the two types of cutoff. The expected distribution for 10 percent probable error (dashed curve) was based on the magnetic pair spectrometer results of Walker and McDaniel,[12] who gave the $Be^8$ gamma-ray energies at the 440 kev $(p,\gamma)$Li resonance as 14.8±0.3 Mev and 17.6±0.2 Mev, with a ratio of 0.5 for the intensity of the lower energy line to the higher energy line, and a natural width of about 2.1 Mev for the 14.8-Mev gamma-ray.

The arithmetic mean quantum energy for the 100 pairs was found, after adding the 2 percent correction for inelastic electronic collisions, to be 17.4±0.5 Mev for both types of cutoff. The probable error of 0.5 Mev is that estimated for statistical uncertainties in scattering data (about 1 percent for the mean of 100 events), and an allowance of 2 percent probable error for corrections due to noise level and energy losses. This experimental mean energy is in fairly good agreement with an expected mean $Be^8$ gamma-ray energy of 16.7±0.3 Mev, calculated from the results of Walker and McDaniel. We therefore conclude that the present experiment indicates the reliability (within about 5 percent) of the theoretical values for the "scattering constant" $K$ for the cell length region 20 to 70 microns for fast particles, i.e., in the region $K_{co}=21.5$, $K_3=21$.

TABLE VI. Energy division in electron pairs.

| $p\beta_1/(p\beta_1+p\beta_2)$ | 0–0.2 | 0.2–0.4 | 0.4–0.6 | 0.6–0.8 | 0.8–1.0 |
|---|---|---|---|---|---|
| No. of events | 7 | 26 | 28 | 29 | 10 |

### B. Resolution of Scattering Technique

The general agreement between the expected distribution and the shapes of the experimental histograms in Fig. 6 indicates that the experimental resolution of energy determinations from our multiple scattering data is very near 10 percent. There is a slight indication of a double humped distribution at about 15 and 18 Mev in both experimental histograms, but these apparent groupings are not considered to be statistically significant. The experimental data shown in Fig. 6 are based on 64 electron pairs due to gamma-radiation emitted in the forward proton direction, and 36 pairs due to radiation emitted at nearly 90° to the direction of the bombarding proton beam. Since the recent magnetic pair spectrometer results of Stearns and McDaniel[40] have indicated that $Be^8$ gamma-rays are truly isotropic near the 440-kev proton resonance, it was decided not to pursue the possibility of gamma-ray

[40] M. B. Stearns and B. D. McDaniel, Phys. Rev. 82, 299 (1951).

anisotropy which was postulated in our preliminary report[13] on this experiment.

To show that the 100 pairs analyzed are fairly representative pair production events, energy division results are given in Table VI. Here $p\beta_1/(p\beta_1+p\beta_2)$ is the ratio of the energy for the upper track (see Fig. 1) to the total energy for the pair, and the $p\beta$ values are those obtained with a cutoff at $\alpha_{ms}$. The distribution is seen to fall off for extremely asymmetric energy division, and is roughly constant in the intermediate region. These results are in reasonable agreement with the experimental and theoretical energy division results for 17-Mev gamma-rays given by Delsasso, Fowler, and Lauritsen.[41]

The approximate angular separation (as projected on the $x$-$y$ plane of the microscope field of view) was also measured for each pair with a simple eyepiece goniometer, with results similar to those given by King[42] for 25-Mev synchrotron x-rays. The distribution of angular separation for the 100 pairs was nearly constant between 0 and 8 degrees, with a rapid decrease at larger angles, resulting in a mean angular separation of $5.0\pm0.3$ degrees. In about 15 cases, both tracks appeared to emerge on the same side with respect to the direction of the original gamma-ray beam, but a more accurate irradiation geometry and goniometer are necessary for reliable data on this effect, which is ascribed to the momentum taken up by the local nucleus in pair production.

### C. Distribution of Scattering Deflections

Since the present experiment is based on electron tracks with a wide energy spread, and a different scattering distribution is expected for each electron energy and each cell length, only an approximate test of the distribution in scattering deflections is possible. However, by concentrating on those tracks for which deflections were measured at overlapping cell lengths of 40 scale divisions (35.2 microns), we are able to obtain reasonable statistics for a comparison with theoretical scattering distributions. This cell length for fast particles ($\beta\simeq1$) corresponds to a value of $M\sim20-25$.

The approximate distribution function $P(\alpha)=G(\alpha)+S(\alpha)$, which was discussed in Sec. II is shown in Fig. 7 for the case $M=21$. The functions $P(\alpha)$, $G(\alpha)$, and $S(\alpha)$ are plotted here against the projected deflection $\alpha$ expressed in $\delta$-units (bottom scale). An experimental distribution in scattering deflections was obtained from a total of 814 deflections using 24 tracks with electron energies between 8.0 and 9.0 as determined with a cutoff at $\alpha_{ms}$. These results are also shown in Fig. 7, plotted as the number of deflections per 0.5 scale division interval *versus* the projected deflection, $|D|$ (scale at top). The theoretical curves are normalized to the same total number of deflections, and the abscissas also correspond if the experimental distribution is considered to be equivalent to that for monoenergetic electrons of energy 8.5 Mev.

A comparison of the theoretical and experimental distributions in Fig. 7 indicates that at very small angles the experimental values are too low, which is thought to be due in large part to spurious errors in measurement, or noise level. At larger deflections the two results are felt to be in reasonable agreement, except for an apparent experimental excess over $P(\alpha)$ in the single scattering region beyond $\alpha_{ms}$. A qualitative examination of Williams' more exact distribution function,[18] as well as some theoretical curves shown by Goldschmidt-Clermont[6] which were based on the theories of Molière and of Snyder and Scott, all indicate a similar excess over the approximate function $P(\alpha)$ in the region of single and plural scattering.

A more direct quantitative test of scattering distributions in the single scattering region may be made by considering the frequency of deflections greater than the cutoffs $\alpha_{ms}$ and $4\bar{\alpha}_{co}$. Since the frequencies, or relative numbers of deflections, depend only on the value of $M$ and not on particle energy, we have analyzed a total of 5000 deflections obtained from overlapping cell lengths of 35.2 microns on 150 tracks whose energies varied from 3 Mev to 15 Mev. In this way, the experimental frequency of deflections greater than $\alpha_{ms}$ was found to be 3.2 percent, and the frequency of deflections greater than $4\bar{\alpha}_{co}$ was 1.1 percent. From the approximate function $S(\alpha)=\pi/\alpha^3$, shown in Fig. 7, the corresponding theoretical frequencies for cutoffs at $\alpha_{ms}$ and $4\bar{\alpha}_{co}$ were calculated to be 2.5 and 1.5 percent for the case $M=21$. Similarly, using the more exact[18] single scattering contribution $S(\alpha)=(\pi/\alpha^3)(1+3\pi\alpha_m^2/\alpha^2)$, the expected frequencies were calculated to be 3.5 and 1.8

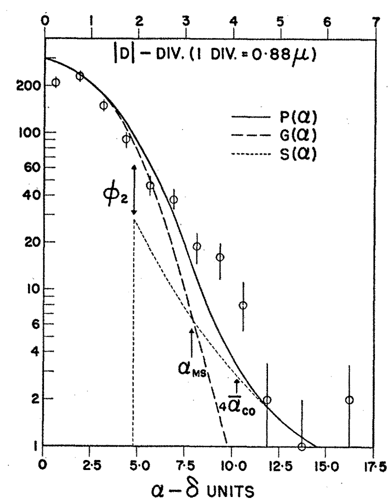


FIG. 7. Distribution of scattering deflections. The theoretical curves show Williams' approximate functions $G(\alpha)$, $S(\alpha)$, and $P(\alpha)=G(\alpha)+S(\alpha)$ for the case $M=21$, plotted against the projected deflection $\alpha$ (bottom scale). The experimental points, with their indicated standard deviations, are plotted against the absolute deflection, $|D|$ (top scale).

[41] Delsasso, Fowler, and Lauritsen, Phys. Rev. **51**, 391 (1937).
[42] D. T. King, Nature **165**, 526 (1950).

percent for the two types of cutoff. Within the uncertainties due to statistics and experimental errors, these results are also considered to indicate approximate agreement between theory and experiment, but experiments with known monoenergetic particles are desirable for more accurate tests of scattering distributions with emulsion techniques.

## VIII. GENERAL CONCLUSIONS AND COMPARISON BETWEEN THEORY AND EXPERIMENTS

(1) There appears to be general agreement within about 1 percent for the mean scattering angle (without cutoff) given by the theories of Snyder and Scott,[43] Williams and Molière for the cases where these theories are applicable. The agreement between Williams and Molière is very close when the Molière $\gamma$-factor is introduced into Williams' theory.

TABLE VII. Comparison of theoretical and experimental results for $K_{co}$ for the scattering of singly charged particles in Ilford G5 emulsions.

| Experiment | $\beta^2$ | Cell length microns | $K_{co}$ (exp) | $K_{co}$ (theor.) |
|---|---|---|---|---|
| Gottstein *et al.*[a] | | | | |
| Positrons 105 Mev | 1 | 200 | 26.2±0.6 | 25.3 |
| Positrons 185 Mev | 1 | 400 | 24.0±0.8 | 26.4 |
| Protons 336 Mev | 0.46 | 600 | 29.2±1.0 | 27.7 |
| Protons and mesons 5–50 Mev | 0.14 | 80 | 26.1±0.7 | 25.6 |
| Protons 10–20 Mev | 0.02 | 72 | 27.5±0.5 | 25.9 |
| Corson[b] | | | | |
| Electrons 40–280 Mev | 1 | not given | 26 ±1 | |
| Berger, Lord, and Schein[c] | | | | |
| Protons 337 Mev | 0.46 | 250 | 24.4±0.8 | 26.5 |
| | | 500 | 24.5±0.8 | 27.4 |
| | | 750 | 24.6±0.9 | 28.0 |
| Voyvodic and Pickup[d] | | | | |
| Electron pairs (Be[8] gamma-rays Mean energy 16.7 Mev) | 1 | 45 (20–70) | 21.2±0.7 | 22.1 |

[a] See reference 16.
[b] See reference 14.
[c] See reference 15.
[d] See reference 13.

(2) The theoretical results of Williams and Molière can be closely approximated to by a relatively simple equation [(14) or (15)] for Ilford G5 emulsions and singly charged particles, which gives the results to within about 1 percent over a wide range of cell lengths and for different values of $\beta$. It may be noted that this accuracy is also within the tolerance with which the emulsion content is given by the maker.

(3) Comparison of theory with the results of metallic foil experiments is not very decisive, and further work would seem necessary. There may be a fairly large discrepancy for the case of lead.

(4) In a $Be^8$ gamma-ray calibration experiment with G5 emulsions, the mean energy for the two types of experimental cutoff discussed in this paper was found to be 17.4±0.5 Mev in both cases, which is in fairly good agreement with an expected mean energy of 16.7±0.3 Mev. Theoretical "scattering constants" and cell lengths from 20 to 70 microns (average≃45 microns) were used to obtain this experimental result. Expressing the result in terms of the "scattering constant," $K_{co}$ (45 microns)=22.1 from Fig. 3, whereas $K_{co}$ (experimental), using the mean energy value 16.7 Mev, $=(16.7/17.4)\times 22.1=21.2$.

(5) Since the present work was completed, several other calibration experiments with singly charged particles of different energies and using G5 emulsions have been published. The results are collected together in Table VII. They are expressed in terms of the "scattering constant" using an experimental cutoff of four times the mean and, for comparison, the corresponding values of $K_{co}$ from the present theory are included. These have all been increased by 2 percent to allow for the effect of inelastic electronic scattering, except for the $Be^8$ result, where the correction was made in deducing the mean gamma-ray energy.

The results shown in Table VII are in satisfactory agreement with theory within 5–10 percent. The results of the $Be^8$ gamma-experiment seem to be in fairly good agreement with theory for the smaller values of $K$, although it would be of interest to check this by measurements using monochromatic electrons with energies of 10–20 Mev. The measurements by Gottstein *et al.* and by Berger, Lord, and Schein on protons do not agree very well. In general it would appear that further experimental work is necessary in order to verify the theory in detail.

(6) Gottstein *et al.* gave measurements of $K$ (without cutoff) for 105-Mev positrons, 185-Mev positrons, and 336-Mev protons, which may be compared with Snyder's recent calculations.[28] The experimental values are 26.7±0.6, 24.9±0.8, and 30.7±1.0 respectively to be compared with values, from Snyder's curve of 27.8, 28.7, and 30.2 respectively. These latter values should probably be increased by about 2 percent to correct for inelastic electronic scattering.

We are much indebted to Dr. B. G. Whitmore (University of Manitoba) for help in connection with Molière's theory and other calculations, and to B. Bigham (Queen's University) for numerical computations.

[43] The agreement here is between the angles essentially as measured between successive tangents. As we have noted, Snyder (reference 28) has recently given values for $K$ (without cutoff), which are about 5 percent greater than the corresponding values in our Fig. 3, calculated using the $(2/3)^{\frac{1}{2}}$ relationship between chord and tangent angles.